Robust Control Toolbox™ 3
User's Guide

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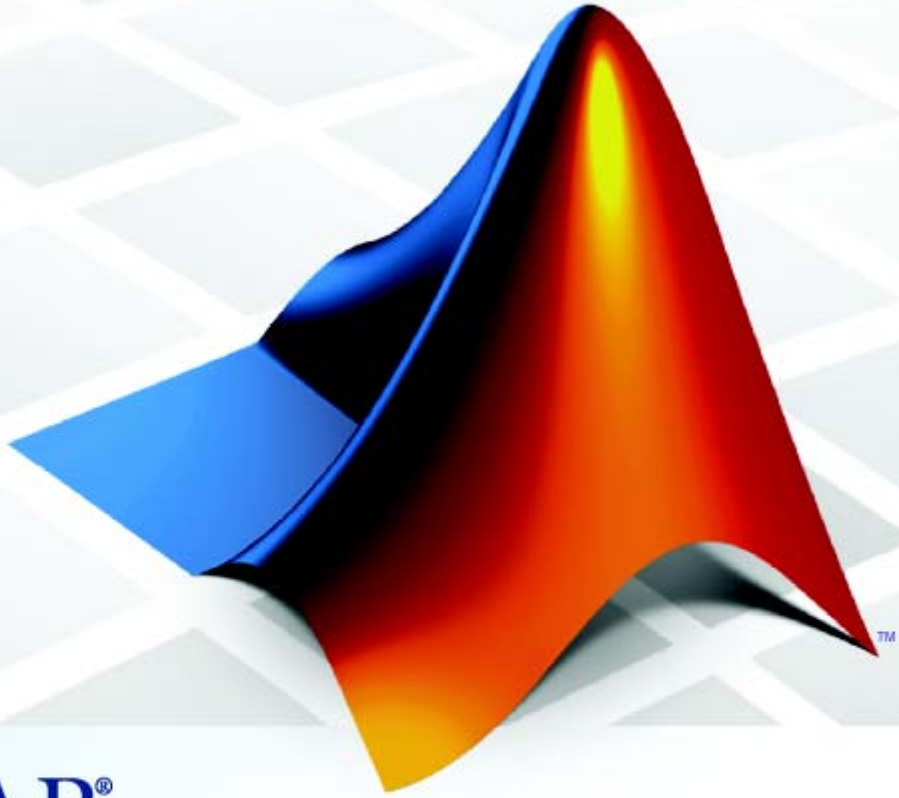

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Robust Control Toolbox™ User's Guide

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1

Building Uncertain Models

Introduction to Uncertain Atoms

Uncertain atoms are the building blocks used to form uncertain matrix objects and uncertain system objects. There are 5 classes of uncertain atoms:

Function	Description
`ureal`	Uncertain real parameter
`ultidyn`	Uncertain, linear, time-invariant dynamics
`ucomplex`	Uncertain complex parameter
`ucomplexm`	Uncertain complex matrix
`udyn`	Uncertain dynamic system

All of the atoms have properties, which are accessed through `get` and `set` methods. This `get` and `set` interface mimics the Control System Toolbox™ and MATLAB Handle Graphics® behavior. For instance, `get(a,'PropertyName')` is the same as `a.PropertyName`, and `set(b,'PropertyName',Value)` is the same as `b.PropertyName = value`. Functionality also includes tab-completion and case-insensitive, partial name property matching.

For `ureal`, `ucomplex` and `ucomplexm` atoms, the syntax is

```
p1 = ureal(name, NominalValue, Prop1, val1, Prop2, val2,...);
p2 = ucomplex(name, NominalValue, Prop1, val1, Prop2, val2,...);
p3 = ucomplexm(name, NominalValue, Prop1, val1, Prop2,
val2,...);
```

For `ultidyn` and `udyn`, the `NominalValue` is fixed, so the syntax is

```
p4 = ultidyn(name, ioSize,  Prop1, val1, Prop2, val2,...);
p5 = udyn(name, ioSize,  Prop1, val1, Prop2, val2,...);
```

For `ureal`, `ultidyn`, `ucomplex` and `ucomplexm` atoms, the command `usample` will generate a random instance (i.e., not uncertain) of the atom, within its modeled range. For example,

```
usample(p1)
```

creates a random instance of the uncertain real parameter `p1`. With an integer argument, whole arrays of instances can be created. For instance

```
usample(p4,100)
```

generates an array of 100 instances of the `ultidyn` object `p4`. See "Sampling Uncertain Objects" on page 1-42 to learn more about `usample`.

Uncertain Real Parameters

An uncertain real parameter is used to represent a real number whose value is uncertain. Uncertain real parameters have a name (the `Name` property), and a nominal value (the `NominalValue` property). Several other properties (`PlusMinus`, `Range`, `Percentage`) describe the uncertainty in the parameter's value.

All properties of a `ureal` can be accessed through `get` and `set`. The properties are:

Properties	Meaning	Class
`Name`	Internal Name	`char`
`NominalValue`	Nominal value of atom	`double`
`Mode`	Signifies which description (from `'PlusMinus','Range','Percentage'`) of uncertainty is invariant when `NominalValue` is changed	`char`
`PlusMinus`	Additive variation	`scalar or 1x2 double`
`Range`	Numerical range	`1x2 double`
`Percentage`	Additive variation (% of absolute value of nominal)	`scalar or 1x2 double`
`AutoSimplify`	`'off' | {'basic'} |'full'`	`char`

The properties Range, Percentage and PlusMinus are all automatically synchronized. If the nominal value is 0, then the Mode cannot be Percentage. The Mode property controls what aspect of the uncertainty remains unchanged when NominalValue is changed. Assigning to any of Range/Percentage/PlusMinus changes the value, *but does not* change the mode.

The AutoSimplify property controls how expressions involving the real parameter are simplified. Its default value is 'basic', which means elementary methods of simplification are applied as operations are completed. Other values for AutoSimplify are 'off' (no simplification performed) and 'full' (model-reduction-like techniques are applied). See "Simplifying Representation of Uncertain Objects" on page 1-38 to learn more about the AutoSimplify property and the command simplify.

If no property/value pairs are specified, default values are used. The default Mode is PlusMinus, and the default value of PlusMinus is [-1 1]. Some examples are shown below. In many cases, the full property name is not specified, taking advantage of the case-insensitive, partial name property matching.

Create an uncertain real parameter, nominal value 3, with default values for all unspecified properties (including plus/minus variability of 1). View the properties and their values, and note that the Range and Percentage descriptions of variability are automatically maintained.

```
a = ureal('a',3)

Uncertain Real Parameter: Name a, NominalValue 3, variability =
[-1  1]
get(a)
             Name: 'a'
     NominalValue: 3
             Mode: 'PlusMinus'
            Range: [2 4]
        PlusMinus: [-1 1]
       Percentage: [-33.3333 33.3333]
     AutoSimplify: 'basic'
```

Create an uncertain real parameter, nominal value 2, with 20% variability. Again, view the properties, and note that the Range and PlusMinus descriptions of variability are automatically maintained.

```
b = ureal('b',2,'percentage',20)
Uncertain Real Parameter: Name b, NominalValue 2, variability =
[-20  20]%
get(b)
            Name: 'b'
    NominalValue: 2
            Mode: 'Percentage'
           Range: [1.6000 2.4000]
       PlusMinus: [-0.4000 0.4000]
      Percentage: [-20.0000 20.0000]
    AutoSimplify: 'basic'
```

Change the range of the parameter. All descriptions of variability are automatically updated, while the nominal value remains fixed. Although the change in variability was accomplished by specifying the `Range`, the `Mode` is unaffected, and remains `Percentage`.

```
b.Range = [1.9 2.3];
get(b)
            Name: 'b'
    NominalValue: 2
            Mode: 'Percentage'
           Range: [1.9000 2.3000]
       PlusMinus: [-0.1000 0.3000]
      Percentage: [-5.0000 15.0000]
    AutoSimplify: 'basic'
```

As mentioned, the `Mode` property signifies what aspect of the uncertainty remains unchanged when `NominalValue` is modified. Hence, if a real parameter is in `Percentage` mode, then the `Range` and `PlusMinus` properties are determined from the `Percentage` property and `NominalValue`. Changing `NominalValue` preserves the `Percentage` property, and automatically updates the `Range` and `PlusMinus` properties.

```
b.NominalValue = 2.2;
get(b)
            Name: 'b'
    NominalValue: 2.2000
            Mode: 'Percentage'
           Range: [2.0900 2.5300]
       PlusMinus: [-0.1100 0.3300]
```

```
      Percentage: [-5.0000 15.0000]
    AutoSimplify: 'basic'
```

Create an uncertain parameter with an unsymmetric variation about its nominal value.

```
c = ureal('c',-5,'per',[-20 30]);
get(c)
            Name: 'c'
    NominalValue: -5
            Mode: 'Percentage'
           Range: [-6 -3.5000]
       PlusMinus: [-1 1.5000]
      Percentage: [-20 30]
    AutoSimplify: 'basic'
```

Create an uncertain parameter, specifying variability with `Percentage`, but force the `Mode` to be `Range`.

```
d = ureal('d',-1,'mode','range','perc',[-40 60]);
get(d)
            Name: 'd'
    NominalValue: -1
            Mode: 'Range'
           Range: [-1.4000 -0.4000]
       PlusMinus: [-0.4000 0.6000]
      Percentage: [-40.0000 60]
    AutoSimplify: 'basic'
```

Finally, create an uncertain real parameter, and set the `AutoSimplify` property to `'full'`.

```
e = ureal('e',10,'plusminus',[-23],'mode','perce',...
'autosimplify','full')
Uncertain Real Parameter: Name e, NominalValue 10, variability
= [-20  30]%
get(e)
            Name: 'e'
    NominalValue: 10
            Mode: 'Percentage'
           Range: [8 13]
       PlusMinus: [-2 3]
```

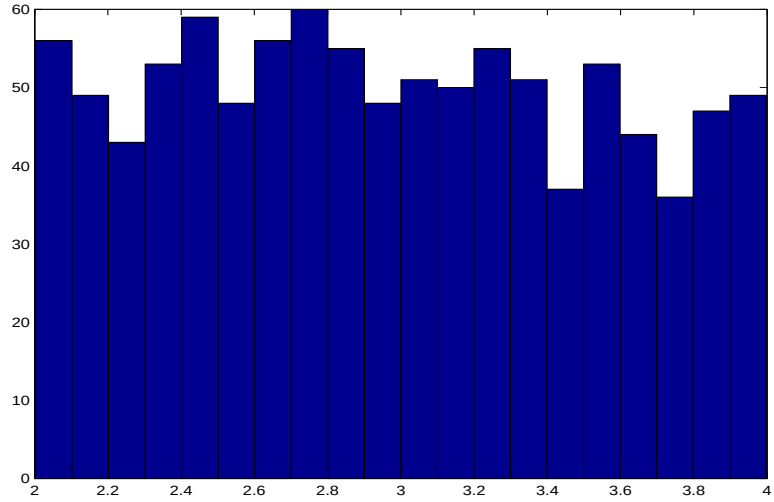
```
      Percentage: [-20 30]
    AutoSimplify: 'full'
```

Specifying conflicting values for `Range/Percentage/PlusMinus` in a multiple property/value `set` is not an error. In this case, the last (in list) specified property is used. This last occurrence also determines the `Mode`, unless `Mode` is explicitly specified, in which case that is used, regardless of the property/value pairs ordering.

```
f = ureal('f',3,'plusminus',[-2 1],'perce',40)
Uncertain Real Parameter: Name f, NominalValue 3, variability =
[-40  40]%
g = ureal('g',2,'plusminus',[-2 1],'mode','range','perce',40)
Uncertain Real Parameter: Name g, NominalValue 2, Range [1.2
2.8]
g.Mode
ans =
Range
```

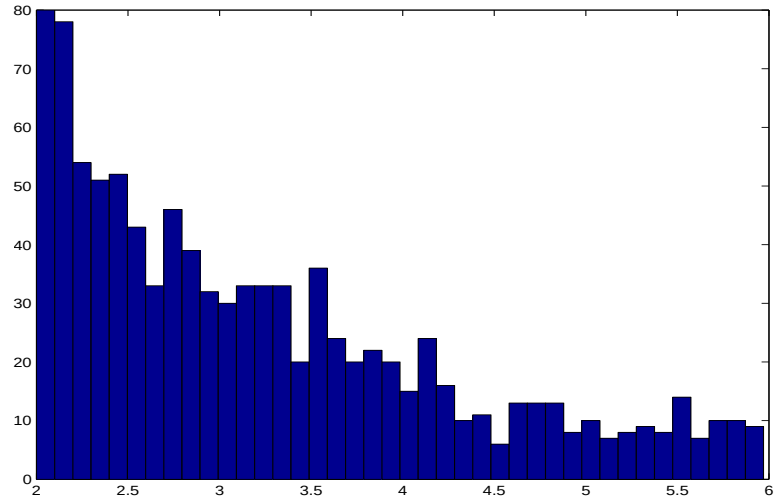
Create an uncertain real parameter, use `usample` to generate 1000 instances (resulting in a 1-by-1-by-1000 array), reshape the array, and plot a histogram, with 20 bins (within the range of 2-to-4).

```
h = ureal('h',3);
hsample = usample(h,1000);
hist(reshape(hsample,[1000 1]),20);
```



Make the range unsymmetric about the nominal value, and repeat the sampling, and histogram plot (with 40 bins over the range of 2-to-6)

```
h.Range = [2 6];
hsample = usample(h,1000);
hist(reshape(hsample,[1000 1]),40);
```



Note that the distribution is skewed. However, the number of samples less than the nominal value and the number of samples greater than the nominal value is equal (on average). Verify this.

```
length(find(hsample(:)<h.NominalValue))
ans =
   509
length(find(hsample(:)>h.NominalValue))
ans =
   491
```

The distribution used in `usample` is uniform in the normalized description of the uncertain real parameter. See “Decomposing Uncertain Objects (for Advanced Users)” on page 1-60 to learn more about the normalized description.

There is no notion of an empty `ureal` (or any other atom, for that matter). `ureal`, by itself, creates an unnamed atom, with default property values. The given name is `'UNNAMED'`. This can be observed with `get` and `set`.

```
get(ureal)
            Name: 'UNNAMED'
    NominalValue: 0
            Mode: 'PlusMinus'
```

```
            Range: [-1 1]
        PlusMinus: [-1 1]
       Percentage: [-Inf Inf]
     AutoSimplify: 'basic'
set(ureal)
             Name: 'String'
     NominalValue: '1x1 real DOUBLE'
             Mode: 'Range | PlusMinus'
            Range: '1x2 DOUBLE'
        PlusMinus: '1x2 or scalar DOUBLE'
       Percentage: 'Not settable since Nominal==0'
     AutoSimplify: '['off' | 'basic' | 'full']'
```

Uncertain LTI Dynamics Atoms

Uncertain linear, time-invariant objects, `ultidyn`, are used to represent unknown linear, time-invariant dynamic objects, whose only known attributes are bounds on their frequency response. Uncertain linear, time-invariant objects have an internal name (the `Name` property), and are created by specifying their size (number of outputs and number of inputs).

The property `Type` specifies whether the known attributes about the frequency response are related to gain or phase. The property `Type` may be `'GainBounded'` or `'PositiveReal'`. The default value is `'GainBounded'`.

The property `Bound` is a single number, which along with `Type`, completely specifies what is known about the uncertain frequency response. Specifically, if Δ is an `ultidyn` atom, and if γ denotes the value of the `Bound` property, then the atom represents the set of all stable, linear, time-invariant systems whose frequency response satisfies certain conditions:

If `Type` is `'GainBounded'`, $\dot{\bar{\sigma}}[\Delta(\omega)] \le \gamma$ for all frequencies. When `Type` is `'GainBounded'`, the default value for `Bound` (i.e., γ) is 1. The `NominalValue` of Δ is always the 0-matrix.

If `Type` is `'PositiveReal'`, $\Delta(\omega)+\Delta^*(\omega) \ge 2\cdot\gamma$ for all frequencies. When `Type` is `'PositiveReal'`, the default value for `Bound` (i.e., γ) is 0. The `NominalValue` is always $(\gamma+1+2|\gamma|)I$.

All properties of a `ultidyn` are can be accessed with `get` and `set` (although the `NominalValue` is determined from `Type` and `Bound`, and not accessible with `set`). The properties are

Properties	Meaning	Class
`Name`	Internal Name	`char`
`NominalValue`	Nominal value of atom	`See above`
`Type`	`'GainBounded' |'PositiveReal'`	`char`
`Bound`	Norm bound or minimum real	`scalar double`
`SampleStateDim`	State-space dimension of random samples of this uncertain element	`scalar double`
`AutoSimplify`	`'off' | {'basic'} |'full'`	`char`

The `SampleStateDim` property specifies the state dimension of random samples of the atom when using `usample`. The default value is 1. The `AutoSimplify` property serves the same function as in the uncertain real parameter.

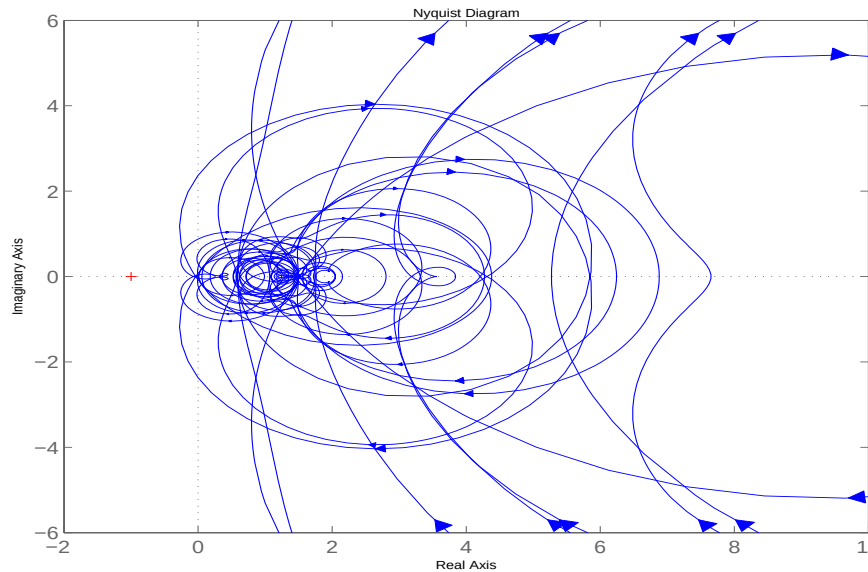
You can create a 2-by-3 gain-bounded uncertain linear dynamics atom. Verify its size, and check the properties.

```
f = ultidyn('f',[2 3]);
size(f)
ans =
     2     3
get(f)
              Name: 'f'
      NominalValue: [2x3 double]
              Type: 'GainBounded'
             Bound: 1
    SampleStateDim: 1
      AutoSimplify: 'basic'
```

You can create a 1-by-1 (scalar) positive-real uncertain linear dynamics atom, whose frequency response always has real part greater than -0.5. Set the

SampleStateDim property to 5. View the properties, and plot a Nyquist plot of 30 instances of the atom.

```
g = ultidyn('g',[1 1],'type','positivereal','bound',-0.5);
g.SampleStateDim = 5;
get(g)
              Name: 'g'
      NominalValue: 1.5000
              Type: 'PositiveReal'
             Bound: -0.5000
    SampleStateDim: 5
      AutoSimplify: 'basic'
nyquist(usample(g,30))
xlim([-2 10])
ylim([-6 6]);
```



Time-Domain of `ultidyn` Atoms

On its own, every `ultidyn` atom is interpreted as a continuous-time, system with uncertain behavior, quantified by bounds (gain or real-part) on its

frequency response. To see this, create a `ultidyn`, and view the sample time of several random samples of the atom.

```
h = ultidyn('h',[1 1]);
get(usample(h),'Ts')
ans =
     0
get(usample(h),'Ts')
ans =
     0
get(usample(h),'Ts')
ans =
     0
```

However, when a `ultidyn` atom is an uncertain element of an uncertain state space model (`uss`), then the time-domain characteristic of the atom is determined from the time-domain characteristic of the system. The bounds (gain-bounded or positivity) apply to the frequency-response of the atom. This is explained and demonstrated in "Interpreting Uncertainty in Discrete Time" on page 1-30.

Complex Parameter Atoms

The `ucomplex` atom represents an uncertain complex number, whose value lies in a disc, centered at `NominalValue`, with radius specified by the `Radius` property. The size of the disc can also be specified by `Percentage`, which means the radius is derived from the absolute value of the `NominalValue`. The properties of `ucomplex` objects are

Properties	Meaning	Class
`Name`	Internal Name	`char`
`NominalValue`	Nominal value of atom	`double`
`Mode`	`'Range' | 'Percentage'`	`char`
`Radius`	Radius of disk	`double`

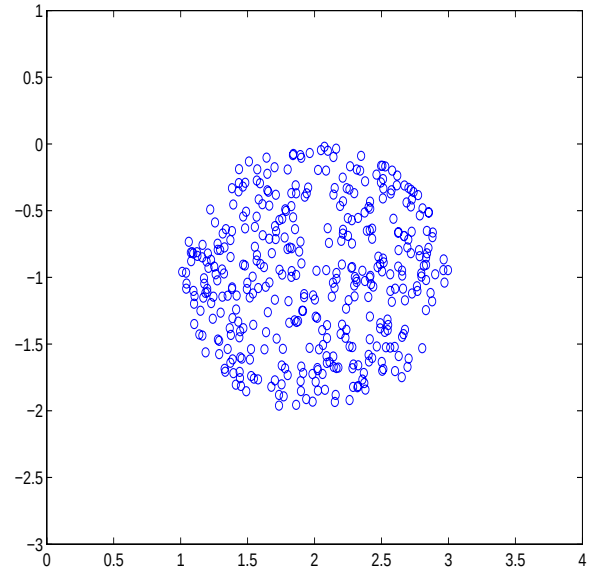
Properties	Meaning	Class
`Percentage`	Additive variation (percent of `Radius`)	`double`
`AutoSimplify`	`'off' | {'basic'} | 'full'`	`char`

The simplest construction requires only a name and nominal value. The default `Mode` is `Radius`, and the default radius is 1.

```
a = ucomplex('a',2-j)
Uncertain Complex Parameter: Name a, NominalValue 2-1i, Radius 1
get(a)
            Name: 'a'
    NominalValue: 2.0000- 1.0000i
            Mode: 'Radius'
          Radius: 1
      Percentage: 44.7214
    AutoSimplify: 'basic'
set(a)
            Name: 'String'
    NominalValue: '1x1 DOUBLE'
            Mode: 'Radius | Percentage'
          Radius: 'scalar DOUBLE'
      Percentage: 'scalar DOUBLE'
    AutoSimplify: '['off' | 'basic' | 'full']'
```

Sample the uncertain complex parameter at 400 values, and plot in the complex plane. Clearly, the samples appear to be from a disc of radius 1, centered in the complex plane at the value 2–*j*.

```
asample = usample(a,400);
plot(asample(:),'o'); xlim([0 4]); ylim([-3 1]);
```



Complex Matrix Atoms

The uncertain complex matrix class, `ucomplexm`, represents the set of matrices given by the formula

$N + W_L \Delta W_R$

where N, W_L, W_R are known matrices, and Δ is any complex matrix with

$\bar{\sigma}(\Delta) \le 1$. All properties of a `ucomplexm` are can be accessed with `get` and `set`. The properties are

Properties	Meaning	Class
`Name`	Internal Name	`char`
`NominalValue`	Nominal value of atom	`double`
`WL`	Left weight	`double`

Properties	Meaning	Class
WR	Right weight	double
AutoSimplify	'off' | {'basic'} | 'full'	char

The simplest construction requires only a name and nominal value. The default left and right weight matrices are identity.

You can create a 4-by-3 ucomplexm element, and view its properties.

```
m = ucomplexm('m',[1 2 3;4 5 6;7 8 9;10 11 12])
Uncertain Complex Matrix: Name m, 4x3
get(m)
            Name: 'm'
    NominalValue: [4x3 double]
              WL: [4x4 double]
              WR: [3x3 double]
    AutoSimplify: 'basic'
m.NominalValue
ans =
     1     2     3
     4     5     6
     7     8     9
    10    11    12
m.WL
ans =
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

Sample the uncertain matrix, and compare to the nominal value. Note the element-by-element sizes of the difference are generally equal, indicative of the default (identity) weighting matrices that are in place.

```
abs(usample(m)-m.NominalValue)
ans =
    0.2948    0.1001    0.2867
    0.3028    0.2384    0.2508
    0.3376    0.1260    0.2506
```

```
    0.2200    0.3472    0.1657
```

Change the left and right weighting matrices, making the uncertainty larger as you move down the rows, and across the columns.

```
m.WL = diag([0.2 0.4 0.8 1.6]);
m.WR = diag([0.1 1 4]);
```

Sample the uncertain matrix, and compare to the nominal value. Note the element-by-element sizes of the difference, and the general trend that the smallest differences are near the (1,1) element, and the largest differences are near the (4,3) element, which is completely expected by choice of the diagonal weighting matrices.

```
abs(usample(m)-m.NominalValue)
ans =
    0.0091    0.0860    0.2753
    0.0057    0.1717    0.6413
    0.0304    0.2756    1.4012
    0.0527    0.4099    1.8335
```

Unstructured Uncertain Dynamic Systems

The unstructured uncertain dynamic system class, `udyn`, represents completely unknown multivariable, time-varying nonlinear systems.

For practical purposes, these uncertain elements represent noncommuting symbolic variables (placeholders). All algebraic operations, such as addition, subtraction, multiplication (i.e., cascade) operate properly, and substitution (with `usubs`) is allowed. However, all of the analysis tools (e.g., `robuststab`) do not handle these types of uncertain elements. As such, these elements do not provide a significant amount of usability, and their role in the user's guide is small.

You can create a `2-by-3 udyn` element. Check its size, and properties.

```
m = udyn('m',[2 3])
Uncertain Dynamic System: Name m, size 2x3
size(m)
ans =
     2     3
get(m)
            Name: 'm'
```

```
    NominalValue: [2x3 double]
    AutoSimplify: 'basic'
```

Uncertain Matrices

Uncertain matrices (class `umat`) are built from doubles, and uncertain atoms, using traditional MATLAB® matrix building syntax. Uncertain matrices can be added, subtracted, multiplied, inverted, transposed, etc., resulting in uncertain matrices. The rows and columns of an uncertain matrix are referenced in the same manner that MATLAB references rows and columns of an array, using parenthesis, and integer indices. The `NominalValue` of a uncertain matrix is the result obtained when all uncertain atoms are replaced with their own `NominalValue`. The uncertain atoms making up a `umat` are accessible through the `Uncertainty` gateway, and the properties of each atom within a `umat` can be changed directly.

Using `usubs`, specific values may be substituted for any of the uncertain atoms within a `umat`. The command `usample` generates a random sample of the uncertain matrix, substituting random samples (within their ranges) for each of the uncertain atoms.

The command `wcnorm` computes tight bounds on the worst-case (maximum over the uncertain elements' ranges) norm of the uncertain matrix.

Standard MATLAB numerical matrices (i.e., `double`) naturally can be viewed as uncertain matrices without any uncertainty.

Creating Uncertain Matrices from Uncertain Atoms

You can create 2 uncertain real parameters, and then a 3-by-2 uncertain matrix using these uncertain atoms.

```
a = ureal('a',3);
b = ureal('b',10,'pe',20);
M = [-a 1/b;b a+1/b;1 3]
UMAT: 3 Rows, 2 Columns
  a: real, nominal = 3, variability = [-1  1], 2 occurrences
  b: real, nominal = 10, variability = [-20  20]%, 3 occurrences
```

The size and class of `M` are as expected

```
size(M)
ans =
     3     2
class(M)
ans =
```

```
umat
```

Accessing Properties of a umat

Use get to view the accessible properties of a umat.

```
get(M)
    NominalValue: [3x2 double]
     Uncertainty: [1x1 atomlist]
```

The NominalValue is a double, obtained by replacing all uncertain elements with their nominal values.

```
M.NominalValue
ans =
   -3.0000    0.1000
   10.0000    3.1000
    1.0000    3.0000
```

The Uncertainty property is a atomlist object, which is simply a gateway from the umat to the uncertain atoms.

```
class(M.Uncertainty)
ans =
atomlist
M.Uncertainty
    a: [1x1 ureal]
    b: [1x1 ureal]
```

Direct access to the atoms is facilitated through Uncertainty. Check the Range of the uncertain element named 'a' within M, then change it.

```
M.Uncertainty.a.Range
ans =
     2     4
M.Uncertainty.a.Range = [2.5 5];
M
UMAT: 3 Rows, 2 Columns
  a: real, nominal = 3, variability = [-0.5  2], 2 occurrences
  b: real, nominal = 10, variability = [-20  20]%, 3 occurrences
```

The change to the uncertain real parameter a only took place within M. Verify that the variable a in the workspace is no longer the same as the variable a within M.

```
isequal(M.Uncertainty.a,a)
ans =
     0
```

Note that combining atoms which have a common internal name, but different properties leads to an error. For instance, subtracting the two atoms gives an error, not 0.

```
M.Uncertainty.a - a
??? Error using ==> ndlft.lftmask
```

Atoms named 'a' have different properties.

Row and Column Referencing

Standard Row/Column referencing is allowed. Note, however, that single-indexing is only allowed if the umat is a column or a row.

Reconstruct M (if need be), and make a 2-by-2 selection from M

```
a = ureal('a',3);
b = ureal('b',10,'pe',20);
M = [-a 1/b;b a+1/b;1 3];
M.Uncertainty.a.Range = [2.5 5];
M(2:3,:)
UMAT: 2 Rows, 2 Columns
  a: real, nominal = 3, variability = [-0.5  2], 1 occurrence
  b: real, nominal = 10, variability = [-20  20]%, 2 occurrences
```

Make a single column selection from M, and use single-index references to access elements of it.

```
h = M([2 1 2 3],2)
UMAT: 4 Rows, 1 Columns
  a: real, nominal = 3, variability = [-0.5  2], 1 occurrence
  b: real, nominal = 10, variability = [-20  20]%, 1 occurrence
h(2)
UMAT: 1 Rows, 1 Columns
  b: real, nominal = 10, variability = [-20  20]%, 1 occurrence
```

```
h(3)
UMAT: 1 Rows, 1 Columns
  a: real, nominal = 3, variability = [-0.5  2], 1 occurrence
  b: real, nominal = 10, variability = [-20  20]%, 1 occurrence
```

Finally, make the (3,2) entry of `M` uncertain.

```
M(3,2) = ureal('c',3,'perc',40)
UMAT: 3 Rows, 2 Columns
  a: real, nominal = 3, variability = [-0.5  2], 2 occurrences
  b: real, nominal = 10, variability = [-20  20]%, 2 occurrences
  c: real, nominal = 3, variability = [-40  40]%, 1 occurrence
```

Matrix Operation on umat Objects

Many matrix operations are allowed, such as matrix-multiply, transpose, and inverse. Combinations of certain (i.e., not uncertain) matrices and uncertain matrices are allowed.

Premultiply `M` by a 1-by-3 constant matrix, resulting in a 1-by-2 `umat`.

```
M1 = [2 3 1]*M
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 3, variability = [-0.5  2], 1 occurrence
  b: real, nominal = 10, variability = [-20  20]%, 2 occurrences
  c: real, nominal = 3, variability = [-40  40]%, 1 occurrence
```

Verify that the 1st entry of `M1` is `-2*a + 3*b + 1`. Direct subtraction yields a `umat` without any dependence on uncertain elements. Simplifying the class shows that the result is zero as expected.

```
d = M1(1) - (-2*M.Uncertainty.a + 3*M.Uncertainty.b + 1)
UMAT: 1 Rows, 1 Columns
simplify(d,'class')
ans =
     0
```

Transpose `M`, form a product, an inverse, and sample the uncertain result. As expected, the result is the 2-by-2 identity matrix.

```
H = M.'*M;
K = inv(H);
usample(K*H,3)
```

```
ans(:,:,1) =
    1.0000   -0.0000
   -0.0000    1.0000
ans(:,:,2) =
    1.0000   -0.0000
   -0.0000    1.0000
ans(:,:,3) =
    1.0000   -0.0000
   -0.0000    1.0000
```

Substituting for Uncertain Atoms

Uncertain atoms can be substituted for using `usubs`. This is described in more detail in the section "Substitution by usubs". Here, we illustrate a few special cases.

Substitute all instances of the uncertain real parameter named `a` with the number 4. This results in a `umat`, with dependence on the uncertain real parameters `b` and `c`.

```
M2 = usubs(M,'a',4)
UMAT: 3 Rows, 2 Columns
  b: real, nominal = 10, variability = [-20  20]%, 2 occurrences
  c: real, nominal = 3, variability = [-40  40]%, 1 occurrence
```

Similarly, we can substitute all instances of the uncertain real parameter named `b` with `M.Uncertainty.a`, resulting in a `umat` with dependence on the uncertain real parameters `a` and `c`.

```
M3 = usubs(M,'b', M.Uncertainty.a)
UMAT: 3 Rows, 2 Columns
  a: real, nominal = 3, variability = [-0.5  2], 4 occurrences
  c: real, nominal = 3, variability = [-40  40]%, 1 occurrence
Nominal and/or random instances can easily be specified.
M4 = usubs(M,'a','nominal','b','random')
UMAT: 3 Rows, 2 Columns
  c: real, nominal = 3, variability = [-40  40]%, 1 occurrence
```

If one value is being substituted for many different atoms, the atom names can be listed in a cell array, and then the value given. For example, substitute `a` and `b` with the number 4, and `c` with the number 5.

```
M5 = usubs(M,{'a' 'b'},4,'c',5)
```

```
M5 =
   -4.0000    0.2500
    4.0000    4.2500
    1.0000    5.0000
```

The command `usample` also generates multiple random instances of a `umat` (and `uss` and `ufrd`). See "Sampling Uncertain Objects" on page 1-42 for details.

Lifting a `double` matrix to a `umat`

A not-uncertain matrix may be interpreted as an uncertain matrix that has no dependence on uncertain atoms. Use the `umat` command to *lift* a `double` to the `umat` class.

```
Md = [1 2 3;4 5 6];
M = umat(Md)
UMAT: 2 Rows, 3 Columns
```

High dimensional `double` matrices can also be lifted. Note from the display that once the matrix is interpreted as a `umat`, the 3rd dimension and beyond are interpreted as array dimensions. See "Array Management for Uncertain Objects" on page 1-49 for more information about how multidimensional arrays of uncertain objects are handled.

```
Md = randn(4,5,6);
M = umat(Md)
UMAT: 4 Rows, 5 Columns [array, 6 x 1]
Md = randn(4,5,6,7);
M = umat(Md)
UMAT: 4 Rows, 5 Columns [array, 6 x 7]
Md = randn(4,5,6,7,8);
M = umat(Md)
UMAT: 4 Rows, 5 Columns [array, 6 x 7 x 8]
```

Uncertain State-Space Systems (uss)

Uncertain systems (`uss`) are linear systems with uncertain state-space matrices and/or uncertain linear dynamics. Like their certain (i.e., not uncertain) counterpart, the `ss` object, they are often built from state-space matrices using the `ss` command. In the case where some of the state-space matrices are uncertain, the result will be a uncertain state-space (`uss`) object.

Combining uncertain systems with uncertain systems (with the `feedback` command, for example) usually leads to an uncertain system. Not-uncertain systems can be combined with uncertain systems. Usually the result is an uncertain system.

The nominal value of an uncertain system is a `ss` object, which is familiar to Control System Toolbox™ software users.

Creating Uncertain Systems

Uncertain systems (class `uss`) are built from of certain and/or uncertain state-space matrices, usually using the `ss` command.

In the example below, the `A, B` and `C` matrices are made up of uncertain real parameters. Packing them together with the `ss` command results in a continuous-time uncertain system.

You can create three uncertain real parameters. Then create 3 uncertain matrices `A, B` and `C`, and one `double` matrix `D`.

```
p1 = ureal('p1',10,'pe',50);
p2 = ureal('p2',3,'plusm',[-.5 1.2]);
p3 = ureal('p3',0);
A = [-p1 p2;0 -p1];
B = [-p2;p2+p3];
C = [1 0;1 1-p3];
D = [0;0];
```

Pack the 4 matrices together using the `ss` command. This results in a continuous-time 2-output, 1-input, 2-state uncertain system.

```
sys = ss(A,B,C,D)
USS: 2 States, 2 Outputs, 1 Input, Continuous System
  p1: real, nominal = 10, variability = [-50  50]%, 2 occurrences
  p2: real, nominal = 3, variability = [-0.5  1.2], 2 occurrences
```

```
    p3: real, nominal = 0, variability = [-1  1], 2 occurrences
```

Properties of uss Objects

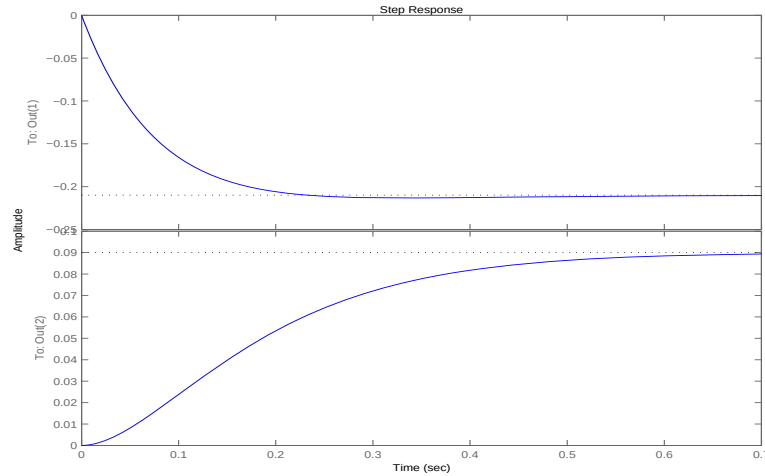
View the properties with the `get` command.

```
get(sys)
               a: [2x2 umat]
               b: [2x1 umat]
               c: [2x2 umat]
               d: [2x1 double]
       StateName: {2x1 cell}
              Ts: 0
       InputName: {''}
      OutputName: {2x1 cell}
      InputGroup: [1x1 struct]
     OutputGroup: [1x1 struct]
    NominalValue: [2x1 ss]
     Uncertainty: [1x1 atomlist]
           Notes: {}
        UserData: []
```

The properties `a, b, c, d`, and `StateName` behave in exactly the same manner as Control System Toolbox `ss` objects. The properties `InputName`, `OutputName`, `InputGroup` and `OutputGroup` behave in exactly the same manner as all of the Control System Toolbox system objects (`ss, zpk, tf`, and `frd`).

The `NominalValue` is a Control System Toolbox `ss` object, and hence all methods for `ss` objects are available. For instance, compute the poles and step response of the nominal system.

```
pole(sys.NominalValue)
ans =
   -10
   -10
step(sys.NominalValue)
```



Just as with the `umat` class, the `Uncertainty` property is a `atomlist` object, acting as a gateway to the uncertain atoms. Direct access to the atoms is facilitated through `Uncertainty`. Check the `Range` of the uncertain element named `'p2'` within `sys`, then change its left endpoint.

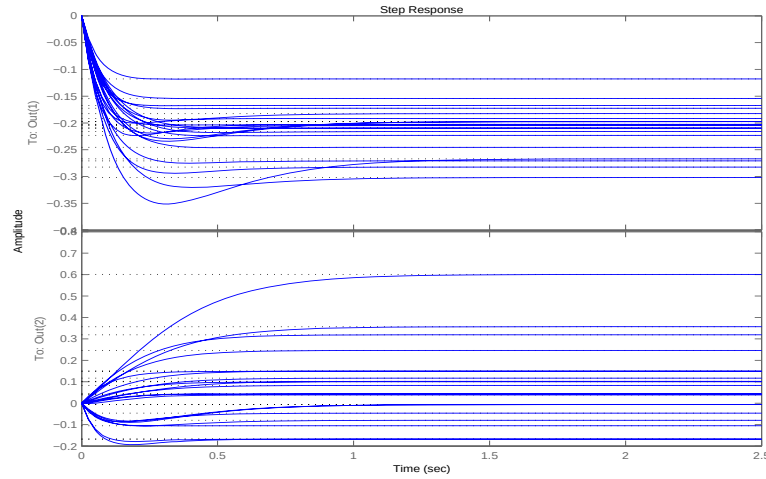
```
sys.Uncertainty.p2.range
ans =
    2.5000    4.2000
sys.Uncertainty.p2.range(1) = 2;
```

Sampling Uncertain Systems

The command `usample` randomly samples the uncertain system at a specified number of points.

Randomly sample the uncertain system at 20 points in its modeled uncertainty range. This gives a 20-by-1 `ss` array. Consequently, all analysis tools from Control System Toolbox software are available.

```
manysys = usample(sys,20);
size(manysys)
20x1 array of state-space models
Each model has 2 outputs, 1 input, and 2 states.
step(manysys)
```



The command `step` can be called directly on a `uss` object. The default behavior samples the `uss` object at 20 instances, and plots the step responses of these 20 models, as well as the nominal value.

The same features are available for `bode, bodemag, impulse, nyquist` and `step`.

Feedback Around an Uncertain Plant

It is possible to form interconnections of `uss` objects. A common example is to form the feedback interconnection of a given controller with an uncertain plant.

First create the uncertain plant. Start with two uncertain real parameters.

```
gamma = ureal('gamma',4);
tau = ureal('tau',.5,'Percentage',30);
```

Next, create an unmodeled dynamics atom, `delta`, and a 1st order weighting function, whose DC value is 0.2, high-frequency gain is 10, and whose crossover frequency is 8 rad/sec.

```
delta = ultidyn('delta',[1 1],'SampleStateDim',5);
W = makeweight(0.2,6,6);
```

Finally, create the uncertain plant consisting of the uncertain parameters and the unmodeled dynamics.

```
P = tf(gamma,[tau 1])*(1+W*delta);
```

You can create an integral controller based on nominal plant parameters. Nominally the closed-loop system will have damping ratio of 0.707 and time constant of `2*tau`.

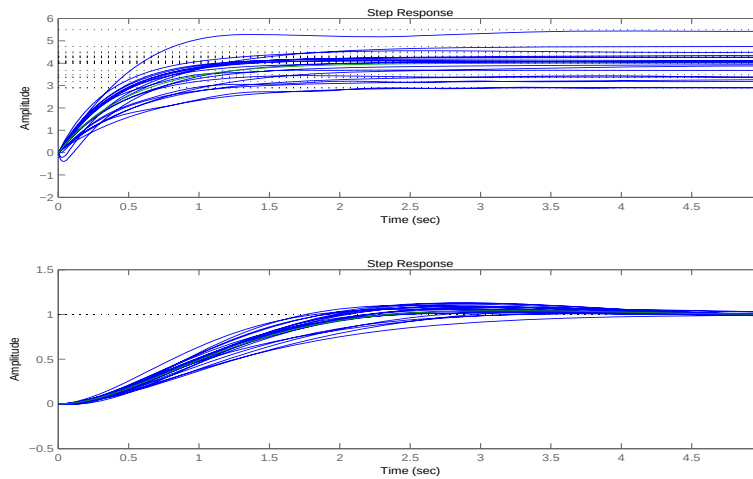
```
KI = 1/(2*tau.Nominal*gamma.Nominal);
C = tf(KI,[1 0]);
```

Create the uncertain closed-loop system using the `feedback` command.

```
CLP = feedback(P*C,1);
```

Using `usample` and `step`, plot samples of the open-loop and closed-loop step responses. As expected the integral controller reduces the variability in the low frequency response.

```
subplot(2,1,1); step(P,5,20)
subplot(2,1,2); step(CLP,5,20)
```



Interpreting Uncertainty in Discrete Time

The interpretation of a `ultidyn` atom as a continuous-time or discrete-time system depends on the nature of the uncertain system (`uss`) within which it is an uncertain element.

For example, create a scalar `ultidyn` object. Then, create two 1-input, 1-output `uss` objects using the `ultidyn` object as their "D" matrix. In one case, create without specifying sample-time, which indicates continuous time. In the second case, force discrete-time, with a sample time of 0.42.

```
delta = ultidyn('delta',[1 1]);
sys1 = uss([],[],[],delta)
USS: 0 States, 1 Output, 1 Input, Continuous System
  delta: 1x1 LTI, max. gain = 1, 1 occurrence
sys2 = uss([],[],[],delta,0.42)
USS: 0 States, 1 Output, 1 Input, Discrete System, Ts = 0.42
  delta: 1x1 LTI, max. gain = 1, 1 occurrence
```

Next, get a random sample of each system. When obtaining random samples using `usample`, the values of the atoms used in the sample are returned in the 2nd argument from `usample` as a structure.

```
[sys1s,d1v] = usample(sys1);
[sys2s,d2v] = usample(sys2);
```

Look at `d1v.delta.Ts` and `d2v.delta.Ts`. In the first case, since `sys1` is continuous-time, the system `d1v.delta` is continuous-time. In the second case, since `sys2` is discrete-time, with sample time 0.42, the system `d2v.delta` is discrete-time, with sample time 0.42.

```
d1v.delta.Ts
ans =
     0
d2v.delta.Ts
ans =
    0.4200
```

Finally, in the case of a discrete-time `uss` object, it is not the case that `ultidyn` objects are interpreted as continuous-time uncertainty in feedback with sampled-data systems. This very interesting hybrid theory has been studied by many authors, see [DullerudGlover], but it is beyond the scope of the toolbox.

Lifting a ss to a uss

A not-uncertain state space object may be interpreted as an uncertain state space object that has no dependence on uncertain atoms. Use the uss command to “lift” a ss to the uss class.

```
sys = rss(3,2,1);
usys = uss(sys)
USS: 3 States, 2 Outputs, 1 Input, Continuous System
```

Arrays of ss objects can also be lifted. See “Array Management for Uncertain Objects” on page 1-49 for more information about how arrays of uncertain objects are handled.

Handling Delays in uss

In the current implementation, delays are not allowed. Delays are omitted and a warning is displayed when ss objects are lifted to uss objects.

```
sys = rss(3,2,1);
sys.inputdelay = 1.3;
usys = uss(sys)
Warning: Omitting DELAYs in conversion to USS
> In uss.uss at 103
USS: 3 States, 2 Outputs, 1 Input, Continuous System
```

This lifting process happens in the background whenever ss objects are combined with any uncertain object. Consequently all delays will be lost in such operations.

Use the command pade to approximately preserve the effect of the time delay in the ss object. Before operations involving ss objects containing delays and uncertain objects, use the pade command to convert the ss object to a delay free object.

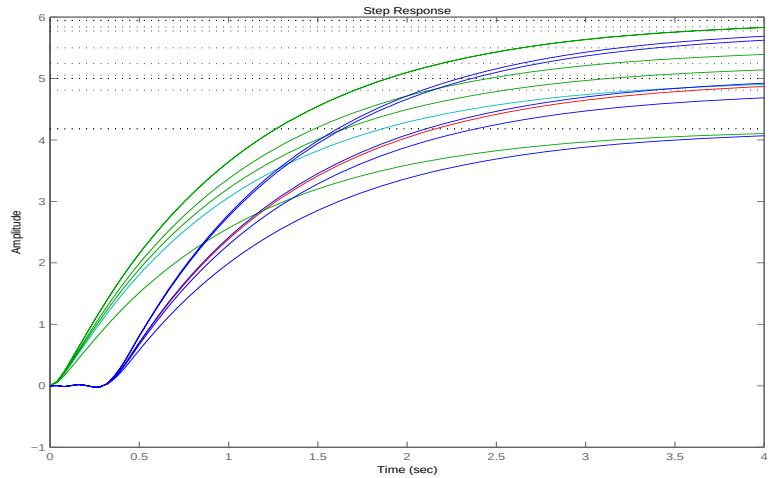
For example, consider an uncertain system with a time constant approximately equal to 1, an extra input delay of 0.3 seconds, second-order rolloff beyond 20 rad/s, and an uncertain steady-state gain ranging from 4 to 6. This can be approximated using the pade command as follows.

```
sys = tf(1,[1 1])*tf(1,[0.05 1]);
sys.inputdelay = 0.3;
gain = ureal('gain',5);
usys = gain*pade(sys,4)
```

```
USS: 6 States, 1 Output, 1 Input, Continuous System
  gain: real, nominal = 5, variability = [-1  1], 1 occurrence
```

If `gain` is multiplied by `sys` directly, the time delay is unfortunately omitted, since this operation involves lifting `sys` to a `uss` as described above. The difference is obvious from the step responses.

```
step(usys,gain*sys,4,5)
Warning: Omitting DELAYs in conversion to USS
> In uss.uss at 103
  In umat.umat at 98
  In atom.mtimes at 7
```



Uncertain frd

Uncertain frequency responses (ufrd) arise naturally when computing the frequency response of an uncertain state-space (uss). They also arise when frequency response data (in an frd object) is combined (added, multiplied, concatenated, etc.) to an uncertain matrix (umat).

Creating Uncertain Frequency Response Objects

The most common manner in which a ufrd arises is taking the frequency response of a uss. The natural command that would do this is frd (an overloaded version in the @uss directory).

Reconstruct sys, if necessary.

```
p1 = ureal('p1',10,'pe',50);
p2 = ureal('p2',3,'plusm',[-.5 1.2]);
p3 = ureal('p3',0);
A = [-p1 p2;0 -p1];
B = [-p2;p2+p3];
C = [1 0;1 1-p3];
D = [0;0];
sys = ss(A,B,C,D)
USS: 2 States, 2 Outputs, 1 Input, Continuous System
  p1: real, nominal = 10, variability = [-50  50]%, 2 occurrences
  p2: real, nominal = 3, variability = [-0.5  1.2], 2 occurrences
  p3: real, nominal = 0, variability = [-1  1], 2 occurrences
```

Compute the uncertain frequency response of the uncertain system. Use the frd command, along with a frequency grid containing 100 points. The result is an uncertain frequency response data object, referred to as a ufrd.

```
sysg = frd(sys,logspace(-2,2,100))
UFRD: 2 Outputs, 1 Input, Continuous System, 100 Frequency
points
  p1: real, nominal = 10, variability = [-50  50]%, 2 occurrences
  p2: real, nominal = 3, variability = [-0.5  1.2], 2 occurrences
  p3: real, nominal = 0, variability = [-1  1], 2 occurrences
```

Properties of ufrd Objects

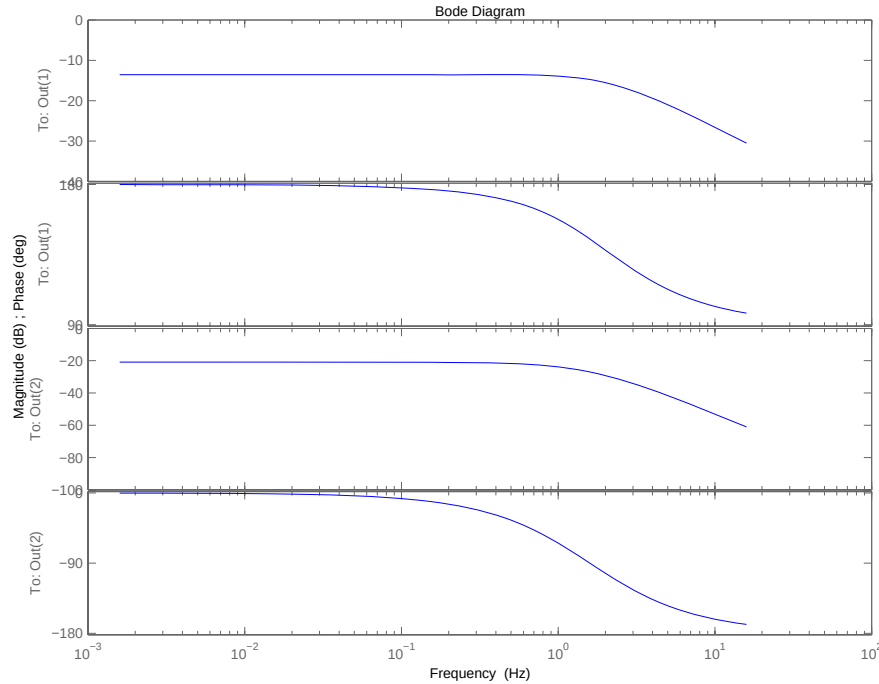
View the properties with the get command.

```
get(sysg)
       Frequency: [100x1 double]
    ResponseData: [2x1x100 umat]
           Units: 'rad/s'
              Ts: 0
       InputName: {''}
      OutputName: {2x1 cell}
      InputGroup: [1x1 struct]
     OutputGroup: [1x1 struct]
    NominalValue: [2x1 frd]
     Uncertainty: [1x1 atomlist]
           Notes: {}
        UserData: []
         Version: 4
```

The properties `ResponseData` and `Frequency` behave in exactly the same manner as Control System Toolbox™ `frd` objects, except that `ResponseData` is a `umat`. The properties `InputName, OutputName, InputGroup` and `OutputGroup` behave in exactly the same manner as all of the Control System Toolbox system objects (`ss, zpk, tf,` and `frd`).

The `NominalValue` is a Control System Toolbox `frd` object, and hence all methods for `frd` objects are available. For instance, plot the Bode response of the nominal system.

```
bode(sysg.nom)
```



Just as with the `umat` and `uss` classes, the `Uncertainty` property is an `atomlist` object, acting as a gateway to the uncertain atoms. Direct access to the atoms is facilitated through `Uncertainty`. Change the nominal value of the uncertain element named `'p1'` within `sysg` to 14, and replot the Bode plot of the (new) nominal system.

```
sysg.unc.p1.nom = 14
UFRD: 2 Outputs, 1 Input, Continuous System, 100 Frequency
points
  p1: real, nominal = 14, variability = [-50  50]%, 2 occurrences
  p2: real, nominal = 3, variability = [-0.5  1.2], 2 occurrences
  p3: real, nominal = 0, variability = [-1  1], 2 occurrences
```

Interpreting Uncertainty in Discrete Time

See "Interpreting Uncertainty in Discrete Time" on page 1-30. The issues are identical.

Lifting an frd to a ufrd

A not-uncertain frequency response object may be interpreted as an uncertain frequency response object that has no dependence on uncertain atoms. Use the `ufrd` command to "lift" an `frd` object to the `ufrd` class.

```
sys = rss(3,2,1);
sysg = frd(sys,logspace(-2,2,100));
usysg = ufrd(sysg)
UFRD: 2 Outputs, 1 Input, Continuous System, 100 Frequency
points
```

Arrays of `frd` objects can also be lifted. See "Array Management for Uncertain Objects" on page 1-49 for more information about how arrays of uncertain objects are handled.

Handling Delays in ufrd

See "Handling Delays in uss" on page 1-31. The issues are identical.

Basic Control System Toolbox™ and MATLAB® Interconnections

This list has all of the basic system interconnection functions defined in Control System Toolbox™ software or in MATLAB®.

- `append`
- `blkdiag`
- `series`
- `parallel`
- `feedback`
- `lft`
- `stack`

These functions work with uncertain objects as well. Uncertain objects may be combined with certain objects, resulting in an uncertain object.

Simplifying Representation of Uncertain Objects

A minimal realization of the transfer function matrix

$$H(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{4}{s+1} \\ \frac{3}{s+1} & \frac{6}{s+1} \end{bmatrix}$$

has only 1 state, obvious from the decomposition

$$H(s) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \frac{1}{s+1} \begin{bmatrix} 1 & 2 \end{bmatrix}$$

However, a "natural" construction, formed by

```
sys11 = ss(tf(2,[1 1]));
sys12 = ss(tf(4,[1 1]));
sys21 = ss(tf(3,[1 1]));
sys22 = ss(tf(6,[1 1]));
sys = [sys11 sys12;sys21 sys22]
a =
       x1  x2  x3  x4
   x1  -1   0   0   0
   x2   0  -1   0   0
   x3   0   0  -1   0
   x4   0   0   0  -1
b =
       u1  u2
   x1   2   0
   x2   0   2
   x3   2   0
   x4   0   2
c =
       x1   x2   x3   x4
   y1   1    2    0    0
   y2   0    0  1.5    3
d =
```

```
        u1  u2
   y1    0   0
   y2    0   0
Continuous-time model
```

has 4 states, and is nonminimal.

In the same manner, the internal representation of uncertain objects built up from uncertain atoms can become nonminimal, depending on the sequence of operations in their construction. The command `simplify` employs ad-hoc simplification and reduction schemes to reduce the complexity of the representation of uncertain objects. There are three levels of simplification: off, basic and full. Each uncertain atom has an `AutoSimplify` property whose value is one of the strings `'off'`, `'basic'` or `'full'`. The default value is `'basic'`.

After (nearly) every operation, the command `simplify` is automatically run on the uncertain object, cycling through all of the uncertain atoms, and attempting to simplify (without error) the representation of the effect of that uncertain object. The `AutoSimplify` property of each atom dictates the types of computations that are performed. In the `'off'` case, no simplification is even attempted. In `'basic'`, fairly simple schemes to detect and eliminate nonminimal representations are used. Finally, in `'full'`, numerical based methods similar to truncated balanced realizations are used, with a very tight tolerance to minimize error.

Effect of Autosimplify Property

Create an uncertain real parameter, view the `AutoSimplify` property of `a`, and then create a 1-by-2 `umat`, both of whose entries involve the uncertain parameter.

```
a = ureal('a',4);
a.AutoSimplify
ans =
basic
m1 = [a+4 6*a]
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 1 occurrence
```

Note that although the uncertain real parameter `a` appears in both (two) entries of the matrix, the resulting uncertain matrix `m1` only depends on "1 occurrence" of `a`.

Set the `AutoSimplify` property of `a` to `'off'` (from `'basic'`). Recreate the 1-by-2 `umat`. Now note that the resulting uncertain matrix `m2` depends on "2 occurrences" of `a`.

```
a.AutoSimplify = 'off';
m2 = [a+4 6*a]
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 2 occurrences
```

The `'basic'` level of autosimplification often detects (and simplifies) duplication created by linear terms in the various entries. Higher order (quadratic, bilinear, etc.) duplication is often not detected by the `'basic'` autosimplify level.

For example, reset the `AutoSimplify` property of `a` to `'basic'` (from `'off'`). Create an uncertain real parameter, and a 1-by-2 `umat`, both of whose entries involve the square of the uncertain parameter.

```
a.AutoSimplify = 'basic';
m3 = [a*(a+4) 6*a*a]
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 4 occurrences
```

Note that the resulting uncertain matrix `m3` depends on "4 occurrences" of `a`.

Set the `AutoSimplify` property of `a` to `'full'` (from `'basic'`). Recreate the 1-by-2 `umat`. Now note that the resulting uncertain matrix `m4` depends on "2 occurrences" of `a`.

```
a.AutoSimplify = 'full';
m4 = [a*(a+4) 6*a*a]
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 2 occurrences
```

Although `m4` has a less complex representation (2 occurrences of `a` rather than 4 as in `m3`), some numerical variations are seen when both uncertain objects are evaluated at (say) 0.

```
usubs(m3,'a',0)
ans =
```

```
     0     0
usubs(m4,'a',0)
ans =
  1.0e-015 *
   -0.4441         0
```

Small numerical differences are also noted at other evaluation points. The example below shows the differences encountered evaluating at a equal to 1.

```
usubs(m3,'a',1)
ans =
     5     6
usubs(m4,'a',1)
ans =
    5.0000    6.0000
```

Direct Use of simplify

The `simplify` command can be used to override all uncertain element's `AutoSimplify` property. The first input to the `simplify` command is an uncertain object. The second input is the desired reduction technique, which can either `'basic'` or `'full'`.

Again create an uncertain real parameter, and a 1-by-2 `umat`, both of whose entries involve the square of the uncertain parameter. Set the `AutoSimplify` property of `a` to `'basic'`.

```
a.AutoSimplify = 'basic';
m3 = [a*(a+4) 6*a*a]
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 4 occurrences
```

Note that the resulting uncertain matrix `m3` depends on “4 occurrences” of `a`.

The `simplify` command can be used to perform a `'full'` reduction on the resulting `umat`.

```
m4 = simplify(m3,'full')
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 2 occurrences
```

The resulting uncertain matrix `m4` depends on only “2 occurrences” of `a` after the reduction.

Sampling Uncertain Objects

The command `usample` is used to randomly sample an uncertain object, giving a not-uncertain instance of the uncertain object.

Generating One Sample

If `A` is an uncertain object, then `usample(A)` generates a single sample of `A`.

For example, a sample of a `ureal` is a scalar `double`.

```
A = ureal('A',6);
B = usample(A)
B =
    5.7298
```

Create a 1-by-3 `umat` with `A` and an uncertain complex parameter `C`. A single sample of this `umat` is a 1-by-3 double.

```
C = ucomplex('C',2+6j);
M = [A C A*A];
usample(M)
ans =
   5.9785             1.4375 + 6.0290i  35.7428
```

Generating Many Samples

If `A` is an uncertain object, then `usample(A,N)` generates `N` samples of `A`.

For example, 20 samples of a `ureal` gives a 1-by-1-20 `double` array.

```
B = usample(A,20);
size(B)
ans =
     1     1    20
```

Similarly, 30 samples of the 1-by-3 `umat` `M` yields a 1-by-3-by-30 array.

```
size(usample(M,30))
ans =
     1     3    30
```

See “Creating Arrays with usample” on page 1-53 for more information on sampling uncertain objects.

Sampling ultidyn Atoms

When sampling a `ultidyn` atom (or an uncertain object that contains a `ultidyn` atom in its `Uncertainty` gateway) the result is always a state-space (`ss`) object. The property `SampleStateDim` of the `ultidyn` class determines the state dimension of the samples.

Create a 1-by-1, gain bounded `ultidyn` object, with gain-bound 3. Verify that the default state dimension for samples is 1.

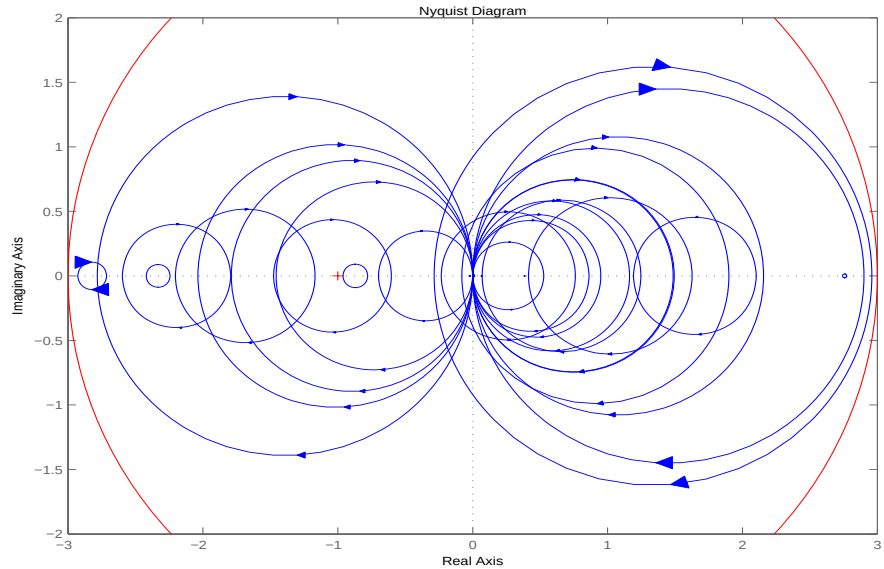
```
del = ultidyn('del',[1 1],'Bound',3);
del.SampleStateDim
ans =
     1
```

Sample the uncertain atom at 30 points. Verify that this creates a 30-by-1 `ss` array of 1-input, 1-output, 1-state systems.

```
delS = usample(del,30);
size(delS)
30x1 array of state-space models
Each model has 1 output, 1 input, and 1 state.
```

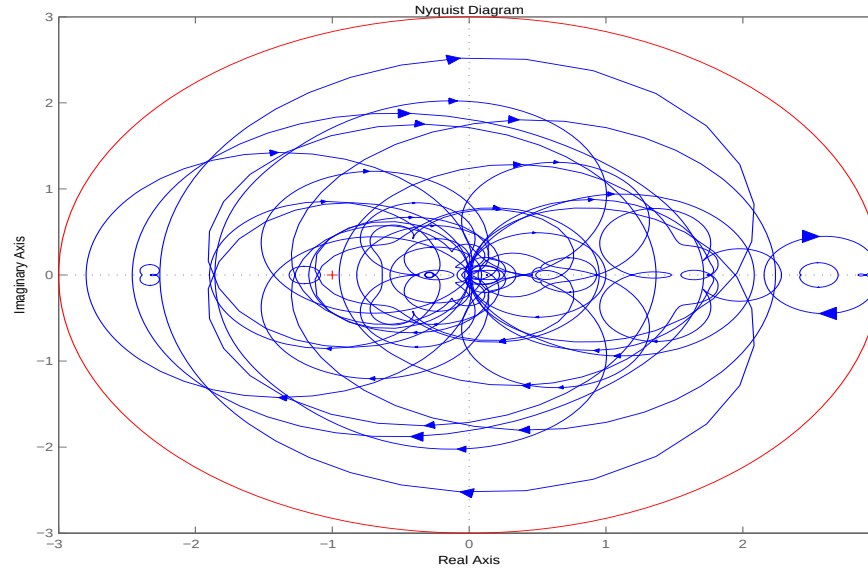
Plot the Nyquist plot of these samples and add a disk of radius 3. Note that the gain bound is satisfied and that the Nyquist plots are all circles, indicative of 1st order systems.

```
nyquist(delS)
hold on;
theta = linspace(-pi,pi);
plot(del.Bound*exp(sqrt(-1)*theta),'r');
hold off;
```



Change the `SampleStateDim` to 4, and repeat entire procedure. The Nyquist plots satisfy the gain bound and as expected are more complex than the circles found in the 1st-order sampling.

```
del.SampleStateDim = 4;
delS = usample(del,30);
nyquist(delS)
hold on;
theta = linspace(-pi,pi);
plot(del.Bound*exp(sqrt(-1)*theta),'r');
hold off;
```


Nyquist Diagram
Imaginary Axis
Real Axis
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3

Substitution by usubs

If an uncertain object (`umat, uss, ufrd`) has many uncertain parameters, it is often necessary to freeze some, but not all, of the uncertain parameters to specific values. The `usubs` command accomplishes this, and also allows more complicated substitutions for an atom.

`usubs` accepts a list of atom names, and respective values to substitute for them. You can create 3 uncertain real parameters and use them to create a 2-by-2 uncertain matrix `A`.

```
delta = ureal('delta',2);
eta = ureal('eta',6);
rho = ureal('rho',-1);
A = [3+delta+eta delta/eta;7+rho rho+delta*eta]
UMAT: 2 Rows, 2 Columns
  delta: real, nominal = 2, variability = [-1  1], 2 occurrences
    eta: real, nominal = 6, variability = [-1  1], 3 occurrences
    rho: real, nominal = -1, variability = [-1  1], 1 occurrence
```

Use `usubs` to substitute the uncertain element named `delta` in `A` with the value 2.3, leaving all other uncertain atoms intact. Note that the result, `B`, is an uncertain matrix with dependence only on `eta` and `rho`.

```
B = usubs(A,'delta',2.3)
UMAT: 2 Rows, 2 Columns
  eta: real, nominal = 6, variability = [-1  1], 3 occurrences
  rho: real, nominal = -1, variability = [-1  1], 1 occurrence
```

To set multiple atoms, list individually, or in cells. The following are the same

```
B1 = usubs(A,'delta',2.3,'eta',A.Uncertainty.rho);
B2 = usubs(A,{'delta';'eta'},{2.3;A.Uncertainty.rho});
```

In each case, `delta` is replaced by 2.3, and `eta` is replaced by `A.Uncertainty.rho`.

If it makes sense, a single replacement value can be used to replace multiple atoms. So

```
B3 = usubs(A,{'delta';'eta'},2.3);
```

replaces both the atoms `delta` and `eta` with the real number 2.3. Any superfluous substitution requests are ignored. Hence

```
B4 = usubs(A,'fred',5);
```

is the same as `A`, and

```
B5 = usubs(A,{'delta';'eta'},2.3,{'fred' 'gamma'},0);
```

is the same as `B3`.

Specifying the Substitution with Structures

An alternative syntax for `usubs` is to specify the substituted values in a structure, whose fieldnames are the names of the atoms being substituted with values.

Create a structure `NV` with 2 fields, `delta` and `eta`. Set the values of these fields to be the desired substituted values. Then perform the substitution with `usubs`.

```
NV.delta = 2.3;
NV.eta = A.Uncertainty.rho;
B6 = usubs(A,NV);
```

Here, `B6` is the same as `B1` and `B2` above. Again, any superfluous fields are ignored. Therefore, adding an additional field `gamma` to `NV`, and substituting does not alter the result.

```
NV.gamma = 0;
B7 = usubs(A,NV);
```

Here, `B7` is the same as `B6`.

The commands `wcgain, robuststab` and `usample` all return substitutable values in this structure format. More discussion can be found in “Creating Arrays with usubs” on page 1-55.

Nominal and Random Values

If the replacement value is the (partial and case-independent) string `'Nominal'`, then the listed atom are replaced with their nominal values. Therefore

```
B8 = usubs(A,fieldnames(A.Uncertainty),'nom')
B8 =
   11.0000    0.3333
```

```
    6.0000   11.0000
B9 = A.NominalValue
B9 =
   11.0000    0.3333
    6.0000   11.0000
```

are the same. It is possible to only set some of the atoms to `NominalValues`, and would be the typical use of `usubs` with the `'nominal'` argument.

Within `A`, set `eta` to its nominal value, `delta` to a random value (within its range) and `rho` to a specific value, say 6.5

```
B10 = usubs(A,'eta','nom','delta','rand','rho',6.5)
B10 =
   10.5183    0.2531
   13.5000   15.6100
```

Unfortunately, the `'Nominal'` and `'Random'` specifiers may not be used in the structure format. However, explicitly setting a field of the structure to an atom's nominal value, and then following (or preceeding) the call to `usubs` with a call to `usample` (to generate the random samples) is acceptable, and achieves the same effect.

Array Management for Uncertain Objects

All of the uncertain system classes (`uss, ufrd`) may be multidimensional arrays. This is intended to provide the same functionality as the LTI-arrays of the Control System Toolbox™ software. The command `size` returns a row vector with the sizes of all dimensions.

The first two dimensions correspond to the outputs and inputs of the system. Any dimensions beyond are referred to as the *array dimensions*. Hence, if `szM = size(M),` then `szM(3:end)` are sizes of the array dimensions of `M`.

For these types of objects, it is clear that the first two dimensions (system output and input) are interpreted differently from the 3rd, 4th, 5th and higher dimensions (which often model parametrized variability in the system input/output behavior).

`umat` objects are treated in the same manner. The first two dimensions are the rows and columns of the uncertain matrix. Any dimensions beyond are the *array dimensions*.

Referencing Arrays

Suppose `M` is a `umat, uss` or `ufrd`, and that `Yidx` and `Uidx` are vectors of integers. Then

```
M(Yidx,Uidx)
```

selects the outputs (rows) referred to by `Yidx` and the inputs (columns) referred to by `Uidx`, preserving all of the array dimensions. For example, if `size(M)` equals `[4 5 3 6 7]`, then (for example) the size of `M([4 2],[1 2 4])` is `[2 3 3 6 7]`.

If `size(M,1)==1` or `size(M,2)==1,` then single indexing on the inputs or outputs (rows or columns) is allowed. If `Sidx` is a vector of integers, then `M(Sidx)` selects the corresponding elements. All array dimensions are preserved.

If there are `K` array dimensions, and `idx1, idx2, ..., idxK` are vectors of integers, then

```
G = M(Yidx,Uidx,idx1,idx2,...,idxK)
```

selects the outputs and inputs referred to by `Yidx` and `Uidx`, respectively, and selects from each array dimension the “slices” referred to by the `idx1,`

idx2,..., idxK index vectors. Consequently, size(G,1) equals length(Yidx), size(G,2) equals length(Uidx), size(G,3) equals length(idx1), size(G,4) equals length(idx2), and size(G,K+2) equals length(idxK).

If M has K array dimensions, and less than K index vectors are used in doing the array referencing, then the MATLAB® convention for single indexing is followed. For instance, suppose size(M) equals [3 4 6 5 7 4]. The expression

```
G = M([1 3],[1 4],[2 3 4],[5 3 1],[8 10 12 2 4 20 18])
```

is valid. The result has size(G) equals [2 2 3 3 7]. The last index vector [8 10 12 2 4 20 18] is used to reference into the 7-by-4 array, preserving the order dictated by MATLAB single indexing (e.g., the 10th element of a 7-by-4 array is the element in the (3,2) position in the array).

Note that if M has either one output (row) or one input (column), *and* M has array dimensions, then it is not allowable to combine single indexing in the output/input dimensions along with indexing in the array dimensions. This will result in an ambiguity in how to interpret the second index vector in the expression (i.e., "does it correspond to the input/output reference, or does it correspond to the first array dimension?").

Creating Arrays with stack and cat Functions

An easy manner to create an array is with stack. Create a [4-by-1] umat array by stacking four 1-by-3 umat objects with the stack command. The first argument of stack specifies in which array dimension the stacking occurs. In the example below, the stacking is done is the 1st array dimension, hence the result is a 1-by-3-by-4-by-1 umat, referred to as a 4-by-1 umat array.

```
a = ureal('a',4);
b = ureal('b',2);
M = stack(1,[a b 1],[-a -b 4+a],[4 5 6],[a 0 0])
UMAT: 1 Rows, 3 Columns [array, 4 x 1]
  a: real, nominal = 4, variability = [-1  1], 1 occurrence
  b: real, nominal = 2, variability = [-1  1], 1 occurrence
size(M)
ans =
     1     3     4
arraysize(M)
```

```
ans =
     4     1
```

Check that result is valid. Use referencing to access parts of the [4-by-1] `umat` array and compare to the expected values. The first 4 examples should all be arrays full of 0 (zeros). The last two should be the value 5, and the uncertain real parameter a, respectively.

```
simplify(M(:,:,1) - [a b 1])
ans =
     0     0     0
simplify(M(:,:,2) - [-a -b 4+a])
ans =
     0     0     0
simplify(M(:,:,3) - [4 5 6])
ans =
     0     0     0
simplify(M(:,:,4) - [a 0 0])
ans =
     0     0     0
simplify(M(1,2,3))  % should be 5
ans =
     5
simplify(M(1,3,2)-4)
Uncertain Real Parameter: Name a, NominalValue 4, variability =
[-1  1]
```

You can create a random 1-by-3-by-4 double matrix and stack this with `M` along the second array dimension, creating a 1-by-3-by-4-by-2 `umat`.

```
N = randn(1,3,4);
M2 = stack(2,M,N);
size(M2)
ans =
     1     3     4     2
arraysize(M2)
ans =
     4     2
```

As expected, both `M` and `N` can be recovered from `M2`.

```
d1 = simplify(M2(:,:,:,1)-M);
```

```
d2 = simplify(M2(:,:,:,2)-N);
[max(abs(d1(:))) max(abs(d2(:)))]
ans =
     0     0
```

It is also possible to stack `M` and `N` along the 1st array dimension, creating a 1-by-3-by-8-by-1 `umat`.

```
M3 = stack(1,M,N);
size(M3)
ans =
     1     3     8
arraysize(M3)
ans =
     8     1
```

As expected, both `M` and `N` can be recovered from `M3`.

```
d3 = simplify(M3(:,:,1:4)-M);
d4 = simplify(M3(:,:,5:8)-N);
[max(abs(d3(:))) max(abs(d4(:)))]
ans =
     0     0
```

Creating Arrays by Assignment

Arrays can be created by direct assignment. As with other MATLAB classes, there is no need to preallocate the variable first. Simply assign elements – all resizing is performed automatically.

For instance, an equivalent construction to

```
a = ureal('a',4);
b = ureal('b',2);
M = stack(1,[a b 1],[-a -b 4+a],[4 5 6],[a 0 0]);
is
Mequiv(1,1,1) = a;
Mequiv(1,2,1) = b;
Mequiv(1,3,1) = 1;
Mequiv(1,:,4) = [a 0 0];
Mequiv(1,:,2:3) = stack(1,[-a -b 4+a],[4 5 6]);
```

The easiest manner for you to verify that the results are the same is to subtract and simplify,

```
d5 = simplify(M-Mequiv);
max(abs(d5(:)))
ans =
     0
```

Binary Operations with Arrays

Most operations simply cycle through the array dimensions, doing pointwise operations. Assume `A` and `B` are `umat` (or `uss`, or `ufrd`) arrays with identical array dimensions (slot 3 and beyond). The operation `C = fcn(A,B)` is equivalent to looping on `k1, k2, ...`, setting

```
C(:,:,k1,k2,...) = fcn(A(:,:,k1,k2,...),B(:,:,k1,k2,...))
```

The result `C` has the same array dimensions as `A` and `B`. The user is required to manage the extra dimensions (i.e., keep track of what they mean). Methods such as `permute`, `squeeze` and `reshape` are included to facilitate this management.

In general, any binary operation requires that the extra-dimensions are compatible. The `umat, uss` and `ufrd` objects allow for slightly more flexible interpretation of this. For illustrative purposes, consider a binary operation involving variables A and B. Suppose the array dimensions of A are $n_1 \times \ldots \times n_{l_A}$ and that the array dimensions of B are $m_1 \times \ldots \times m_{l_B}$. By MATLAB convention, the infinite number of singleton (i.e., 1) trailing dimensions are not listed. The compatibility of the extra dimensions is determined by the following rule: If l_A=l_B, then pad the shorter dimension list with trailing `1`'s. Now compare the extra dimensions: In the *k-th* dimension, it must be that one of 3 conditions hold: n_k=m_k, or n_k=1 or m_k=1. In other words, non-singleton dimensions must exactly match (so that the pointwise operation can be executed), and singleton dimensions match with anything, implicitly through a `repmat`.

Creating Arrays with usample

An extremely common manner in which to generate an array is to sample (in some of the uncertain elements) an uncertain object. Using the `ureal` objects `a` and `b` from above, create a 2-by-3 `umat`.

```
M = [a b;b*b a/b;1-b 1+a*b]
UMAT: 3 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 3 occurrences
  b: real, nominal = 2, variability = [-1  1], 6 occurrences
size(M)
ans =
     3     2
```

Sample (at 20 random points within its range) the uncertain real parameter `b` in the matrix `M`. This results in a 3-by-2-by-20 `umat`, with only one uncertain element, `a`. The uncertain element `b` of `M` has been "sampled out", leaving a new array dimension in its place.

```
[Ms,bvalues] = usample(M,'b',20);
Ms
UMAT: 3 Rows, 2 Columns [array, 20 x 1]
  a: real, nominal = 4, variability = [-1  1], 2 occurrences
size(Ms)
ans =
     3     2    20
```

Continue sampling (at 15 random points within its range) the uncertain real parameter `a` in the matrix `Ms`. This results in a 3-by-2-by-20-by-15 `double`.

```
[Mss,avalues] = usample(Ms,'a',15);
size(Mss)
ans =
     3     2    20    15
class(Mss)
ans =
double
```

The above 2-step sequence can be performed in 1 step,

```
[Mss,values] = usample(M,'b',20,'a',15);
class(Mss)
ans =
double
```

In this case, `values` is a 20-by-15 struct array, with 2 fields `b` and `a`, whose values are the values used in the random sampling. It follows that `usubs(M,values)` is the same as `Mss`.

Rather than sampling the each variable (`a` and `b`) independently, generating a 20-by-15 grid in a 2-dimensional space, the two-dimensional space can be sampled. Sample the 2-dimensional space with 800 points,

```
[Ms,values] = usample(M,{'a' 'b'},800);
size(Ms)
ans =
     3     2   800
size(values)
ans =
   800     1
```

Creating Arrays with usubs

Suppose `Values` is a `struct` array, with the following properties: the dimensions of `Value` "match" the array dimensions of `M`, in the manner described in Section above; the fieldnames of `Values` are some (or all) of the names of the uncertain elements of `M`; and the dimensions of the contents of the fields within `Values` match the sizes of the uncertain elements within `M`. Then `usubs(M,Values)` will substitute the uncertain elements in `M` with the contents found in the respective fields of `Values`.

You can create a 3-by-2 uncertain matrix using 2 uncertain real parameters.

```
a = ureal('a',4);
b = ureal('b',2);
M = [a b;b*b a/b;1-b 1+a*b];
```

Create a 5-by-1 `struct` array with fieldname `a`. Make its values random scalars. Create a 1-by-4 `struct` array with fieldname `b`.

```
Avalue = struct('a',num2cell(rand(5,1)));
Bvalue = struct('b',num2cell(rand(1,4)));
```

Substitute the uncertain real parameter `a` in `M` with the values in `Avalue`, yielding `ma`. Similarly substitute the uncertain real parameter `b` in `M` with the values in `Avalue`, yielding `mb`.

```
ma = usubs(M,Avalue)
UMAT: 3 Rows, 2 Columns [array, 5 x 1]
  b: real, nominal = 2, variability = [-1  1], 6 occurrences
mb = usubs(M,Bvalue)
UMAT: 3 Rows, 2 Columns [array, 1 x 4]
```

```
    a: real, nominal = 4, variability = [-1  1], 2 occurrences
```

Continue, substituting the uncertain real parameter `b` in `ma` with the values in `Bvalue`, yielding `mab`. Do the analogous operation for `mb`, yielding `mba`. Subtract, and note that the difference is 0, as expected.

```
mab = usubs(ma,Bvalue);
mba = usubs(mb,Avalue);
thediff = mab-mba;
max(abs(thediff(:)))
ans =
  4.4409e-016
```

Creating Arrays with gridureal

The command `gridureal` enables uniform sampling of specified uncertain real parameters within an uncertain object. It is a specialized case of `usubs`.

`gridureal` removes a specified uncertain real parameter and adds an array dimension (to the end of the existing array dimensions). The new array dimension represents the uniform samples of the uncertain object along the specified uncertain real parameter's range.

Create a 2-by-2 uncertain matrix with 3 uncertain real parameters.

```
a = ureal('a',3,'Range',[2.5 4]);
b = ureal('b',4,'Percentage',15);
c = ureal('c',-2,'Plusminus',[-1 .3]);
M = [a b;b c]
UMAT: 2 Rows, 2 Columns
  a: real, nominal = 3, range = [2.5  4], 1 occurrence
  b: real, nominal = 4, variability = [-15  15]%, 2 occurrences
  c: real, nominal = -2, variability = [-1  0.3], 1 occurrence
```

Grid the uncertain real parameter `b` in `M` with 100 points. The result is a `umat` array, with dependence on uncertain real parameters `a` and `c`.

```
Mgrid1 = gridureal(M,'b',100)
UMAT: 2 Rows, 2 Columns [array, 100 x 1]
  a: real, nominal = 3, range = [2.5  4], 1 occurrence
  c: real, nominal = -2, variability = [-1  0.3], 1 occurrence
```

Operating on the uncertain matrix M, grid the uncertain real parameter a with 20 points, the uncertain real parameter b with 12 points, and the uncertain real parameter c with 7 points, The result is a 2-by-2-by20-by-12-by7 double array.

```
Mgrid3 = gridureal(M,'a',20,'b',12,'c',7);
size(Mgrid3)
ans =
     2     2    20    12     7
```

Creating Arrays with repmat

The MATLAB command repmat is used to replicate and tile arrays. It works on the built-in objects of MATLAB, namely double, char, as well as the generalized container objects cell and struct. The identical functionality is provided for replicating and tiling uncertain elements (ureal, ultidyn, etc.) and umat objects.

You can create an uncertain real parameter, and replicate it in a 2-by-3 uncertain matrix. Compare to generating the same uncertain matrix through multiplication.

```
a = ureal('a',5);
Amat = repmat(a,[2 3])
UMAT: 2 Rows, 3 Columns
  a: real, nominal = 5, variability = [-1  1], 1 occurrence
Amat2 = a*ones(2,3);
simplify(Amat-Amat2)
ans =
     0     0     0
     0     0     0
```

Create (as in section a [4-by-1] umat array by stacking four 1-by-3 umat objects with the stack command. Use repmat to tile this 1-by-3-by-4-by-1 umat, into a 2-by-3-by-8-by-5 umat.

```
a = ureal('a',4);
b = ureal('b',2);
M = stack(1,[a b 1],[-a -b 4+a],[4 5 6],[a 0 0]);
size(M)
ans =
     1     3     4
```

```
Mtiled = repmat(M,[2 1 2 5])
UMAT: 2 Rows, 3 Columns [array, 8 x 5]
  a: real, nominal = 4, variability = [-1  1], 1 occurrence
  b: real, nominal = 2, variability = [-1  1], 1 occurrence
Verify the equality of M and a few certain tiles of Mtiled.
d1 = simplify(M-Mtiled(2,:,5:8,3));
d2 = simplify(M-Mtiled(1,:,1:4,2));
d3 = simplify(M-Mtiled(2,:,1:4,5));
[max(abs(d1(:))) max(abs(d2(:))) max(abs(d3(:)))]
ans =
     0     0     0
```

Note that `repmat` never increases the complexity of the representation of an uncertain object. The number of occurrences of each uncertain element remains the same, regardless of the extent of the replication and tiling.

Creating Arrays with repsys

Replicating and tiling uncertain state-space systems (`uss`, and uncertain frequency response data (`ufrd`) is done with `repsys`. The syntax and behavior are the same as the manner in which `repmat` is used to replicate and tile matrices. The syntax and behavior of `repsys` for `uss` and `ufrd` objects are the same as the traditional `repsys` which operates on `ss` and `frd` object. Just as in those cases, the uncertain version of `repsys` also allows for diagonal tiling.

Using permute and ipermute

The commands `permute` and `ipermute` are generalizations of `transpose`, which exchanges the rows and columns of a two-dimensional matrix.

`permute(A,ORDER)` rearranges the dimensions of `A` so that they are in the order specified by the vector `ORDER`. The array produced has the same values of `A` but the order of the subscripts needed to access any particular element are rearranged as specified by `ORDER`. The elements of `ORDER` must be a rearrangement of the numbers from 1 to `N`.

All of the uncertain objects are essentially 2-dimensional (output and input) operators with array dependence. This means that the first 2 dimensions are treated differently from dimensions 3 and beyond. It is not permissible to permute across these groups.

For `uss` and `ufrd`, the restriction is built into the syntax. The elements of the `ORDER` vector ***o***nly refer to array dimensions. Therefore, there is no possibility of permute across these dimensions. In you need to permute the first two dimensions, use the command `transpose` instead.

For `umat`, the restriction is enforced in the software. The elements of the `ORDER` vector refer to all dimensions. However, the first two elements of `ORDER` must be a rearrangement of the numbers 1 and 2. The remaining elements of `ORDER` must be a rearrangement of the numbers 3 through N. If either of those conditions fail, an error is generated. Hence, for `umat` arrays, either `permute` or `transpose` can be used to effect the transpose operation.

Decomposing Uncertain Objects (for Advanced Users)

Each uncertain object (`umat, uss, ufrd`) is a generalized feedback connection (`lft`) of a not-uncertain object (e.g., `double, ss, frd`) with a diagonal augmentation of uncertain atoms (`ureal, ultidyn, ucomplex, ucomplexm, udyn`). In robust control jargon, if the uncertain elements are normalized, this decomposition is often called "the M/D form."

The purpose of the uncertain objects (`ureal, ultidyn, umat, uss`, etc.) is to hide this underlying decomposition, and allow the user to focus on modeling and analyzing uncertain systems, rather than the details of correctly propagating the M/D representation in manipulations. Nevertheless, advanced users may want access to the familiar M/D form. The command `lftdata` accomplishes this decomposition.

Since `ureal`, `ucomplex` and `ucomplexm` do not have their `NominalValue` necessarily at zero, and in the case of `ureal` objects, are not symmetric about the `NominalValue`, some details are required in describing the decomposition.

Normalizing Functions for Uncertain Atoms

Associated with each uncertain element is a normalizing function. The normalizing function maps the uncertain element into a normalized uncertain element.

If ρ is an uncertain real parameter, with range `[L R]` and nominal value `N`, then the normalizing function F is

$$F(\rho) = \frac{A + B\rho}{C + D\rho}$$

with the property that for all ρ satisfying $L \le \rho \le R$, it follows that $-1 \le F(\rho) \le 1$, moreover, $F(L)=-1$, $F(N)=0$, and $F(R)=1$. If the nominal value is centered in the range, then it is easy to conclude that

$$A = -\frac{R+L}{R-L} \ , \ B = \frac{2}{R-L} \ , \texttt{C=1, D=0}$$

It is left as an algebra exercise for the user to work out the various values for `A, B, C` and `D` when the nominal value is not centered.

If E is an uncertain gain-bounded, linear, time-invariant dynamic uncertainty, with gain-bound β, then the normalizing function F is

$$F(E) = \frac{1}{\beta}E$$

If E is an uncertain positive-real, linear, time-invariant dynamic uncertainty, with positivity bound β, then the normalizing function F is

$$F(E) = \left[I - \alpha\left(E - \frac{\beta}{2}I\right)\right]\left[I + \alpha\left(E - \frac{\beta}{2}I\right)\right]^{-1}$$

where $\alpha=2|\beta|+1$.

The normalizing function for an uncertain complex parameter ξ, with nominal value C and radius γ is

$$F(\xi) = \frac{1}{\gamma}(\xi - C)$$

The normalizing function for uncertain complex matrices H, with nominal value N and weights W_L and W_R is

$$F(H) = W_L^{-1}(H - N)W_R^{-1}$$

In each case, as the uncertain atom varies over its range, the absolute value of the normalizing function (or norm, in the matrix case) varies from 0 and 1.

Properties of the Decomposition

Take an uncertain object A, dependent on uncertain real parameters $\rho_1,\ldots,\rho_N$, uncertain complex parameters $\xi_1,\ldots,\xi_K$, uncertain complex matrices $H_1,\ldots,H_B$, uncertain gain-bounded linear, time-invariant dynamics $E_1,\ldots,E_D$, and uncertain positive-real linear, time-invariant dynamics $P_1,\ldots,P_Q$.

Write $A(\rho,\xi,H,E,P)$ to indicate this dependence. Using `lftdata`, A can be decomposed into two separate pieces: M and $\Delta(\rho,\xi,H,E,P)$ with the following properties:

- M is certain (i.e., if A is `uss`, then M is `ss`; if A is `umat`, then M is `double`; if A is `ufrd`, then M is `frd`).
- Δ is always a `umat`, depending on the same uncertain elements as A, with ranges, bounds, weights, etc., unaltered.

- The form of Δ is block diagonal, with elements made up of the normalizing functions acting on the individual uncertain elements.

$$\Delta(\rho, \xi, H, E, P) = \begin{bmatrix} F(\rho) & 0 & 0 & 0 & 0 \\ 0 & F(\xi) & 0 & 0 & 0 \\ 0 & 0 & F(H) & 0 & 0 \\ 0 & 0 & 0 & F(E) & 0 \\ 0 & 0 & 0 & 0 & F(P) \end{bmatrix}$$

- $A(\rho,\xi,H,E,P)$ is given by a linear fractional transformation of M and $\Delta(\rho,\xi,H,E,P)$,

$$A(\rho, \xi) = M_{22} + M_{21}\Delta(\rho, \xi, H, E, P)[I - M_{11}\Delta(\rho, \xi, H, E, P)]^{-1}M_{12}$$

The order of the normalized atoms making up `A` is not the simple order shown above. It is actually the same order as given by the command `fieldnames(M.Uncertainty)`. See "Advanced Syntax of lftdata" on page 1-64 for more information.

Syntax of lftdata

The decomposition is carried out by the command `lftdata`.

You can create a 2-by-2 `umat` named `A` using three uncertain real parameters.

```
delta = ureal('delta',2);
eta = ureal('eta',6);
rho = ureal('rho',-1);
A = [3+delta+eta delta/eta;7+rho rho+delta*eta]
UMAT: 2 Rows, 2 Columns
  delta: real, nominal = 2, variability = [-1  1], 2 occurrences
    eta: real, nominal = 6, variability = [-1  1], 3 occurrences
    rho: real, nominal = -1, variability = [-1  1], 1 occurrence
```

Note that `A` depends on two occurrences of `delta`, three occurrences of `eta` and one occurrence of `rho`.

Decompose `A` into `M` and `Delta`. Note that `M` is a `double`, and `Delta` has the same uncertainty dependence as `A`.

```
[M,Delta] = lftdata(A);
class(M)
```

```
ans =
double
Delta
UMAT: 6 Rows, 6 Columns
  delta: real, nominal = 2, variability = [-1  1], 2 occurrences
    eta: real, nominal = 6, variability = [-1  1], 3 occurrences
    rho: real, nominal = -1, variability = [-1  1], 1 occurrence
```

Sample `Delta` at 5 points. Things to note are: it is diagonal; the values range between -1 and 1; there are 3 independent values, and duplication of the entries is consistent with the dependence of `Delta` and `A` on the 3 uncertain real parameters.

```
usample(Delta,5)
ans(:,:,1) =
   -0.7106         0         0         0         0         0
         0   -0.7106         0         0         0         0
         0         0    0.6374         0         0         0
         0         0         0    0.6374         0         0
         0         0         0         0    0.6374         0
         0         0         0         0         0   -0.1258
ans(:,:,2) =
   -0.5850         0         0         0         0         0
         0   -0.5850         0         0         0         0
         0         0   -0.3021         0         0         0
         0         0         0   -0.3021         0         0
         0         0         0         0   -0.3021         0
         0         0         0         0         0    0.0803
ans(:,:,3) =
    0.7013         0         0         0         0         0
         0    0.7013         0         0         0         0
         0         0   -0.6749         0         0         0
         0         0         0   -0.6749         0         0
         0         0         0         0   -0.6749         0
         0         0         0         0         0    0.3967
ans(:,:,4) =
    0.4262         0         0         0         0         0
         0    0.4262         0         0         0         0
         0         0    0.0795         0         0         0
         0         0         0    0.0795         0         0
         0         0         0         0    0.0795         0
```

```
         0         0         0         0         0   -0.9959
ans(:,:,5) =
   -0.8392         0         0         0         0         0
         0   -0.8392         0         0         0         0
         0         0    0.8467         0         0         0
         0         0         0    0.8467         0         0
         0         0         0         0    0.8467         0
         0         0         0         0         0    0.6732
```

In fact, verify that the maximum gain of `Delta` is indeed 1.

```
maxnorm = wcnorm(Delta)
maxnorm =
    LowerBound: 1.0000
    UpperBound: 1.0004
```

Finally, verify that `lft(Delta,M)` is the same as `A`. Subtract (and use the `'full'` option in `simplify`)

```
simplify(lft(Delta,M)-A,'full')
ans =
     0     0
     0     0
```

Advanced Syntax of lftdata

Even for the advanced user, the variable `Delta` will actually not be that useful, as it is still a complex object. On the other hand, its internal structure is described completely using a 3rd (and 4th) output argument.

```
[M,Delta,BlkStruct,NormUnc] = lftdata(A);
```

The rows of `BlkStruct` correspond to the uncertain atoms named in `fieldnames(A.Uncertainty)`. Note that the range/bound information about each uncertain atom is not included in `BlkStruct`.

The elements of `BlkStruct` describe the size, type and number-of-copies of the uncertain atoms in `A`, and implicitly delineate the exact block-diagonal structure of `Delta`. Note that the range/bound information about each uncertain atom is not included in `BlkStruct`.

```
BlkStruct(1)
ans =
```

```
           Name: 'delta'
           Size: [1 1]
           Type: 'ureal'
    Occurrences: 2
BlkStruct(2)
ans =
           Name: 'eta'
           Size: [1 1]
           Type: 'ureal'
    Occurrences: 3
BlkStruct(3)
ans =
           Name: 'rho'
           Size: [1 1]
           Type: 'ureal'
    Occurrences: 1
```

Together, these mean `Delta` is a block diagonal augmentation of the normalized version of 3 uncertain atoms.

- The first atom is named `'delta'`. It is 1-by-1; it is of class `ureal`; and there are 2 copies diagonally augmented.
- The second atom is named `'eta'`. It is 1-by-1; it is of class `ureal`; and there are 3 copies diagonally augmented.
- The third atom is named 'rho'. It is 1-by-1; it is of class `ureal`; and there is 1 copy,

The 4th output argument contains a cell array of normalized uncertain elements. The cell array contains as many occurrences of each element as there are occurrences in the original uncertain object A.

```
size(NormUnc)
ans =
     6     1
NormUnc{1}
Uncertain Real Parameter: Name deltaNormalized, NominalValue 0,
variability = [-1  1]
isequal(NormUnc{2},NormUnc{1})
```

```
ans =
     1
NormUnc{3}
Uncertain Real Parameter: Name etaNormalized, NominalValue 0,
variability = [-1  1]
isequal(NormUnc{4},NormUnc{3})
ans =
     1
isequal(NormUnc{5},NormUnc{3})
ans =
     1
NormUnc{6}
Uncertain Real Parameter: Name rhoNormalized, NominalValue 0,
variability = [-1  1]
```

Each normalized element has the string `'Normalized'` appended to its original name to avoid confusion. By normalized,

- `ureal` objects have nominal value of 0, and range from -1 to 1.
- `ultidyn` objects are norm bounded, with norm bound of 1.
- `ucomplex` objects have nominal value of 0, and radius 1.
- `ucomplexm` objects have nominal value of 0, and identity matrices for each of the `WL` and `WR` weights.

The possible behaviors of `Delta` and `blkdiag(NormUnc{:})` are the same. Consequently, the possible behaviors of `A` and `lft(blkdiag(NormUnc{:}),M)` are the same.

Hence, by manipulating `M, BlkStruct` and `NormUnc`, a power-user has direct access to all of the linear fractional transformation details, and can easily work at the level of the theorems and algorithms that underlie the methods.

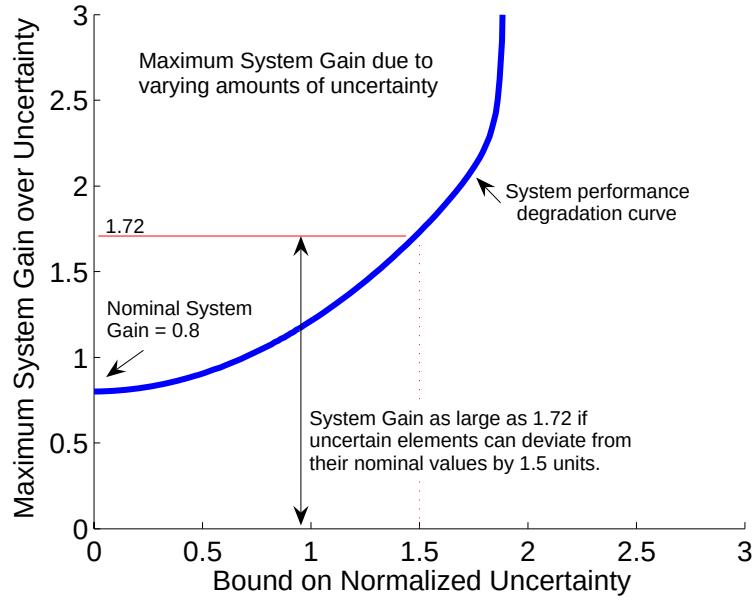
2

Generalized Robustness Analysis

Introduction to Generalized Robustness Analysis

The performance of a nominally stable uncertain system model will generally degrade for specific values of its uncertain elements. Moreover, the maximum possible degradation increases as the uncertain elements are allowed to further and further deviate from their nominal values.

The graph below shows the typical tradeoff curve between allowable deviation of uncertain elements from their nominal values and the worst-case degradation in system performance. Here, system performance is characterized by system gain (e.g., peak magnitude on Bode plot). Interpreting the system as the relationship mapping disturbances/commands to errors, small system gains are desirable, and large gains are undesirable.



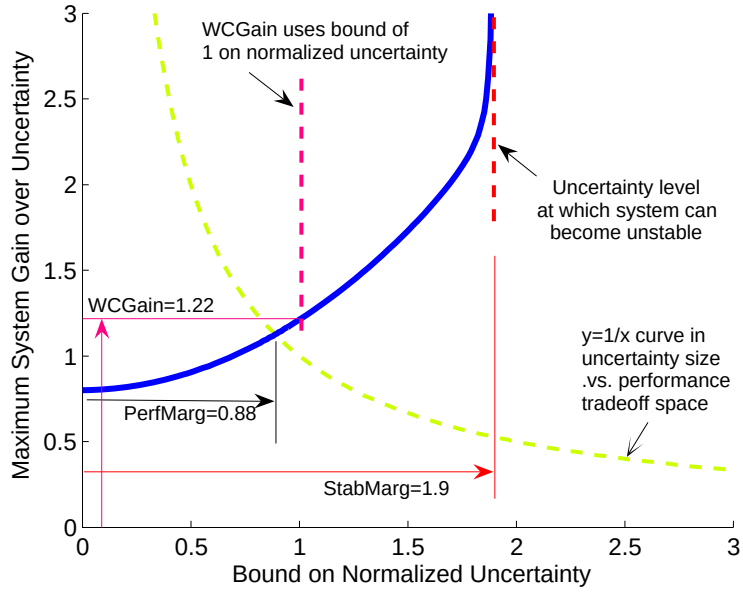
When all uncertain elements are set to their nominal values (i.e., zero deviation from their nominal values) the input/output gain of the system is its nominal value. In the figure, the nominal system gain is about 0.8. As the uncertainties are allowed to deviate from nominal, the maximum (over the

uncertain elements) system gain increases. The heavy blue line represents the maximum system gain due to uncertainty of various sizes (the horizontal axis). and is called the *system performance degradation curve*. It is monotonically increasing.

Determining specific attributes of the system performance degradation curve are referred to as robustness computations.

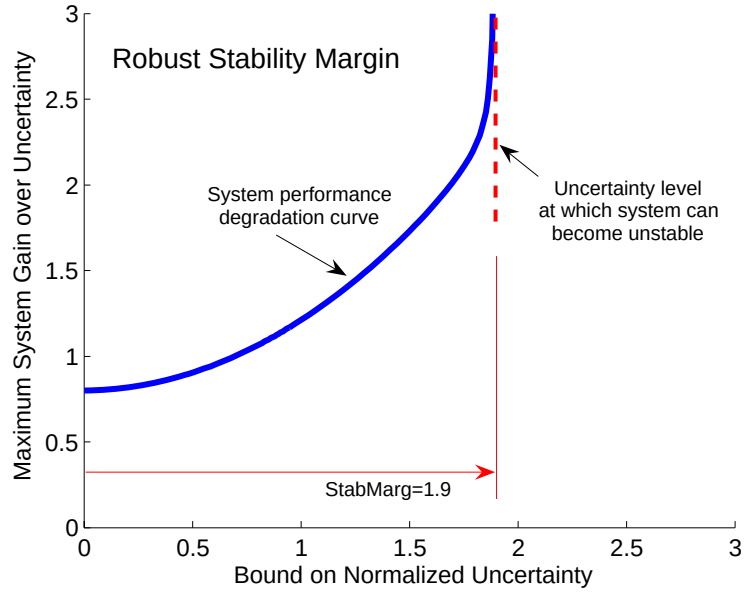
Generally, "robustness computations" refer to determining specific attributes of the system performance degradation curve. The commands `robuststab`, `robustperf` and `wcgain` all compute single scalar attributes of the system performance degradation curve.

Redraw the *system performance degradation curve* with 3 additional curves: a hyperbola defined by *xy*=1; a vertical line drawn at the uncertainty bound = 1; and a vertical line tangent to the asymptotic behavior of the performance degradation curve at large uncertainty bounds. These are used to define three robustness measures, explained next.



Robust Stability Margin

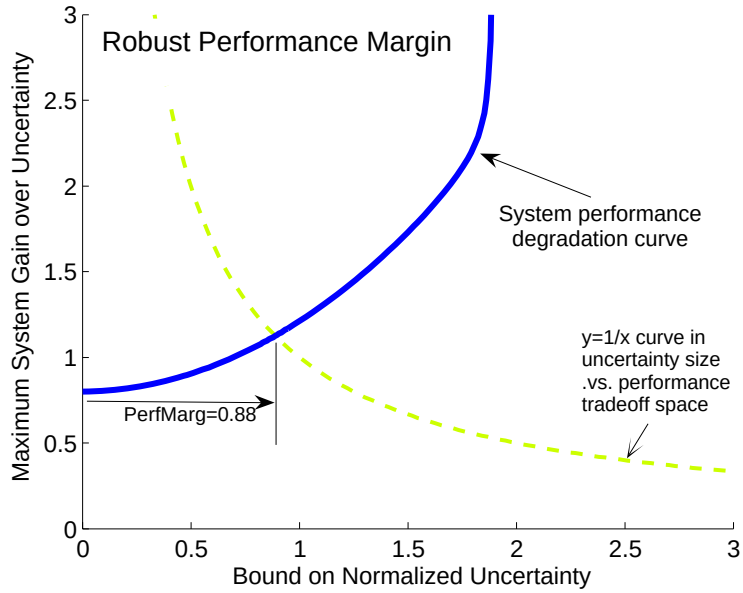
The *robust stability margin*, `StabMarg`, is the size of the smallest deviation from nominal of the uncertain elements that leads to system instability.



System instability is equivalent to the system gain becoming arbitrarily large, and hence characterized by the vertical line tangent to the asymptotic behavior of the performance degradation curve.

Robust Performance Margin

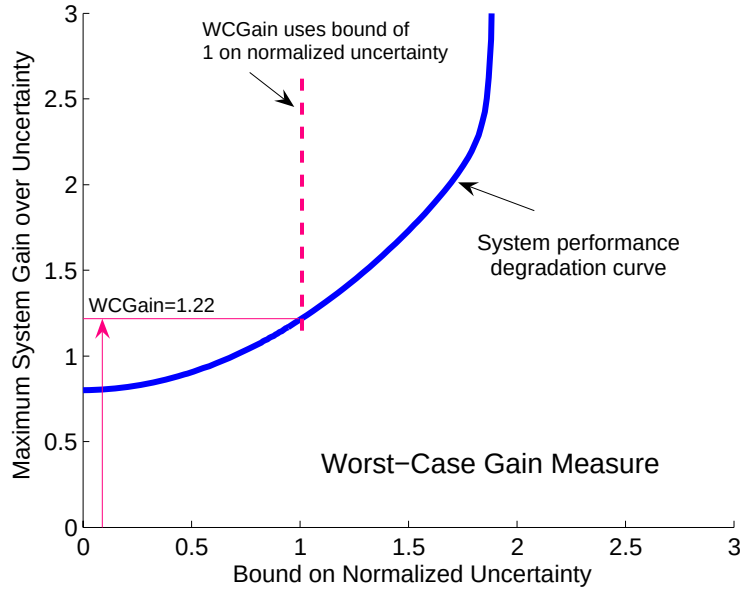
The hyperbola is used to define the performance margin. Systems whose performance degradation curve intersects high on the hyperbola curve represent “non-robustly performing systems” in that very small deviations of the uncertain elements from their nominal values can result in very large system gains. Conversely, an intersection low on the hyperbola represent “robustly performing systems.”



The point where the system performance degradation curve crosses the green line is used as a scalar measure of the robustness of a system to uncertainty. The horizontal coordinate of the crossing point is the *robust performance margin*, `PerfMarg`.

Worst-Case Gain Measure

The worst-case gain measure is the maximum achievable system gain over all uncertain elements whose normalized size is bounded by 1.



On the graph, this is the vertical coordinate of the performance degradation curve as it crosses the vertical line drawn at the uncertainty bound = 1.

Each measure captures a single scalar attribute of the *system performance degradation curve*. Mathematically, they are independent quantities, answering subtlely different questions. Consequently, for two uncertain systems, `sysA` and `sysB`, it is possible that the `StabMarg` of `sysA` is larger than the `StabMarg` of `sysB`, though the `PerfMarg` of `sysA` is smaller than the `PerfMarg` of `sysB`. Nevertheless, they are useful metrics for concise description of the robustness of a system (`uss` or `ufrd`) due to various uncertain elements.

3

Introduction to Linear Matrix Inequalities

Linear Matrix Inequalities

Linear Matrix Inequalities (LMIs) and LMI techniques have emerged as powerful design tools in areas ranging from control engineering to system identification and structural design. Three factors make LMI techniques appealing:

- A variety of design specifications and constraints can be expressed as LMIs.
- Once formulated in terms of LMIs, a problem can be solved *exactly* by efficient convex optimization algorithms (see "LMI Solvers" on page 1-18).
- While most problems with multiple constraints or objectives lack analytical solutions in terms of matrix equations, they often remain tractable in the LMI framework. This makes LMI-based design a valuable alternative to classical "analytical" methods.

See [9] for a good introduction to LMI concepts. Robust Control Toolbox™ software is designed as an easy and progressive gateway to the new and fast-growing field of LMIs:

- For users who occasionally need to solve LMI problems, the LMI Editor and the tutorial introduction to LMI concepts and LMI solvers provide for quick and easy problem solving.
- For more experienced LMI users, Chapter 4, "The LMI Lab", offers a rich, flexible, and fully programmable environment to develop customized LMI-based tools.

LMI Features

Robust Control Toolbox LMI functionality serves two purposes:

- Provide state-of-the-art tools for the LMI-based analysis and design of robust control systems
- Offer a flexible and user-friendly environment to specify and solve general LMI problems (the LMI Lab)

Examples of LMI-based analysis and design tools include

- Functions to analyze the robust stability and performance of uncertain systems with varying parameters (`popov`, `quadstab`, `quadperf` ...)

- Functions to design robust control with a mix of H_2, H_∞, and pole placement objectives (`h2hinfsyn`)
- Functions for synthesizing robust gain-scheduled H_∞ controllers (`hinfgs`)

For users interested in developing their own applications, the LMI Lab provides a general-purpose and fully programmable environment to specify and solve virtually any LMI problem. Note that the scope of this facility is by no means restricted to control-oriented applications.

Note Robust Control Toolbox software implements state-of-the-art interior-point LMI solvers. While these solvers are significantly faster than classical convex optimization algorithms, you should keep in mind that the complexity of LMI computations can grow quickly with the problem order (number of states). For example, the number of operations required to solve a Riccati equation is $o(n^3)$ where n is the state dimension, while the cost of solving and equivalent "Riccati inequality" LMI is $o(n^6)$.

LMIs and LMI Problems

A linear matrix inequality (LMI) is any constraint of the form

$$A(x) := A_0 + x_1A_1 + \ldots + x_NA_N < 0$$

where

- $x = (x_1, \ldots, x_N)$ is a vector of unknown scalars (the *decision* or *optimization* variables)
- $A_0, \ldots, A_N$ are given *symmetric* matrices
- < 0 stands for "negative definite," i.e., the largest eigenvalue of $A(x)$ is negative

Note that the constraints $A(x) > 0$ and $A(x) < B(x)$ are special cases of (1-1) since they can be rewritten as $-A(x) < 0$ and $A(x) - B(x) < 0$, respectively.

The LMI (1-1) is a convex constraint on x since $A(y) < 0$ and $A(z) < 0$ imply that $A\left(\frac{y+z}{2}\right) < 0$. As a result,

- Its solution set, called the *feasible set*, is a convex subset of $\mathbf{R}^N$
- Finding a solution x to (1-1), if any, is a convex optimization problem.

Convexity has an important consequence: even though (1-1) has no analytical solution in general, it can be solved numerically with guarantees of finding a solution when one exists. Note that a system of LMI constraints can be regarded as a single LMI since

$$\left\{\begin{array}{l} A_1(x) < 0 \\ \quad\vdots \quad \text{is equivalent to } A(x) := \mathrm{diag}(A_1(x), \ldots, A_K(x)) < 0 \\ A_K(x) < 0 \end{array}\right.$$

where diag $(A_1(x), \ldots, A_K(x))$ denotes the block-diagonal matrix with $A_1(x), \ldots, A_K(x)$ on its diagonal. Hence multiple LMI constraints can be imposed on the vector of decision variables x without destroying convexity.

In most control applications, LMIs do not naturally arise in the canonical form (1-1), but rather in the form

$$L(X_1, \ldots, X_n) < R(X_1, \ldots, X_n)$$

where $L(\cdot)$ and $R(\cdot)$ are affine functions of some structured *matrix* variables X_1, $\ldots$, X_n. A simple example is the Lyapunov inequality

$$A^T X + XA < 0 \qquad \textbf{(3-1)}$$

where the unknown X is a symmetric matrix. Defining $x_1, \ldots, x_N$ as the independent scalar entries of X, this LMI could be rewritten in the form (1-1). Yet it is more convenient and efficient to describe it in its natural form (1-2), which is the approach taken in the LMI Lab.

Three Generic LMI Problems

Finding a solution x to the LMI system

$$A(x) < 0 \qquad \textbf{(3-2)}$$

is called the feasibility problem. Minimizing a convex objective under LMI constraints is also a convex problem. In particular, the *linear objective minimization problem*

$$\text{Minimize } c^T x \text{ subject to } A(x) < 0 \qquad \textbf{(3-3)}$$

plays an important role in LMI-based design. Finally, the *generalized eigenvalue minimization problem*

$$\text{Minimize } \lambda \text{ subject to } \begin{cases} A(x) < \lambda B(x) \\ B(x) > 0 \\ C(x) < 0 \end{cases} \qquad \textbf{(3-4)}$$

is quasi-convex and can be solved by similar techniques. It owes its name to the fact that is related to the largest generalized eigenvalue of the pencil $(A(x),B(x))$.

Many control problems and design specifications have LMI formulations [9]. This is especially true for Lyapunov-based analysis and design, but also for optimal LQG control, H_∞ control, covariance control, etc. Further applications of LMIs arise in estimation, identification, optimal design, structural design

[6], [7], matrix scaling problems, and so on. The main strength of LMI formulations is the ability to combine various design constraints or objectives in a numerically tractable manner.

A nonexhaustive list of problems addressed by LMI techniques includes the following:

- Robust stability of systems with LTI uncertainty (μ-analysis) ([24], [21], [27])
- Robust stability in the face of sector-bounded nonlinearities (Popov criterion) ([22], [28], [13], [16])
- Quadratic stability of differential inclusions ([15], [8])
- Lyapunov stability of parameter-dependent systems ([12])
- Input/state/output properties of LTI systems (invariant ellipsoids, decay rate, etc.) ([9])
- Multi-model/multi-objective state feedback design ([4], [17], [3], [9], [10])
- Robust pole placement
- Optimal LQG control ([9])
- Robust H_∞ control ([11], [14])
- Multi-objective H_∞ synthesis ([18], [23], [10], [18])
- Design of robust gain-scheduled controllers ([5], [2])
- Control of stochastic systems ([9])
- Weighted interpolation problems ([9])

To hint at the principles underlying LMI design, let's review the LMI formulations of a few typical design objectives.

Stability

The stability of the dynamic system

$$\dot{x} = Ax$$

is equivalent to the feasibility of

$$\text{Find } P = P^T \text{ such that } A^T P + P A < 0, P > I.$$

This can be generalized to linear differential inclusions (LDI)

$$\dot{x} = A(t)x$$

where $A(t)$ varies in the convex envelope of a set of LTI models:

$$A(t) \in \mathrm{Co}\{A_1,\ldots,A_n\} = \left\{ \sum_{i=1}^{n} a_i A_i : a_i \geq 0,\ \sum_{i=1}^{N} a_i = 1 \right\}$$

A sufficient condition for the asymptotic stability of this LDI is the feasibility of

$$\text{Find } P = P^T \text{ such that } A_i^T P + P A_i < 0,\ \ P > I.$$

RMS Gain

The random-mean-squares (RMS) gain of a stable LTI system

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

is the largest input/output gain over all bounded inputs $u(t)$. This gain is the global minimum of the following linear objective minimization problem [1], [25], [26].

Minimize γ over $X = X^T$ and γ such that

$$\begin{pmatrix} A^T X + XA & XB & C^T \\ B^T X & -\gamma I & D^T \\ C & D & -\gamma I \end{pmatrix} < 0$$

$$X > 0$$

LQG Performance

For a stable LTI system

$$G\begin{cases} \dot{x} = Ax + Bw \\ y = Cx \end{cases}$$

where w is a white noise disturbance with unit covariance, the LQG or H_2 performance $\|G\|_2$ is defined by

$$\|G\|_2^2 := \lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T y^T(t)y(t)dt\right\}$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} G^H(j\omega)G(j\omega)d\omega$$

It can be shown that

$$\|G\|_2^2 = \inf\{\text{Trace}\,(CPC^T) : AP + PA^T + BB^T < 0\}$$

Hence $\|G\|_2^2$ is the global minimum of the LMI problem. Minimize Trace (Q) over the symmetric matrices P,Q such that

$$AP + PA^T + BB^T < 0$$
$$\begin{pmatrix} Q & CP \\ PC^T & P \end{pmatrix} > 0$$

Again this is a linear objective minimization problem since the objective Trace (Q) is linear in the decision variables (free entries of P,Q).

Further Mathematical Background

Efficient interior-point algorithms are now available to solve the three generic LMI problems (8-2)–(8-4) defined in "Three Generic LMI Problems" on page 3-5. These algorithms have a polynomial-time complexity. That is, the number $N(\varepsilon)$ of flops needed to compute an ε-accurate solution is bounded by

$$M\,N^3\log(V/\varepsilon)$$

where M is the total row size of the LMI system, N is the total number of scalar decision variables, and V is a data-dependent scaling factor. Robust Control Toolbox™ software implements the Projective Algorithm of Nesterov and Nemirovski [20], [19]. In addition to its polynomial-time complexity, this algorithm does not require an initial feasible point for the linear objective minimization problem (8-3) or the generalized eigenvalue minimization problem (8-4).

Some LMI problems are formulated in terms of inequalities rather than strict inequalities. For instance, a variant of (8-3) is

$$\text{Minimize } c^Tx \text{ subject to } A(x) < 0.$$

While this distinction is immaterial in general, it matters when $A(x)$ can be made negative semi-definite but not negative definite. A simple example is

$$\text{Minimize } c^Tx \text{ subject to } \begin{pmatrix} x & x \\ x & x \end{pmatrix} \geq 0 \qquad \textbf{(3-5)}$$

Such problems cannot be handled directly by interior-point methods which require strict feasibility of the LMI constraints. A well-posed reformulation of (8-5) would be

$$\text{Minimize } c^Tx \text{ subject to } x \geq 0.$$

Keeping this subtlety in mind, we always use strict inequalities in this manual.

References

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4

The LMI Lab

Introduction

The LMI Lab is a high-performance package for solving general LMI problems. It blends simple tools for the specification and manipulation of LMIs with powerful LMI solvers for three generic LMI problems. Thanks to a structure-oriented representation of LMIs, the various LMI constraints can be described in their natural block-matrix form. Similarly, the optimization variables are specified directly as *matrix variables* with some given structure. Once an LMI problem is specified, it can be solved numerically by calling the appropriate LMI solver. The three solvers `feasp`, `mincx`, and `gevp` constitute the computational engine of the LMI portion of Robust Control Toolbox™ software. Their high performance is achieved through C-MEX implementation and by taking advantage of the particular structure of each LMI.

The LMI Lab offers tools to

- Specify LMI systems either symbolically with the LMI Editor or incrementally with the `lmivar` and `lmiterm` commands
- Retrieve information about existing systems of LMIs
- Modify existing systems of LMIs
- Solve the three generic LMI problems (feasibility problem, linear objective minimization, and generalized eigenvalue minimization)
- Validate results

This chapter gives a tutorial introduction to the LMI Lab as well as more advanced tips for making the most out of its potential. The tutorial material is also covered by the demo `lmidem`.

Some Terminology

Any linear matrix inequality can be expressed in the canonical form

$$L(x) = L_0 + x_1 L_1 + \ldots + x_N L_N < 0$$

where

- $L_0, L_1, \ldots, L_N$ are given symmetric matrices
- $x = (x_1, \ldots, x_N)^T \in \mathbf{R}^N$ is the vector of scalar variables to be determined. We refer to $x_1, \ldots, x_N$ as the *decision variables*. The names “design variables” and “optimization variables” are also found in the literature.

Even though this canonical expression is generic, LMIs rarely arise in this form in control applications. Consider for instance the Lyapunov inequality

$$A^T X + XA < 0 \qquad \textbf{(4-1)}$$

where $A = \begin{pmatrix} -1 & 2 \\ 0 & -2 \end{pmatrix}$ and the variable $X = \begin{pmatrix} x_1 & x_2 \\ x_2 & x_3 \end{pmatrix}$ is a symmetric matrix.

Here the decision variables are the free entries x_1, x_2, x_3 of X and the canonical form of this LMI reads

$$x_1 \begin{pmatrix} -2 & 2 \\ 2 & 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 & -3 \\ -3 & 4 \end{pmatrix} + x_3 \begin{pmatrix} 0 & 0 \\ 0 & -4 \end{pmatrix} < 0 \qquad \textbf{(4-2)}$$

Clearly this expression is less intuitive and transparent than (8-1). Moreover, the number of matrices involved in (8-2) grows roughly as $n^2/2$ if n is the size of the A matrix. Hence, the canonical form is very inefficient from a storage viewpoint since it requires storing $o(n^2/2)$ matrices of size n when the single n-by-n matrix A would be sufficient. Finally, working with the canonical form is also detrimental to the efficiency of the LMI solvers. For these various reasons, the LMI Lab uses a *structured representation* of LMIs. For instance, the expression $A^TX + XA$ in the Lyapunov inequality (8-1) is explicitly described as a function of the matrix variable X, and only the A matrix is stored.

In general, LMIs assume a block matrix form where each block is an affine combination of the matrix variables. As a fairly typical illustration, consider the following LMI drawn from H_∞ theory

$$N^T \begin{pmatrix} A^TX + XA & XC^T & B \\ CX & -\gamma I & D \\ B^T & D^T & -\gamma I \end{pmatrix} N < 0 \qquad \textbf{(4-3)}$$

where A, B, C, D, N are given matrices and the problem variables are $X = X^T \in \mathbf{R}^{n \times n}$ and $\gamma \in \mathbf{R}$. We use the following terminology to describe such LMIs:

- N is called the *outer factor*, and the block matrix

$$L(X,\gamma) = \begin{pmatrix} A^TX + XA & XC^T & B \\ CX & -\gamma I & D \\ B^T & D^T & -\gamma I \end{pmatrix}$$

  is called the *inner factor*. The outer factor *needs not be square* and is *often absent*.
- X and γ are the *matrix variables* of the problem. Note that scalars are considered as 1-by-1 matrices.
- The inner factor $L(X, \gamma)$ is a symmetric *block matrix*, its block structure being characterized by the sizes of its diagonal blocks. By symmetry, $L(X, \gamma)$ is entirely specified by the blocks on or above the diagonal.
- Each block of $L(X, \gamma)$ is an affine expression in the matrix variables X and γ. This expression can be broken down into a sum of elementary *terms*. For instance, the block (1,1) contains two elementary terms: A^TX and XA.
- Terms are either *constant* or *variable*. Constant terms are fixed matrices like B and D above. Variable terms involve one of the matrix variables, like XA, XC^T, and $-\gamma I$ above.

The LMI (Equation (4-3)) is specified by the list of terms in each block, as is any LMI regardless of its complexity.

As for the matrix variables X and γ, they are characterized by their dimensions and structure. Common structures include rectangular unstructured, symmetric, skew-symmetric, and scalar. More sophisticated structures are sometimes encountered in control problems. For instance, the matrix variable X could be constrained to the block-diagonal structure

$$X = \left(\begin{array}{c|cc} x_1 & 0 & 0 \\ \hline 0 & x_2 & x_3 \\ 0 & x_3 & x_4 \end{array}\right)$$

Another possibility is the symmetric Toeplitz structure

$$X = \begin{pmatrix} x_1 & x_2 & x_3 \\ x_2 & x_1 & x_2 \\ x_3 & x_2 & x_1 \end{pmatrix}$$

Summing up, structured LMI problems are specified by declaring the matrix variables and describing the term content of each LMI. This term-oriented description is systematic and accurately reflects the specific structure of the LMI constraints. There is no built-in limitation on the number of LMIs that you can specify or on the number of blocks and terms in any given LMI. LMI systems of arbitrary complexity can therefore, be defined in the LMI Lab.

Overview of the LMI Lab

The LMI Lab offers tools to specify, manipulate, and numerically solve LMIs. Its main purpose is to

- Allow for straightforward description of LMIs in their natural block-matrix form
- Provide easy access to the LMI solvers (optimization codes)
- Facilitate result validation and problem modification

The structure-oriented description of a given LMI system is stored as a single vector called the *internal representation* and generically denoted by `LMISYS` in the sequel. This vector encodes the structure and dimensions of the LMIs and matrix variables, a description of all LMI terms, and the related numerical data. It must be stressed that you need not attempt to read or understand the content of `LMISYS` since all manipulations involving this internal representation can be performed in a transparent manner with LMI-Lab tools.

The LMI Lab supports the following functionalities:

Specification of a System of LMIs

LMI systems can be either specified as symbolic matrix expressions with the interactive graphical user interface `lmiedit`, or assembled incrementally with the two commands `lmivar` and `lmiterm`. The first option is more intuitive and transparent while the second option is more powerful and flexible.

Information Retrieval

The interactive function `lmiinfo` answers qualitative queries about LMI systems created with `lmiedit` or `lmivar` and `lmiterm`. You can also use `lmiedit` to visualize the LMI system produced by a particular sequence of `lmivar`/`lmiterm` commands.

Solvers for LMI Optimization Problems

General-purpose LMI solvers are provided for the three generic LMI problems defined in "Three Generic LMI Problems" on page 3-5. These solvers can handle very general LMI systems and matrix variable structures. They return a feasible or optimal vector of decision variables x^*. The corresponding values $X_1^*, \ldots, X_K^*$ of the matrix variables are given by the function `dec2mat`.

Result Validation

The solution x^* produced by the LMI solvers is easily validated with the functions `evallmi` and `showlmi`. This allows a fast check and/or analysis of the results. With `evallmi`, all variable terms in the LMI system are evaluated for the value x^* of the decision variables. The left- and right-hand sides of each LMI then become constant matrices that can be displayed with `showlmi`.

Modification of a System of LMIs

An existing system of LMIs can be modified in two ways:

- An LMI can be removed from the system with `dellmi`.
- A matrix variable X can be deleted using `delmvar`. It can also be instantiated, that is, set to some given matrix value. This operation is performed by `setmvar` and allows, for example, to fix some variables and solve the LMI problem with respect to the remaining ones.

Specifying a System of LMIs

The LMI Lab can handle any system of LMIs of the form

$$N^T L(X_1, \ldots, X_K)\, N < MT\, R(X_1, \ldots, X_K)\, M$$

where

- $X_1, \ldots, X_K$ are matrix variables with some prescribed structure
- The left and right outer factors N and M are given matrices with *identical* dimensions
- The left and right inner factors $L(\cdot)$ and $R(\cdot)$ are symmetric block matrices with identical block structures, each block being an affine combination of X_1, $\ldots$, X_K and their transposes.

Note Throughout this chapter, "left-hand side" refers to what is on the "smaller" side of the inequality, and "right-hand side" to what is on the "larger" side. Accordingly, X is called the right-hand side and 0 the left-hand side of the LMI

$$0 < X$$

even when this LMI is written as $X > 0$.

The specification of an LMI system involves two steps:

1 Declare the dimensions and structure of each matrix variable $X_1, \ldots, X_K$.

2 Describe the term content of each LMI.

This process creates the so-called *internal representation* of the LMI system. This computer description of the problem is used by the LMI solvers and in all subsequent manipulations of the LMI system. It is stored as a single vector called `LMISYS`.

There are two ways of generating the internal description of a given LMI system: (1) by a sequence of `lmivar`/`lmiterm` commands that build it incrementally, or (2) via the LMI Editor `lmiedit` where LMIs can be specified directly as symbolic matrix expressions. Though somewhat less flexible and powerful than the command-based description, the LMI Editor is more straightforward to use, hence particularly well-suited for beginners. Thanks to

its coding and decoding capabilities, it also constitutes a good tutorial introduction to `lmivar` and `lmiterm`. Accordingly, beginners may elect to skip the subsections on `lmivar` and `lmiterm` and to concentrate on the GUI-based specification of LMIs with `lmiedit`.

A Simple Example

The following tutorial example is used to illustrate the specification of LMI systems with the LMI Lab tools. Run the demo `lmidem` to see a complete treatment of this example.

Example 9.1

Consider a stable transfer function

$$G(s) = C(sI - A)^{-1}B \tag{4-4}$$

with four inputs, four outputs, and six states, and consider the set of input/output scaling matrices D with block-diagonal structure

$$D = \begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_1 & 0 & 0 \\ 0 & 0 & d_2 & d_3 \\ 0 & 0 & d_4 & d_5 \end{pmatrix} \tag{4-5}$$

The following problem arises in the robust stability analysis of systems with time-varying uncertainty [4]:

> Find, if any, a scaling D of structure (Equation (4-5)) such that the largest gain across frequency of $D\,G(s)\,D^{-1}$ is less than one.

This problem has a simple LMI formulation: there exists an adequate scaling D if the following feasibility problem has solutions:

> Find two symmetric matrices $X \in \mathbf{R}^{6\times 6}$ and $S = \mathrm{D}^{\mathrm{T}}\,D \in \mathbf{R}^{4\times 4}$ such that

$$\begin{pmatrix} A^TX + XA + C^TSC & XB \\ B^TX & -S \end{pmatrix} < 0 \tag{4-6}$$

$$X > 0 \tag{4-7}$$

$$S > 1 \tag{4-8}$$

The LMI system (Equation (4-6), Equation (4-7), and Equation (4-8)) can be described with the LMI Editor as outlined below. Alternatively, its internal description can be generated with `lmivar` and `lmiterm` commands as follows:

```
setlmis([])
X=lmivar(1,[6 1])
S=lmivar(1,[2 0;2 1])

% 1st LMI
lmiterm([1 1 1 X],1,A,'s')
lmiterm([1 1 1 S],C',C)
lmiterm([1 1 2 X],1,B)
lmiterm([1 2 2 S], 1,1)

% 2nd LMI
lmiterm([ 2 1 1 X],1,1)

% 3rd LMI
lmiterm([ 3 1 1 S],1,1)
lmiterm([3 1 1 0],1)

LMISYS = getlmis
```

Here the `lmivar` commands define the two matrix variables X and S while the `lmiterm` commands describe the various terms in each LMI. Upon completion, `getlmis` returns the internal representation `LMISYS` of this LMI system. The following subsections give more details on the syntax and usage of these various commands:

- "Initializing the LMI System" on page 4-10
- "Specifying the LMI Variables" on page 4-10
- "Specifying Individual LMIs" on page 4-13

More information on how the internal representation is updated by `lmivar`/`lmiterm` can also be found in "How It All Works" on page 4-19.

Initializing the LMI System

The description of an LMI system should begin with `setlmis` and end with `getlmis`. The function `setlmis` initializes the LMI system description. When specifying a new system, type

```
setlmis([])
```

To add on to an existing LMI system with internal representation `LMISO`, type

```
setlmis(LMISO)
```

Specifying the LMI Variables

The matrix variables are declared one at a time with `lmivar` and are characterized by their structure. To facilitate the specification of this structure,

the LMI Lab offers two predefined structure types along with the means to describe more general structures:

Type 1 Symmetric block diagonal structure. This corresponds to matrix variables of the form

$$X = \begin{pmatrix} D_1 & 0 & \dots & 0 \\ 0 & D_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & D_r \end{pmatrix}$$

where each diagonal block D_j is square and is either zero, a *full symmetric* matrix, or a *scalar* matrix

$$D_j = d \times I, \ d \in \mathbf{R}$$

This type encompasses ordinary symmetric matrices (single block) and scalar variables (one block of size one).

Type 2 **Rectangular** structure. This corresponds to arbitrary rectangular matrices without any particular structure.

Type 3 **General** structures. This third type is used to describe more sophisticated structures and/or correlations between the matrix variables. The principle is as follows: each entry of X is specified independently as either 0, x_n, or $-x_n$ where x_n denotes the n-th decision variable in the problem. For details on how to use Type 3, see "Structured Matrix Variables" on page 4-32 as well as the `lmivar` entry in the reference pages.

In "Example 9.1" on page 4-8, the matrix variables X and S are of Type 1. Indeed, both are symmetric and S inherits the block-diagonal structure from Equation (4-5) of D. Specifically, S is of the form

$$S = \begin{pmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_1 & 0 & 0 \\ 0 & 0 & s_2 & s_3 \\ 0 & 0 & s_3 & s_4 \end{pmatrix}$$

After initializing the description with the command `setlmis([])`, these two matrix variables are declared by

```
lmivar(1,[6 1]) % X
lmivar(1,[2 0;2 1]) % S
```

In both commands, the first input specifies the structure type and the second input contains additional information about the structure of the variable:

- For a matrix variable X of Type 1, this second input is a matrix with two columns and as many rows as diagonal blocks in X. The first column lists the sizes of the diagonal blocks and the second column specifies their nature with the following convention:

  $1 \rightarrow$ full symmetric block

  $0 \rightarrow$ scalar block

  $-1 \rightarrow$ zero block

  In the second command, for instance, `[2 0;2 1]` means that S has two diagonal blocks, the first one being a 2-by-2 scalar block and the second one a 2–βψ–2 full block.
- For matrix variables of Type 2, the second input of `lmivar` is a two-entry vector listing the row and column dimensions of the variable. For instance, a 3-by-5 rectangular matrix variable would be defined by

  ```
  lmivar(2,[3 5])
  ```

For convenience, `lmivar` also returns a "tag" that identifies the matrix variable for subsequent reference. For instance, X and S in "Example 9.1" could be defined by

```
X = lmivar(1,[6 1])
S = lmivar(1,[2 0;2 1])
```

The identifiers X and S are integers corresponding to the ranking of X and S in the list of matrix variables (in the order of declaration). Here their values would be `X=1` and `S=2`. Note that these identifiers still point to X and S after deletion or instantiation of some of the matrix variables. Finally, `lmivar` can also return the total number of decision variables allocated so far as well as the entry-wise dependence of the matrix variable on these decision variables (see the `lmivar` entry in the reference pages for more details).

Specifying Individual LMIs

After declaring the matrix variables with `lmivar`, we are left with specifying the term content of each LMI. Recall that LMI terms fall into three categories:

- The *constant terms*, i.e., fixed matrices like I in the left-hand side of the LMI $S > I$
- The *variable terms*, i.e., terms involving a matrix variable. For instance, A^TX and C^TSC in (8-6). Variable terms are of the form PXQ where X is a variable and P, Q are given matrices called the left and right *coefficients*, respectively.
- The *outer factors*

The following rule should be kept in mind when describing the term content of an LMI:

Important: Specify only the terms in the blocks on or above the diagonal. The inner factors being symmetric, this is sufficient to specify the entire LMI. *Specifying all blocks results in the duplication of off-diagonal terms, hence in the creation of a different LMI.* Alternatively, you can describe the blocks on or below the diagonal.

LMI terms are specified one at a time with `lmiterm`. For instance, the LMI

$$\begin{pmatrix} A^TX + XA + C^TSC & XB \\ B^TX & -S \end{pmatrix} < 0$$

is described by

```
lmiterm([1 1 1 1],1,A,'s')
```

```
lmiterm([1 1 1 2],C',C)
lmiterm([1 1 2 1],1,B)
lmiterm([1 2 2 2], 1,1)
```

These commands successively declare the terms $A^TX + XA$, C^TSC, XB, and $-S$. In each command, the first argument is a four-entry vector listing the term characteristics as follows:

- The first entry indicates to which LMI the term belongs. The value `m` means "left-hand side of the m-th LMI," and `m` means "right-hand side of the m-th LMI".
- The second and third entries identify the block to which the term belongs. For instance, the vector `[1 1 2 1]` indicates that the term is attached to the (1, 2) block.
- The last entry indicates which matrix variable is involved in the term. This entry is `0` for constant terms, `k` for terms involving the k-th matrix variable X_k, and `k` for terms involving X_k^T (here X and S are first and second variables in the order of declaration).

Finally, the second and third arguments of `lmiterm` contain the numerical data (values of the constant term, outer factor, or matrix coefficients P and Q for variable terms PXQ or PX^TQ). These arguments must refer to existing MATLAB® variables and be *real-valued*. See "Complex-Valued LMIs" on page 4-34 for the specification of LMIs with complex-valued coefficients.

Some shorthand is provided to simplify term specification. First, blocks are zero by default. Second, in *diagonal blocks* the extra argument `'s'` allows you to specify the conjugated expression $AXB + B^TX^TA^T$ with a *single* `lmiterm` command. For instance, the first command specifies $A^TX + XA$ as the "symmetrization" of XA. Finally, scalar values are allowed as shorthand for s*calar matrices*, i.e., matrices of the form αI with α scalar. Thus, a constant term of the form αI can be specified as the "scalar" α. This also applies to the coefficients P and Q of variable terms. The dimensions of scalar matrices are inferred from the context and set to 1 by default. For instance, the third LMI $S > I$ in "Example 9.3" on page 4-32 is described by

```
lmiterm([ 3 1 1 2],1,1)      % 1*S*1 = S
lmiterm([3 1 1 0],1)         % 1*I = I
```

Recall that by convention `S` is considered as the right-hand side of the inequality, which justifies the –3 in the first command.

Finally, to improve readability it is often convenient to attach an identifier (tag) to each LMI and matrix variable. The variable identifiers are returned by `lmivar` and the LMI identifiers are set by the function `newlmi`. These identifiers can be used in `lmiterm` commands to refer to a given LMI or matrix variable. For the LMI system of "Example 9.1", this would look like:

```
setlmis([])
X = lmivar(1,[6 1])
S = lmivar(1,[2 0;2 1])

BRL = newlmi
lmiterm([BRL 1 1 X],1,A,'s')
lmiterm([BRL 1 1 S],C',C)
lmiterm([BRL 1 2 X],1,B)
lmiterm([BRL 2 2 S], 1,1)

Xpos = newlmi
lmiterm([-Xpos 1 1 X],1,1)

Slmi = newlmi
lmiterm([-Slmi 1 1 S],1,1)
lmiterm([Slmi 1 1 0],1)
```

When the LMI system is completely specified, type

```
LMISYS = getlmis
```

This returns the internal representation `LMISYS` of this LMI system. This MATLAB description of the problem can be forwarded to other LMI-Lab functions for subsequent processing. The command `getlmis` must be used *only once* and after declaring *all* matrix variables and LMI terms.

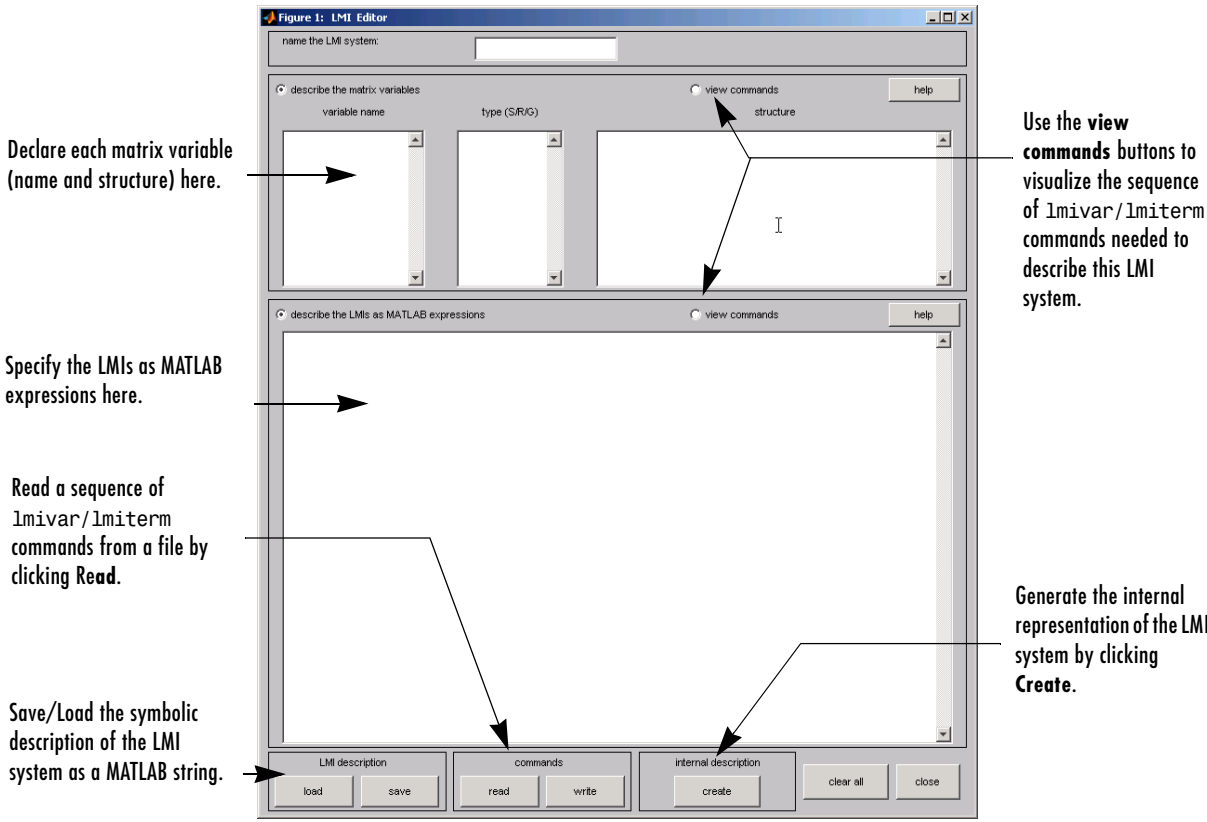
Here the identifiers `X` and `S` point to the variables X and S while the tags `BRL`, `Xpos`, and `Slmi` point to the first, second, and third LMI, respectively. Note that `Xpos` refers to the right-hand side of the second LMI. Similarly, `X'` would indicate transposition of the variable X.

Specifying LMIs with the LMI Editor

The LMI Editor `lmiedit` is a graphical user interface (GUI) to specify LMI systems in a straightforward symbolic manner. Typing

```
lmiedit
```

calls up a window with several editable text areas and various buttons.



In more detail, to specify your LMI system,

1. Declare each matrix variable (name and structure) in the upper half of the worksheet. The structure is characterized by its type (`S` for symmetric block diagonal, `R` for unstructured, and `G` for other structures) and by an additional "structure" matrix. This matrix contains specific information about the structure and corresponds to the second argument of `lmivar` (see "Specifying the LMI Variables" on page 4-10 for details).

   Please use *one line per matrix variable* in the text editing areas.

2 Specify the LMIs as MATLAB expressions in the lower half of the worksheet. For instance, the LMI

$$\begin{pmatrix} A^T X + XA & XB \\ B^T X & -I \end{pmatrix} < 0$$

is entered by typing

```
[a'*x+x*a x*b; b'*x  1] < 0
```

if `x` is the name given to the matrix variable X in the upper half of the worksheet. The left- and right-hand sides of the LMIs should be *valid* MATLAB expressions.

Once the LMI system is fully specified, the following tasks can be performed by clicking the corresponding button:

- Visualize the sequence of `lmivar`/`lmiterm` commands needed to describe this LMI system (`view commands` button). Conversely, the LMI system defined by a particular sequence of `lmivar`/`lmiterm` commands can be displayed as a MATLAB expression by clicking on the **describe...** buttons.

  Beginners can use this facility as a tutorial introduction to the `lmivar` and `lmiterm` commands.
- Save the symbolic description of the LMI system as a MATLAB string (`save` button). This description can be reloaded later on by clicking the **load** button.
- Read a sequence of `lmivar`/`lmiterm` commands from a file (**read** button). You can then click on **describe the matrix variables** or **describe the LMIs** to visualize the symbolic expression of the LMI system specified by these commands. The file should describe a single LMI system but may otherwise contain any sequence of MATLAB commands.

  This feature is useful for code validation and debugging.

  Write in a file the sequence of `lmivar`/`lmiterm` commands needed to describe a particular LMI system (**write** button).

  This is helpful to develop code and prototype MATLAB functions based on the LMI Lab.

- Generate the internal representation of the LMI system by clicking **create**. The result is written in a MATLAB variable named after the LMI system (if the "name of the LMI system" is set to `mylmi`, the internal representation is written in the MATLAB variable `mylmi`). Note that all LMI-related data should be defined in the MATLAB workspace at this stage.

  The internal representation can be passed directly to the LMI solvers or any other LMI Lab function.

Keyboard Shortcuts

As with `lmiterm`, you can use various shortcuts when entering LMI expressions at the keyboard. For instance, zero blocks can be entered simply as `0` and need not be dimensioned. Similarly, the identity matrix can be entered as `1` without dimensioning. Finally, *upper diagonal* LMI blocks need not be fully specified. Rather, you can just type `(*)` in place of each such block.

Limitations

Though fairly general, `lmiedit` is not as flexible as `lmiterm` and the following limitations should be kept in mind:

- Parentheses cannot be used around matrix variables. For instance, the expression

  ```
  (a*x+b)'*c + c'*(a*x+b)
  ```

  is invalid when x is a variable name. By contrast,

  ```
  (a+b)'*x + x'*(a+b)
  ```

  is perfectly valid.
- Loops and `if` statements are ignored.
- When turning `lmiterm` commands into a symbolic description of the LMI system, an error is issued if the first argument of `lmiterm` cannot be evaluated. Use the LMI and variable identifiers supplied by `newlmi` and `lmivar` to avoid such difficulties.

Figure 8-1 shows how to specify the feasibility problem of "Example 9.1" on page 4-8 with `lmiedit`.

How It All Works

Users familiar with MATLAB may wonder how `lmivar` and `lmiterm` physically update the internal representation `LMISYS` since `LMISYS` is not an argument to these functions. In fact, all updating is performed through global variables for maximum speed. These global variables are initialized by `setlmis`, cleared by `getlmis`, and are not visible in the workspace. Even though this artifact is transparent from the user's viewpoint, be sure to

- Invoke `getlmis` only once and after completely specifying the LMI system
- Refrain from using the command `clear global` before the LMI system description is ended with `getlmis`

Querying the LMI System Description

Recall that the full description of an LMI system is stored as a single vector called the internal representation. The user should not attempt to read or retrieve information directly from this vector. Robust Control Toolbox™ software provides three functions called `lmiinfo`, `lminbr`, and `matnbr` to extract and display all relevant information in a user-readable format.

lmiinfo

`lmiinfo` is an interactive facility to retrieve qualitative information about LMI systems. This includes the number of LMIs, the number of matrix variables and their structure, the term content of each LMI block, etc. To invoke `lmiinfo`, enter

```
lmiinfo(LMISYS)
```

where `LMISYS` is the internal representation of the LMI system produced by `getlmis`.

lminbr and matnbr

These two functions return the number of LMIs and the number of matrix variables in the system. To get the number of matrix variables, for instance, enter

```
matnbr(LMISYS)
```

LMI Solvers

LMI solvers are provided for the following three generic optimization problems (here x denotes the vector of decision variables, i.e., of the free entries of the matrix variables $X_1, \ldots, X_K$):

- Feasibility problem

  Find $x \in \mathbf{R}^N$ (or equivalently matrices $X_1, \ldots, X_K$ with prescribed structure) that satisfies the LMI system

  $$A(x) < B(x)$$

  The corresponding solver is called `feasp`.

- Minimization of a linear objective under LMI constraints

  Minimize $c^T x$ over $x \in \mathbf{R}^N$ subject to $A(x) < B(x)$

  The corresponding solver is called `mincx`.

- Generalized eigenvalue minimization problem

  Minimize λ over $x \in \mathbf{R}^N$ subject to

  $$C(x) < D(x)$$

  $$0 < B(x)$$

  $$A(x) < \lambda B(x).$$

  The corresponding solver is called `gevp`.

Note that $A(x) < B(x)$ above is a shorthand notation for general structured LMI systems with decision variables $x = (x_1, \ldots, x_N)$.

The three LMI solvers `feasp`, `mincx`, and `gevp` take as input the internal representation `LMISYS` of an LMI system and return a feasible or optimizing value x^* of the decision variables. The corresponding values of the matrix variables $X_1, \ldots, X_K$ are derived from x^* with the function `dec2mat`. These solvers are C-MEX implementations of the polynomial-time Projective Algorithm Projective Algorithm of Nesterov and Nemirovski [3], [2].

For generalized eigenvalue minimization problems, it is necessary to distinguish between the standard LMI constraints $C(x) < D(x)$ and the linear-fractional LMIs

$$A(x) < \lambda B(x)$$

attached to the minimization of the generalized eigenvalue λ. When using `gevp`, you should follow these three rules to ensure proper specification of the problem:

- Specify the LMIs involving λ as $A(x) < B(x)$ (*without* the λ)
- Specify them *last* in the LMI system. `gevp` systematically assumes that the last L LMIs are linear-fractional if L is the number of LMIs involving λ
- Add the constraint $0 < B(x)$ or any other constraint that enforces it. This positivity constraint is required for well-posedness of the problem and is not automatically added by `gevp` (see the reference pages for details).

An initial guess `xinit` for x can be supplied to `mincx` or `gevp`. Use `mat2dec` to derive `xinit` from given values of the matrix variables $X_1, \ldots, X_K$. Finally, various options are available to control the optimization process and the solver behavior. These options are described in detail in the reference pages.

The following example illustrates the use of the `mincx` solver.

Example 9.2

Consider the optimization problem

Minimize Trace(X) subject to

$$A^T X + XA + XBB^T X + Q < 0 \qquad \textbf{(4-9)}$$

with data

$$A = \begin{pmatrix} -1 & -2 & 1 \\ 3 & 2 & 1 \\ 1 & -2 & -1 \end{pmatrix};\ B = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix};\ Q = \begin{pmatrix} 1 & -1 & 0 \\ -1 & -3 & -12 \\ 0 & -12 & -36 \end{pmatrix}$$

It can be shown that the minimizer X^* is simply the stabilizing solution of the algebraic Riccati equation

$$A^T X + XA + XBB^T X + Q = 0$$

This solution can be computed directly with the Riccati solver `care` and compared to the minimizer returned by `mincx`.

From an LMI optimization standpoint, the problem specified in Equation (4-9) is equivalent to the following linear objective minimization problem:

$$\text{Minimize Trace(X) subject to } \begin{pmatrix} A^TX + XA + Q & XB \\ B^TX & -I \end{pmatrix} < 0 \qquad \textbf{(4-10)}$$

Since Trace(X) is a linear function of the entries of X, this problem falls within the scope of the `mincx` solver and can be numerically solved as follows:

1 Define the LMI constraint [4-9] by the sequence of commands

```
setlmis([])
X = lmivar(1,[3 1]) % variable X, full symmetric

lmiterm([1 1 1 X],1,a,'s')
lmiterm([1 1 1 0],q)
lmiterm([1 2 2 0],-1)
lmiterm([1 2 1 X],b',1)

LMIs = getlmis
```

2 Write the objective Trace(X) as c^Tx where x is the vector of free entries of X. Since c should select the diagonal entries of X, it is obtained as the decision vector corresponding to $X = I$, that is,

```
c = mat2dec(LMIs,eye(3))
```

Note that the function `defcx` provides a more systematic way of specifying such objectives (see “Specifying cTx Objectives for mincx” on page 4-37 for details).

3 Call `mincx` to compute the minimizer `xopt` and the global minimum `copt = c'*xopt` of the objective:

```
options = [1e 5,0,0,0,0]
[copt,xopt] = mincx(LMIs,c,options)
```

Here `1e 5` specifies the desired relative accuracy on `copt`.

The following trace of the iterative optimization performed by `mincx` appears on the screen:

```
Solver for linear objective minimization under LMI constraints
```

```
  Iterations : Best objective value so far
1
2               8.511476
3               13.063640
***             new lower bound:          34.023978
4               15.768450
***             new lower bound:          25.005604
5               17.123012
***             new lower bound:          21.306781
6               17.882558
***             new lower bound:          19.819471
7               18-339853
***             new lower bound:          19.189417
8               18.552558
***             new lower bound:          18.919668
9               18.646811
***             new lower bound:          18.803708
10              18.687324
***             new lower bound:          18.753903
11              18.705715
***             new lower bound:          18.732574
12              18.712175
***             new lower bound:          18.723491
13              18.714880
***             new lower bound:          18.719624
14              18.716094
***             new lower bound:          18.717986
15              18.716509
```

```
***            new lower bound:          18.717297
16              18.716695
***            new lower bound:          18.716873

  Result: feasible solution of required accuracy
     best objective value:  18.716695
     guaranteed relative accuracy: 9.50e 06
     f-radius saturation: 0.000% of R = 1.00e+09
```

The iteration number and the best value of $c^T x$ at the current iteration appear in the left and right columns, respectively. Note that no value is displayed at the first iteration, which means that a feasible x satisfying the constraint [4-10] was found only at the second iteration. Lower bounds on the global minimum of $c^T x$ are sometimes detected as the optimization progresses. These lower bounds are reported by the message

```
*** new lower bound: xxx
```

Upon termination, `mincx` reports that the global minimum for the objective Trace(X) is –18.716695 with relative accuracy of at least 9.5-by-10^{-6}. This is the value `copt` returned by `mincx`.

4 `mincx` also returns the optimizing vector of decision variables `xopt`. The corresponding optimal value of the matrix variable X is given by

```
Xopt = dec2mat(LMIs,xopt,X)
```

which returns

$$X_{\text{opt}} = \begin{pmatrix} -6.3542 & -5.8895 & 2.2046 \\ -5.8895 & -6.2855 & 2.2201 \\ 2.2046 & 2.2201 & -6.0771 \end{pmatrix}$$

This result can be compared with the stabilizing Riccati solution computed by `care`:

```
Xst = care(a,b,q, 1)
norm(Xopt-Xst)
```

```
ans =
   6.5390e 05
```

From Decision to Matrix Variables and Vice Versa

While LMIs are specified in terms of their matrix variables $X_1, \ldots, X_K$, the LMI solvers optimize the vector x of free scalar entries of these matrices, called the decision variables. The two functions `mat2dec` and `dec2mat` perform the conversion between these two descriptions of the problem variables.

Consider an LMI system with three matrix variables X_1, X_2, X_3. Given particular values `X1`, `X2`, `X3` of these variables, the corresponding value `xdec` of the vector of decision variables is returned by `mat2dec`:

```
xdec = mat2dec(LMISYS,X1,X2,X3)
```

An error is issued if the number of arguments following `LMISYS` differs from the number of matrix variables in the problem (see `matnbr`).

Conversely, given a value `xdec` of the vector of decision variables, the corresponding value of the k-th matrix is given by `dec2mat`. For instance, the value `X2` of the second matrix variable is extracted from `xdec` by

```
X2 = dec2mat(LMISYS,xdec,2)
```

The last argument indicates that the second matrix variable is requested. It could be set to the matrix variable identifier returned by `lmivar`.

The total numbers of matrix variables and decision variables are returned by `matnbr` and `decnbr`, respectively. In addition, the function `decinfo` provides precise information about the mapping between decision variables and matrix variable entries (see the function reference pages).

Validating Results

The LMI Lab offers two functions to analyze and validate the results of an LMI optimization. The function `evallmi` evaluates all variable terms in an LMI system for a given value of the vector of decision variables, for instance, the feasible or optimal vector returned by the LMI solvers. Once this evaluation is performed, the left- and right-hand sides of a particular LMI are returned by `showlmi`.

In the LMI problem considered in "Example 9.2" on page 4-22, you can verify that the minimizer `xopt` returned by `mincx` satisfies the LMI constraint [4-10] as follows:

```
evlmi = evallmi(LMIs,xopt)
[lhs,rhs] = showlmi(evlmi,1)
```

The first command evaluates the system for the value `xopt` of the decision variables, and the second command returns the left- and right-hand sides of the first (and only) LMI. The negative definiteness of this LMI is checked by

```
eig(lhs-rhs)

ans =
    2.0387e 04
    3.9333e 05
    1.8917e 07
    4.6680e+01
```

Modifying a System of LMIs

Once specified, a system of LMIs can be modified in several ways with the functions `dellmi`, `delmvar`, and `setmvar`.

Deleting an LMI

The first possibility is to remove an entire LMI from the system with `dellmi`. For instance, suppose that the LMI system of "Example 9.1" on page 4-8 is described in `LMISYS` and that we want to remove the positivity constraint on X. This is done by

```
NEWSYS = dellmi(LMISYS,2)
```

where the second argument specifies deletion of the second LMI. The resulting system of two LMIs is returned in `NEWSYS`.

The LMI identifiers (*initial* ranking of the LMI in the LMI system) are not altered by deletions. As a result, the last LMI

$$S > I$$

remains known as the third LMI even though it now ranks second in the modified system. To avoid confusion, it is safer to refer to LMIs via the identifiers returned by `newlmi`. If `BRL`, `Xpos`, and `Slmi` are the identifiers attached to the three LMIs [4-6]–[4-8], `Slmi` keeps pointing to $S > I$ even after deleting the second LMI by

```
NEWSYS = dellmi(LMISYS,Xpos)
```

Deleting a Matrix Variable

Another way of modifying an LMI system is to delete a matrix variable, that is, to remove all variable terms involving this matrix variable. This operation is performed by `delmvar`. For instance, consider the LMI

$$A^TX + XA + BW + W^TB^T + I < 0$$

with variables $X = X^T \in \mathbf{R}^{4\times4}$ and W $\in \mathbf{R}^{2\times4}$. This LMI is defined by

```
setlmis([])
X = lmivar(1,[4 1]) % X
W = lmivar(2,[2 4]) % W
```

```
lmiterm([1 1 1 X],1,A,'s')
lmiterm([1 1 1 W],B,1,'s')
lmiterm([1 1 1 0],1)

LMISYS = getlmis
```

To delete the variable `W`, type the command

```
NEWSYS = delmvar(LMISYS,W)
```

The resulting `NEWSYS` now describes the Lyapunov inequality

$$A^TX + XA + I < 0$$

Note that `delmvar` automatically removes all LMIs that depended only on the deleted matrix variable.

The matrix variable identifiers are not affected by deletions and continue to point to the same matrix variable. For subsequent manipulations, it is therefore advisable to refer to the remaining variables through their identifier. Finally, note that deleting a matrix variable is equivalent to setting it to the zero matrix of the same dimensions with `setmvar`.

Instantiating a Matrix Variable

The function `setmvar` is used to set a matrix variable to some given value. As a result, this variable is removed from the problem and all terms involving it become constant terms. This is useful, for instance, to fix`setmvar` some variables and optimize with respect to the remaining ones.

Consider again “Example 9.1” on page 4-8 and suppose we want to know if the peak gain of G itself is less than one, that is, if

$$\|G\|_\infty < 1$$

This amounts to setting the scaling matrix D (or equivalently, $S = D^TD$) to a multiple of the identity matrix. Keeping in mind the constraint $S > I$, a legitimate choice is $S = 2–\beta\psi–I$. To set S to this value, enter

```
NEWSYS = setmvar(LMISYS,S,2)
```

The second argument is the variable identifier `S`, and the third argument is the value to which S should be set. Here the value 2 is shorthand for 2–by–I. The resulting system `NEWSYS` reads

$$\begin{pmatrix} A^TX + XA + 2C^TC & XB \\ B^TX & -2I \end{pmatrix} < 0$$

$$X > 0$$

$$2I > I$$

Note that the last LMI is now free of variable and trivially satisfied. It could, therefore, be deleted by

```
NEWSYS = dellmi(NEWSYS,3)
```

or

```
NEWSYS = dellmi(NEWSYS,Slmi)
```

if `Slmi` is the identifier returned by `newlmi`.

Advanced Topics

This last section gives a few hints for making the most out of the LMI Lab. It is directed toward users who are comfortable with the basics described above.

Structured Matrix Variables

Fairly complex matrix variable structures and interdependencies can be specified with `lmivar`. Recall that the symmetric block-diagonal or rectangular structures are covered by Types 1 and 2 of `lmivar` provided that the matrix variables are independent. To describe more complex structures or correlations between variables, you must use Type 3 and specify each entry of the matrix variables directly in terms of the free scalar variables of the problem (the so-called decision variables).

With Type 3, each entry is specified as either 0 or $\pm x_n$ where x_n is the n-th decision variable. The following examples illustrate how to specify nontrivial matrix variable structures with `lmivar`. We first consider the case of uncorrelated matrix variables.

Example 9.3

Suppose that the problem variables include a 3-by-3 symmetric matrix X and a 3-by-3 symmetric Toeplitz matrix

$$Y = \begin{pmatrix} y_1 & y_2 & y_3 \\ y_2 & y_1 & y_2 \\ y_3 & y_2 & y_1 \end{pmatrix}$$

The variable Y has three independent entries, hence involves three decision variables. Since Y is independent of X, these decision variables should be labeled $n + 1$, $n + 2$, $n + 3$ where n is the number of decision variables involved in X. To retrieve this number, define the variable X (Type 1) by

```
setlmis([])
[X,n] = lmivar(1,[3 1])
```

The second output argument `n` gives the total number of decision variables used so far (here `n = 6`). Given this number, Y can be defined by

```
Y = lmivar(3,n+[1 2 3;2 1 2;3 2 1])
```

or equivalently by

```
Y = lmivar(3,toeplitz(n+[1 2 3]))
```

where `toeplitz` is a standard MATLAB® function. For verification purposes, we can visualize the decision variable distributions in X and Y with `decinfo`:

```
lmis = getlmis
decinfo(lmis,X)

ans =
   1   2   4
   2   3   5
   4   5   6

decinfo(lmis,Y)

ans =
   7   8   9
   8   7   8
   9   8   7
```

The next example is a problem with interdependent matrix variables.

Example 9.4

Consider three matrix variables X, Y, Z with structure

$$X = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}, \quad Y = \begin{pmatrix} z & 0 \\ 0 & t \end{pmatrix}, \quad Z = \begin{pmatrix} 0 & -x \\ -t & 0 \end{pmatrix}$$

where x, y, z, t are independent scalar variables. To specify such a triple, first define the two independent variables X and Y (both of Type 1) as follows:

```
setlmis([])
[X,n,sX] = lmivar(1,[1 0;1 0])
[Y,n,sY] = lmivar(1,[1 0;1 0])
```

The third output of `lmivar` gives the entry-wise dependence of X and Y on the decision variables $(x_1, x_2, x_3, x_4) := (x, y, z, t)$:

```
sX =
   1   0
   0   2
```

```
sY =
    3    0
    0    4
```

Using Type 3 of `lmivar`, you can now specify the structure of `Z` in terms of the decision variables $x_1 = x$ and $x_4 = t$:

```
[Z,n,sZ] = lmivar(3,[0  sX(1,1);-sY(2,2) 0])
```

Since `sX(1,1)` refers to x_1 while `sY(2,2)` refers to x_4, this defines the variable

$$Z = \begin{pmatrix} 0 & -x_1 \\ -x_4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -x \\ -t & 0 \end{pmatrix}$$

as confirmed by checking its entry-wise dependence on the decision variables:

```
sZ =
    0     1
     4    0
```

Complex-Valued LMIs

The LMI solvers are written for real-valued matrices and cannot directly handle LMI problems involving complex-valued matrices. However, complex-valued LMIs can be turned into real-valued LMIs by observing that a complex Hermitian matrix $L(x)$ satisfies

$$L(x) < 0$$

if and only if

$$\begin{pmatrix} \mathrm{Re}(L(x)) & \mathrm{Im}(L(x)) \\ -\mathrm{Im}(L(x)) & \mathrm{Re}(L(x)) \end{pmatrix} < 0$$

This suggests the following systematic procedure for turning complex LMIs into real ones:

- Decompose every complex matrix variable X as

$X = X_1 + jX_2$

where X_1 and X_2 are real

- Decompose every complex matrix coefficient A as

  $A = A_1 + jA_2$

  where A_1 and A_2 are real
- Carry out all complex matrix products. This yields affine expressions in X_1, X_2 for the real and imaginary parts of each LMI, and an equivalent real-valued LMI is readily derived from the above observation.

For LMIs without outer factor, a streamlined version of this procedure consists of replacing any occurrence of the matrix variable $X = X_1 + jX_2$ by

$$\begin{pmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{pmatrix}$$

and any fixed matrix $A = A_1 + jA_2$, including real ones, by

$$\begin{pmatrix} A_1 & A_2 \\ -A_2 & A_1 \end{pmatrix}$$

For instance, the real counterpart of the LMI system

$$M^H X M < X, \qquad X = X^H > I \tag{4-11}$$

reads (given the decompositions $M = M_1 + jM_2$ and $X = X_1 + jX_2$ with M_j, X_j real):

$$\begin{pmatrix} M_1 & M_2 \\ -M_2 & M_1 \end{pmatrix}^T \begin{pmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{pmatrix} \begin{pmatrix} M_1 & M_2 \\ -M_2 & M_1 \end{pmatrix} < \begin{pmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{pmatrix}$$

$$\begin{pmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{pmatrix} < I$$

Note that $X = X^H$ in turn requires that $X_1 = X_1^H$ and $X_2 + X_2^T = 0$. Consequently, X_1 and X_2 should be declared as symmetric and skew-symmetric matrix variables, respectively.

Assuming, for instance, that $M \in \mathbf{C}^{5\times 5}$, the LMI system [4-11] would be specified as follows:

```
M1=real(M), M2=imag(M)
bigM=[M1 M2;-M2 M1]
setlmis([])

% declare bigX=[X1 X2;-X2 X1] with X1=X1' and X2+X2'=0:

[X1,n1,sX1] = lmivar(1,[5 1])
[X2,n2,sX2] = lmivar(3,skewdec(5,n1))
bigX = lmivar(3,[sX1 sX2;-sX2 sX1])

% describe the real counterpart of (1.12):

lmiterm([1 1 1 0],1)
lmiterm([ 1 1 1 bigX],1,1)
lmiterm([2 1 1 bigX],bigM',bigM)
lmiterm([ 2 1 1 bigX],1,1)

lmis = getlmis
```

Note the three-step declaration of the structured matrix variable `bigX`,

$$bigX = \begin{pmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{pmatrix}$$

1. Specify X_1 as a (real) symmetric matrix variable and save its structure description `sX1` as well as the number `n1` of decision variables used in X_1.

2. Specify X_2 as a skew-symmetric matrix variable using Type 3 of `lmivar` and the utility `skewdec`. The command `skewdec(5,n1)` creates a 5-by–5 skew-symmetric structure depending on the decision variables `n1 + 1`, `n1 + 2`,...

3. Define the structure of `bigX` in terms of the structures `sX1` and `sX2` of X_1 and X_2.

See "Structured Matrix Variables" on page 4-32 for more details on such structure manipulations.

Specifying c^Tx Objectives for mincx

The LMI solver `mincx` minimizes linear objectives of the form c^Tx where x is the vector of decision variables. In most control problems, however, such objectives are expressed in terms of the matrix variables rather than of x. Examples include Trace(X) where X is a symmetric matrix variable, or u^TXu where u is a given vector.

The function `defcx` facilitates the derivation of the c vector when the objective is an affine function of the *matrix variables*. For the sake of illustration, consider the linear objective

$$\text{Trace}(X) + x_0^T P x_0$$

where X and P are two symmetric variables and x_0 is a given vector. If `lmisys` is the internal representation of the LMI system and if x_0, X, P have been declared by

```
x0 = [1;1]
setlmis([])
X = lmivar(1,[3 0])
P = lmivar(1,[2 1])
    :
    :
lmisys = getlmis
```

the c vector such that $c^Tx = \text{Trace}(X) + x_0^T P x_0$ can be computed as follows:

```
n = decnbr(lmisys)
c = zeros(n,1)

for j=1:n,
   [Xj,Pj] = defcx(lmisys,j,X,P)
   c(j) = trace(Xj) + x0'*Pj*x0
end
```

The first command returns the number of decision variables in the problem and the second command dimensions c accordingly. Then the `for` loop performs the following operations:

1. Evaluate the matrix variables X and P when all entries of the decision vector x are set to zero except x_j:= 1. This operation is performed by the function `defcx`. Apart from `lmisys` and `j`, the inputs of `defcx` are the identifiers `X` and `P` of the variables involved in the objective, and the outputs `Xj` and `Pj` are the corresponding values.

2. Evaluate the objective expression for `X:= Xj` and `P:= Pj`. This yields the j-th entry of `c` by definition.

In our example the result is

```
c =
   3
   1
   2
   1
```

Other objectives are handled similarly by editing the following generic skeleton:

```
n = decnbr( LMI system )
c = zeros(n,1)
for j=1:n,
   [ matrix values ] = defcx( LMI system,j,
matrix identifiers)
   c(j) = objective(matrix values)
end
```

Feasibility Radius

When solving LMI problems with `feasp`, `mincx`, or `gevp`, it is possible to constrain the solution x to lie in the ball

$$x^T x < R^2$$

where $R > 0$ is called the *feasibility radius*. This specifies a maximum (Euclidean norm) magnitude for x and avoids getting solutions of very large norm. This may also speed up computations and improve numerical stability. Finally, the feasibility radius bound regularizes problems with redundant variable sets. In rough terms, the set of scalar variables is redundant when an equivalent problem could be formulated with a smaller number of variables.

The feasibility radius R is set by the third entry of the options vector of the LMI solvers. Its default value is $R = 109$. Setting R to a negative value means "no rigid bound," in which case the feasibility radius is increased during the optimization if necessary. This "flexible bound" mode may yield solutions of large norms.

Well-Posedness Issues

The LMI solvers used in the LMI Lab are based on interior-point optimization techniques. To compute feasible solutions, such techniques require that the system of LMI constraints be strictly feasible, that is, the feasible set has a nonempty interior. As a result, these solvers may encounter difficulty when the LMI constraints are feasible but not *strictly feasible*, that is, when the LMI

$$L(x) \leq 0$$

has solutions while

$$L(x) < 0$$

has no solution.

For feasibility problems, this difficulty is automatically circumvented by `feasp`, which reformulates the problem

$$\text{Find } x \text{ such that } L(x) \leq 0 \qquad \textbf{(4-12)}$$

as

$$\text{Minimize } t \text{ subject to } Lx < t \times I. \qquad \textbf{(4-13)}$$

In this modified problem, the LMI constraint is always strictly feasible in x, t and the original LMI Equation (4-12) is feasible if and only if the global minimum $t_{\min}$ of Equation (4-12) satisfies

$$t_{\min} \leq 0$$

For feasible but not strictly feasible problems, however, the computational effort is typically higher as `feasp` strives to approach the global optimum $t_{\min} = 0$ to a high accuracy.

For the LMI problems addressed by `mincx` and `gevp`, nonstrict feasibility generally causes the solvers to fail and to return an "infeasibility" diagnosis. Although there is no universal remedy for this difficulty, it is sometimes

possible to eliminate underlying algebraic constraints to obtain a strictly feasible problem with fewer variables.

Another issue has to do with homogeneous feasibility problems such as

$$A^T P + P A < 0, \qquad P > 0$$

While this problem is technically well-posed, the LMI optimization is likely to produce solutions close to zero (the trivial solution of the nonstrict problem). To compute a nontrivial Lyapunov matrix and easily differentiate between feasibility and infeasibility, replace the constraint $P > 0$-by-$P > \alpha I$ with $\alpha > 0$. Note that this does not alter the problem due to its homogeneous nature.

Semi-Definite *B(x)* in gevp Problems

Consider the generalized eigenvalue minimization problem

$$\text{Minimize } \lambda \text{ subject to } A(x) < \lambda B(x),\ B(x) > 0,\ C(x) < 0. \tag{4-14}$$

Technically, the positivity of $B(x)$ for some $x \in \mathbf{R}^n$ is required for the well-posedness of the problem and the applicability of polynomial-time interior-point methods. Hence problems where

$$B(x) = \begin{pmatrix} B_1(x) & 0 \\ 0 & 0 \end{pmatrix}, \quad \text{with } B_1(x) > 0 \text{ strictly feasible}$$

cannot be directly solved with `gevp`. A simple remedy consists of replacing the constraints

$$A(x) < B(x), \qquad B(x) > 0$$

by

$$A(x) < \begin{pmatrix} Y & 0 \\ 0 & 0 \end{pmatrix}, \quad Y < \lambda B_1(x), \quad B_1(x) > 0$$

where Y is an additional symmetric variable of proper dimensions. The resulting problem is equivalent to Equation (4-14) and can be solved directly with `gevp`.

Efficiency and Complexity Issues

As explained in "Introduction" on page 4-2, the term-oriented description of LMIs used in the LMI Lab typically leads to higher efficiency than the canonical representation

$$A_0 + x_1 A_1 + \ldots + x_N A_N < 0 \qquad \textbf{(4-15)}$$

This is no longer true, however, when the number of variable terms is nearly equal to or greater than the number N of decision variables in the problem. If your LMI problem has few free scalar variables but many terms in each LMI, it is therefore preferable to rewrite it as (9-15) and to specify it in this form. Each scalar variable x_j is then declared independently and the LMI terms are of the form $x_j A_j$.

If M denotes the total row size of the LMI system and N the total number of scalar decision variables, the flop count per iteration for the `feasp` and `mincx` solvers is proportional to

- N^3 when the least-squares problem is solved via. Cholesly factorization of the Hessian matrix (default) [2]
- M-by-N^2 when numerical instabilities warrant the use of QR factorization instead

While the theory guarantees a worst-case iteration count proportional to M, the number of iterations actually performed grows slowly with M in most problems. Finally, while `feasp` and `mincx` are comparable in complexity, `gevp` typically demands more computational effort. Make sure that your LMI problem cannot be solved with `mincx` before using `gevp`.

Solving $M + P^T X Q + Q^T X^T P < 0$

In many output-feedback synthesis problems, the design can be performed in two steps:

1. Compute a closed-loop Lyapunov function via LMI optimization.
2. Given this Lyapunov function, derive the controller state-space matrices by solving an LMI of the form

$$M + P^T X Q + Q^T X^T P < 0 \qquad \textbf{(4-16)}$$

where M, P, Q are given matrices and X is an unstructured m-by-n matrix variable.

It turns out that a particular solution X_c of Equation (4-16) can be computed via simple linear algebra manipulations [1]. Typically, X_c corresponds to the center of the ellipsoid of matrices defined by Equation (4-16).

The function `basiclmi` returns the "explicit" solution X_c:

```
Xc = basiclmi(M,P,Q)
```

Since this central solution sometimes has large norm, `basiclmi` also offers the option of computing an approximate least-norm solution of Equation (4-16). This is done by

```
X = basiclmi(M,P,Q,'Xmin')
```

and involves LMI optimization to minimize $\|X\|$.

References

[1] Gahinet, P., and P. Apkarian, "A Linear Matrix Inequality Approach to H_∞ Control," *Int. J. Robust and Nonlinear Contr*., 4 (1994), pp. 421-448.

[2] Nemirovski, A., and P. Gahinet, "The Projective Method for Solving Linear Matrix Inequalities," *Proc. Amer. Contr. Conf*., 1994, pp. 840-844.

[3] Nesterov, Yu, and A. Nemirovski, *Interior Point Polynomial Methods in Convex Programming: Theory and Applications*, SIAM Books, Philadelphia, 1994.

[4] Shamma, J.S., "Robustness Analysis for Time-Varying Systems," *Proc. Conf. Dec. Contr*., 1992, pp. 3163-3168.

5

Function Reference

Functions - By Category

Uncertain Elements

`ucomplex`	Create uncertain complex parameter
`ucomplexm`	Create uncertain complex matrix
`udyn`	Create unstructured uncertain dynamic system object
`ultidyn`	Create uncertain linear time-invariant object
`ureal`	Create uncertain real parameter

Uncertain Matrices and Systems

`diag`	Diagonalize vector of uncertain matrices and systems
`randatom`	Generate random uncertain `atom` objects
`randumat`	Create random uncertain matrix
`randuss`	Create random uncertain state-space model
`ufrd`	Create uncertain frequency response data (`ufrd`) object, or convert another model type to `ufrd` model
`umat`	Create uncertain matrix
`uss`	Specify uncertain state-space models or convert LTI model to uncertain state-space model

Manipulation of Uncertain Models

`actual2normalized`	Calculate normalized distance between nominal value and given value for uncertain atom
`gridureal`	Grid `ureal` parameters uniformly over range
`isuncertain`	True for uncertain systems
`lftdata`	Decompose uncertain object into fixed normalized and fixed uncertain parts

`normalized2actual`	Convert value for atom in normalized coordinates to corresponding actual value
`repmat`	Replicate and tile uncertain matrix
`simplify`	Simplify representation of uncertain object
`squeeze`	Remove singleton dimensions for `umat` objects
`usample`	Generate random samples of uncertain object
`usubs`	Substitute values for uncertain elements of uncertain objects
`uss/ssbal`	Scale state/uncertainty of uncertain system

Interconnection of Uncertain Models

`iconnect`	Create empty interconnection object
`icsignal`	Create `icsignal` object
`imp2exp`	Convert implicit linear relationship to explicit input-output relationship
`stack`	Construct array by stacking uncertain matrices, models, or arrays
`sysic`	Build interconnections of certain and uncertain matrices and systems

Model Order Reduction

`balancmr`	Balanced model truncation via square root method
`bstmr`	Balanced stochastic model truncation (BST) via Schur method
`hankelmr`	Hankel minimum degree approximation (MDA) without balancing
`hankelsv`	Compute Hankel singular values for stable/unstable or continuous/discrete system
`imp2ss`	Convert impulse response to approximate state-space realization
`modreal`	Modal form realization and projection
`ncfmr`	Balanced normalized coprime factor model reduction
`reduce`	Model order reduction using Hankel singular value based algorithms
`schurmr`	Balanced truncation via Schur method
`slowfast`	State-space slow-fast decomposition
`stabproj`	State-space stable/anti-stable decomposition

Robustness and Worst-Case Analysis

`cpmargin`	Coprime stability margin of plant-controller feedback loop
`gapmetric`	Compute upper bounds on `gap` and `nugap` distances between two systems
`loopmargin`	Analyze feedback loop
`loopsens`	Analyze sensitivity of plant-controller feedback loop
`mussv`	Compute bounds on structured singular value (μ)
`mussvextract`	Extract `muinfo` structure returned by `mussv`
`ncfmargin`	Calculate normalized coprime stability margin of feedback loop
`popov`	Perform Popov robust stability test
`robopt`	Create options object for use with `robuststab` and `robustperf`
`robustperf`	Calculate robust performance margin of uncertain multivariable system
`robuststab`	Calculate robust stability margin of uncertain multivariable system
`wcgain`	Calculate bounds on worst-case gain of uncertain system
`wcgopt`	Create options object for use with `wcgain`, `wcsens`, and `wcmargin`
`wcmargin`	Calculate worst-case gain/phase margins for feedback loop
`wcnorm`	Calculate worst-case norm of uncertain matrix
`wcsens`	Calculate worst-case sensitivity and complementary sensitivity functions of feedback loop

Robustness Analysis for Parameter-Dependent Systems (P-Systems)

`aff2pol`	Convert affine parameter-dependent models to polytopic models
`decay`	Quadratic decay rate of polytopic or affine P-systems
`ispsys`	True for parameter-dependent systems
`pdlstab`	Assess robust stability of polytopic or parameter-dependent system
`pdsimul`	Time response of parameter-dependent system along parameter trajectory
`polydec`	Compute polytopic coordinates with respect to box corners
`psinfo`	Query characteristics of polytopic or parameter-dependent systems
`psys`	Specify polytopic or parameter-dependent linear system
`pvec`	Specify vector of uncertain or time-varying parameters
`pvinfo`	Query characteristics of parameter vector
`quadperf`	Compute quadratic H_∞ performance of polytopic or parameter-dependent system
`quadstab`	Assess quadratic stability of polytopic or affine parameter-dependent systems

Controller Synthesis

`augw`	Create augmented plant model for weighted mixed-sensitivity loop shaping design
`h2hinfsyn`	Mixed H_2/H_∞ synthesis with pole placement constraints

`h2syn`	H_2 controller synthesis for LTI plant
`hinfsyn`	H_∞ optimal controller synthesis for LTI plant
`loopsyn`	H_∞ loop shaping controller synthesis
`ltrsyn`	LQG loop-transfer recovery controller synthesis
`mkfilter`	Generate Bessel, Butterworth, Chebyshev, or RC filter
`mixsyn`	H_∞ mixed-sensitivity controller synthesis
`ncfsyn`	H_∞ normalized coprime factor controller synthesis

μ-Synthesis

`cmsclsyn`	Constant matrix μ-synthesis
`dksyn`	Synthesis of robust controller via μ-synthesis D-K iteration
`dkitopt`	Create options object for use with `dksyn`
`drawmag`	Interactive mouse-based sketching and fitting tool
`fitfrd`	Fit D-scaling frequency response data with state-space model
`fitmagfrd`	Fit scaling magnitude data with stable, minimum-phase state-space model
`genphase`	Fit single-input/single-output magnitude data with real, rational, minimum-phase transfer function

Sampled-Data Systems

`sdhinfnorm`	Compute L2 norm of sampled-data system
`sdhinfsyn`	Sampled-data H_∞ controller synthesis
`sdlsim`	Time response of sampled-data feedback system

Gain Scheduling

`hinfgs`	Gain-scheduled H_∞ controller synthesis

Frequency-Response Data (FRD) Models

`frd/loglog`	Log-log scale plot of `frd` objects
`frd/semilogx`	Semilog scale plot of `frd` object
`frd/semilogy`	Semilog scale plot of `frd` object
`frd/rcond`	Reciprocal condition estimator of `frd` object

`frd/schur`	Schur decomposition of `frd` object
`frd/svd`	Singular value decomposition of `frd` object

Supporting Utilities

`bilin`	Multivariable binlinear transform of frequency (s or z)
`dmplot`	Interpret disk gain and phase margins
`mktito`	Partition LTI system into two-input-two-output system
`sectf`	State-space sector bilinear transformation
`skewdec`	Create skew-symmetric matrix
`symdec`	Form symmetric matrix

LMIs

"LMI Systems" on page 5-12	Functions for specifying systems of LMIs
"LMI Characteristics" on page 5-14	Functions for extracting information about systems of LMIs
"LMI Solvers" on page 5-14	Functions for solving systems of LMIs
"Validation of Results" on page 5-14	Functions for analyzing and validating LMI optimization results
"Modification of Systems of LMIs" on page 5-14	Functions for altering systems of LMIs

LMI Systems

`getlmis`	Internal description of LMI system
`lmiedit`	Specify or display systems of LMIs as MATLAB expressions
`lmiterm`	Add new term to existing LMI
`lmivar`	Specify matrix variables in existing LMI system

`newlmi`	Add new LMI to LMI system
`setlmis`	Initialize description of LMI system

LMI Characteristics

`dec2mat`	Extract matrix variable value from vector of decision variables
`decinfo`	Describe how entries of matrix variable X relate to decision variables
`decnbr`	Number of decision variables in LMI system
`lmiinfo`	Information about existing system of LMIs
`lminbr`	Number of LMIs in LMI system
`mat2dec`	Extract vector of decision variables from matrix variable values
`matnbr`	Number of matrix variables in LMI system

LMI Solvers

`defcx`	Specify $c^T x$ objectives of `mincx`
`feasp`	Compute solution to given system of LMIs
`gevp`	Generalized eigenvalue minimization under LMI constraints
`mincx`	Minimize linear objective under LMI constraints

Validation of Results

`evallmi`	Evaluate LMI for given values of decision variables
`showlmi`	Left- and right-hand sides of evalauted LMI

Modification of Systems of LMIs

`dellmi`	Remove LMI from system of LMIs
`delmvar`	Remove matrix variable from LMI problem
`setmvar`	Instantiate matrix variable and evaluate LMI terms

Functions — Alphabetical List

actual2normalized

Purpose

Compute normalized-distance between uncertain atom nominal value and specified value

Syntax

```
NDIST = actual2normalized(A,V)
```

Description

`NDIST = actual2normalized(A,V)` is the normalized-distance between the nominal value of the uncertain atom `A` and the given value `V`. If `A` is a `ureal`, then `NDIST` may be positive or negative, reflecting that `V` is greater than, or less than the nominal value. If `A` is any other class of uncertain atom, then `ndist` is nonnegative.

If `V` is an array of values, then `NDIST` is an array of normalized distances.

The robustness margins computed in `robuststab` and `robustperf` serve as bounds for the normalized distances in `NDIST`. For example, if an uncertain system has a stability margin of 1.4, this system is stable when the normalized distance of the uncertain element values from the nominal is less than 1.4.

Example

Uncertain Real Parameter With Symmetric Range

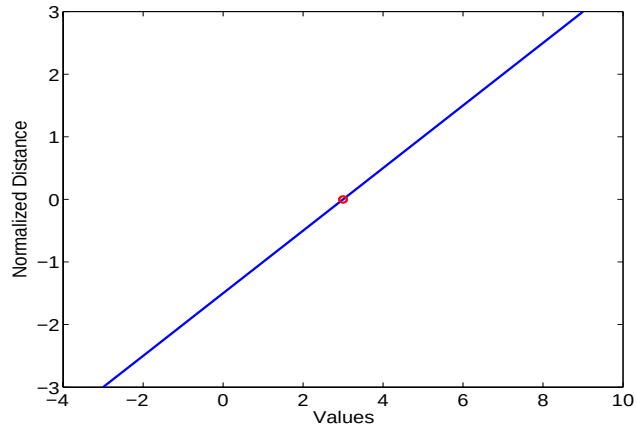
For uncertain real parameters whose range is symmetric about their nominal value, the normalized distance is intuitive, scaling linearly with the numerical difference from the uncertain real parameter's nominal value

Create uncertain real parameters with a range that is symmetric about the nominal value, where each end point is 1 unit from the nominal. Points that lie inside the range are less than 1 unit from the nominal, while points that lie outside the range are greater than 1 unit from the nominal.

```
a = ureal('a',3,'range',[1 5]);
actual2normalized(a,[1 3 5])
ans =
   -1.0000   -0.0000    1.0000
actual2normalized(a,[2 4])
ans =
   -0.5000    0.5000
actual2normalized(a,[0 6])
ans =
   -1.5000    1.5000
```

Graph the normalized distance for several values. The nominal point is shown as a red circle. Note that the relationship between a normalized distance and a numerical difference is linear.

```
values = linspace(-3,9,250);
ndist = actual2normalized(a,values);
plot(values,ndist)
```



Uncertain Real Parameter With Nonsymmetric Range

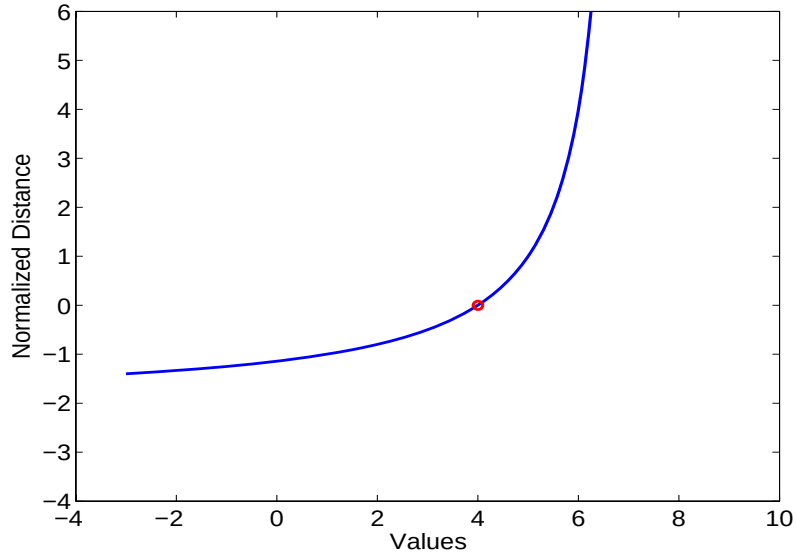
Next, create an unasymmetric parameter. It still is true that the end points are 1 normalized unit from nominal, and the nominal is 0 normalized units from nominal, moreover points inside the range are less than 1 unit from nominal, and points outside the range are greater than 1 unit from nominal. However, the relationship between the normalized distance and numerical difference is nonlinear.

```
au = ureal('a',4,'range',[1 5]);
actual2normalized(a,[1 4 5])
ans =
   -1.0000    0.5000    1.0000
actual2normalized(a,[2 4.5])
ans =
   -0.5000    0.7500
actual2normalized(a,[0 6])
ans =
```

```
    -1.5000     1.5000
```

Graph the normalized distance for several values. Note that the relationship between normalized distance and numerical difference is very nonlinear.

```
ndistu = actual2normalized(au,values);
plot(values,ndistu,au.NominalValue,0,'ro')
```



Algorithm

For details on the normalize distance, see the section ““Normalizing Functions for Uncertain Atoms” on page 1-60.

See Also

`normalized2actual`	Converts normalized to actual value
`robuststab`	Calculate robust stability margin
`robustperf`	Calculate robust performance margin

Purpose

Convert affine parameter-dependent models to polytopic models

Syntax

```
polsys = aff2pol(affsys)
```

Description

`aff2pol` derives a polytopic representation `polsys` of the *affine* parameter-dependent system

$$E(p)\dot{x} = A(p)x + B(p)u \qquad \textbf{(5-1)}$$

$$y = C(p)x + D(p)u \qquad \textbf{(5-2)}$$

where $p = (p_1, \ldots, p_n)$ is a vector of uncertain or time-varying real parameters taking values in a box or a polytope. The description `affsys` of this system should be specified with `psys`.

The vertex systems of `polsys` are the instances of Equation 5-1 and Equation 5-2 at the vertices p_{ex} of the parameter range, i.e., the `SYSTEM` matrices

$$\begin{pmatrix} A(p_{ex}) + jE(p_{ex}) & B(p_{ex}) \\ C(p_{ex}) & D(p_{ex}) \end{pmatrix}$$

for all corners p_{ex} of the parameter box or all vertices p_{ex} of the polytope of parameter values.

See Also

`psys`	Specification of uncertain state-space model
`pvec`	Quantification of uncertainty on physical parameters
`uss`	Create an uncertain state-space model

Purpose

State-space or transfer function plant augmentation for use in weighted mixed-sensitivity H_∞ and H_2 loopshaping design

Syntax

```
P = AUGW(G,W1,W2,W3)
```

Description

`P = AUGW(G,W1,W2,W3)` computes a state-space model of an augmented LTI plant $P(s)$ with weighting functions $W_1(s)$, $W_2(s)$, and $W_3(s)$ penalizing the error signal, control signal and output signal respectively (see block diagram) so that the closed-loop transfer function matrix is the weighted mixed sensitivity

$$T_{y_1 u_1} \triangleq \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix}$$

where S, R and T are given by

$$S = (I + GK)^{-1}$$
$$R = K(I + GK)^{-1}$$
$$T = GK(I + GK)^{-1}$$

The LTI systems S and T are called the *sensitivity* and *complementary sensitivity,* respectively.

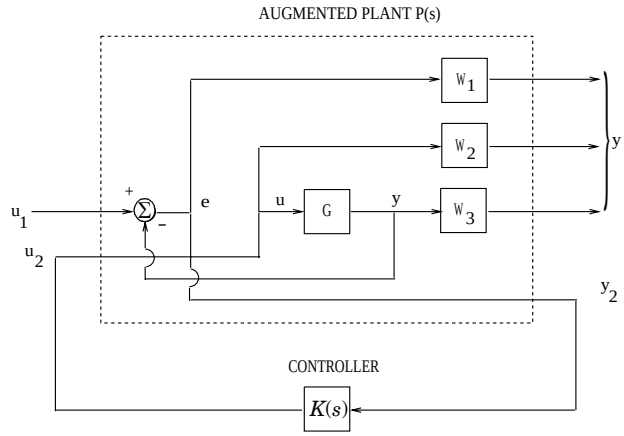


Figure 5-1: Plant Augmentation

For dimensional compatibility, each of the three weights W_1, W_2 and W_3 must be either empty, a scalar (SISO) or have respective input dimensions N_Y, N_U, and N_Y where G is N_Y-by-N_U. If one of the weights is not needed, you may simply assign an empty matrix [ ]; e.g., `P = AUGW(G,W1,[],W3)` is $P(s)$ as in the "Algorithm" section below, but without the second row (without the row containing `W2`).

Algorithm

The augmented plant *P(s)* produced by is

$$P(s) = \left[\begin{array}{c|c} W_1 & -W_1 G \\ 0 & W_2 \\ 0 & W_3 G \\ \hline I & -G \end{array}\right]$$

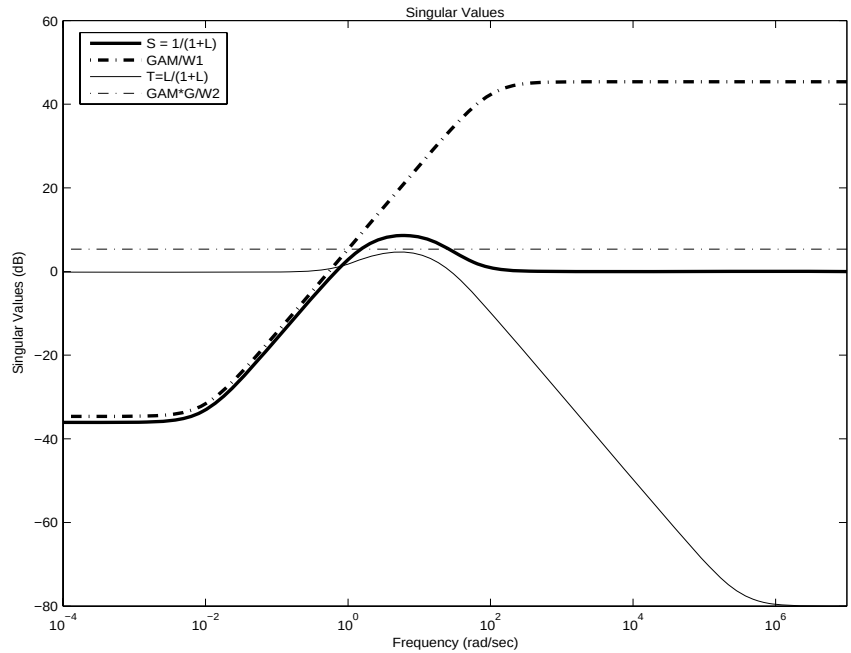
Partitioning is embedded via `P=mktito(P,NY,NU)`, which sets the InputGroup and OutputGroup properties of P as follows

```
[r,c]=size(P);
P.InputGroup  = struct('U1',1:c-NU,'U2',c-NU+1:c);
P.OutputGroup = struct('Y1',1:r-NY,'Y2',r-NY+1:r);
```

Example

```
s=zpk('s'); G=(s-1)/(s+1);
```

```
W1=0.1*(s+100)/(100*s+1); W2=0.1; W3=[];
P=augw(G,W1,W2,W3);
[K,CL,GAM]=hinfsyn(P); [K2,CL2,GAM2]=h2syn(P);
L=G*K; S=inv(1+L); T=1-S;
sigma(S,'k',GAM/W1,'k-.',T,'r',GAM*G/W2,'r-.')
legend('S = 1/(1+L)','GAM/W1', 'T=L/(1+L)','GAM*G/W2',2)
```



Limitations

The transfer functions G, W_1, W_2 and W_3 must be proper, i.e., bounded as $s \to \infty$ or, in the discrete-time case, as $z \to \infty$. Additionally, W_1, W_2 and W_3 should be stable. The plant G should be stabilizable and detectable; else, `P` will not be stabilizable by any `K`.

See Also

`h2syn`	H_2 controller synthesis
`hinfsyn`	H_∞ controller synthesis
`mixsyn`	H_∞ mixed-sensitivity controller synthesis
`mktito`	Partition LTI system via Input and Output Groups

Purpose

Balanced model truncation via square root method

Syntax

```
GRED = balancmr(G)
GRED = balancmr(G,order)
[GRED,redinfo] = balancmr(G,key1,value1,...)
[GRED,redinfo] = balancmr(G,order,key1,value1,...)
```

Description

`balancmr` returns a reduced order model `GRED` of `G` and a struct array `redinfo` containing the error bound of the reduced model and Hankel singular values of the original system.

The error bound is computed based on Hankel singular values of `G`. For a stable system these values indicate the respective state energy of the system. Hence, reduced order can be directly determined by examining the system Hankel SV's, σι.

With only one input argument `G`, the function will show a Hankel singular value plot of the original model and prompt for model order number to reduce.

This method guarantees an error bound on the infinity norm of the *additive error* || `G-GRED` ||∞ for well-conditioned model reduced problems [1]:

$$\|G - Gred\|_\infty \le 2 \sum_{k+1}^{n} \sigma_i$$

This table describes input arguments for `balancmr`.

Argument	Description
`G`	LTI model to be reduced (without any other inputs will plot its Hankel singular values and prompt for reduced order)
`ORDER`	(Optional) an integer for the desired order of the reduced model, or optionally a vector packed with desired orders for batch runs

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y`, or a vector of positive integers. By default, all the

anti-stable part of a system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system.

'*MaxError*' can be specified in the same fashion as an alternative for 'Order'. In this case, reduced order will be determined when the sum of the tails of the Hankel sv's reaches the '*MaxError*'.

This table lists the input arguments 'key' and its 'value'.

Argument	Value	Description
'*MaxError*'	Real number or vector of different errors	Reduce to achieve H∞ error. When present, '*MaxError*' overides ORDER input.
'*Weights*'	{Wout,Win} cell array	Optimal 1-by-2 cell array of LTI weights Wout (output) and Win (input). Defaults are both identity. Weights must be invertible.
'*Display*'	'*on*' or '*off*'	Display Hankel singular plots (default '*off*').
'*Order*'	Integer, vector or cell array	Order of reduced model. Use only if not specified as 2nd argument.

Weights on the original model input and/or output can make the model reduction algorithm focus on some frequency range of interests. But weights have to be stable, minimum phase and invertible.

This table describes output arguments.

Argument	Description
`GRED`	LTI reduced order model. Becomes multidimensional array when input is a serial of different model order array
`REDINFO`	A STRUCT array with three fields: • `REDINFO.ErrorBound (bound on` $\|\|$ G-GRED $\|\|_\infty$`)` • `REDINFO.StabSV (Hankel SV of stable part of G)` • `REDINFO.UnstabSV (Hankel SV of unstable part of G)`

`G` can be stable or unstable, continuous or discrete.

Algorithm

Given a state space *(A,B,C,D)* of a system and *k,* the desired reduced order, the following steps will produce a similarity transformation to truncate the original state space system to the k^{th} order reduced model.

1 Find the SVD of the controllability and observability grammians

$$P = U_p \, \Sigma_p \, V_p{}^T$$

$$Q = U_q \, \Sigma_q \, V_q{}^T$$

2 Find the square root of the grammians (left/right eigenvectors)

$$L_p = U_p \, \Sigma_p{}^{1/2}$$

$$L_o = U_q \, \Sigma_q{}^{1/2}$$

3 Find the SVD of $(L_o{}^T L_p)$

$$L_o{}^T \, L_p = U \, \Sigma \, V^T$$

4 Then the left and right transformation for the final k^{th} order reduced model is

$$S_{L,BIG} = L_o \, U(:,1{:}k) \, \Sigma(1;k,1{:}k))^{-1/2}$$

$$S_{R,BIG} = L_p \, V(:,1{:}k) \, \Sigma(1;k,1{:}k))^{-1/2}$$

5 Finally,

$$\left[\begin{array}{c|c} \hat{A} & \hat{B} \\ \hline \hat{C} & \hat{D} \end{array}\right] = \left[\begin{array}{c|c} S^T{}_{L,BIG} A S_{R,BIG} & S^T{}_{L,BIG} B \\ \hline C S_{R,BIG} & D \end{array}\right]$$

The proof of the square root balance truncation algorithm can be found in [2].

Example

Given a continuous or discrete, stable or unstable system, G, the following commands can get a set of reduced order models based on your selections:

```
rand('state',1234); randn('state',5678);G = rss(30,5,4);
[g1, redinfo1] = balancmr(G); % display Hankel SV plot
                              % and prompt for order (try 15:20)
[g2, redinfo2] = balancmr(G,20);
[g3, redinfo3] = balancmr(G,[10:2:18]);
[g4, redinfo4] = balancmr(G,'MaxError',[0.01, 0.05]);
rand('state',12345); randn('state',6789);
wt1 = rss(6,5,5); wt1.d = eye(5)*2;
wt2 = rss(6,4,4); wt2.d = 2*eye(4);
[g5, redinfo5] = balancmr(G, [10:2:18], 'weight',{wt1,wt2});
for i = 1:5
    figure(i); eval(['sigma(G,g' num2str(i) ');']);
end
```

Reference

[1] Glover, K., "All Optimal Hankel Norm Approximation of Linear Multivariable Systems, and Their **L**μ - error Bounds," Int. J. Control, Vol. 39, No. 6, 1984, p. 1145-1193

[2] Safonov, M.G., and R.Y. Chiang, "A Schur Method for Balanced Model Reduction," *IEEE Trans. on Automat. Contr.*, Vol. 34, No. 7, July 1989, p. 729-733

See Also

reduce	Top level model reduction function
schurmr	Balanced model truncation via Schur method
hankelmr	Hankel minimum degree approximation
bstmr	Balanced stochastic model truncation via Schur method
ncfmr	Balanced model truncation for normalized coprime factors
hankelsv	Hankel singular values

Purpose

Multivariable bilinear transform of frequency (s or z)

Syntax

```
GT = bilin(G,VERS,METHOD,AUG)
```

Description

`bilin` computes the effect on a system of the frequency-variable substitution,

$$s = \frac{\alpha z + \delta}{\gamma z + \beta}$$

The variable `VERS` denotes the transformation direction:

`VERS= 1`, forward transform $(s \to z)$ or $(s \to \tilde{s})$.

`VERS=-1`, reverse transform $(z \to s)$ or $(s \to \tilde{s})$.

This transformation maps lines and circles to circles and lines in the complex plane. People often use this transformation to do sampled-data control system design [1] or, in general, to do shifting of $j\omega$ modes [2], [3], [4].

`Bilin` computes several state-space bilinear transformations such as backward rectangular, etc., based on the `METHOD` you select

Table 5-1: Bilinear transform types.

Method	Type of bilinear transform
`'BwdRec'`	backward rectangular: $s = \frac{z-1}{Tz}$ `AUG` = T, the sampling period.
`'FwdRec'`	forward rectangular: $s = \frac{z-1}{T}$ `AUG` = T, the sampling period.

Table 5-1: Bilinear transform types.

Method	Type of bilinear transform
`'S_Tust'`	shifted Tustin: $s = \frac{2}{T}\left(\frac{z-1}{\frac{z}{h}+1}\right)$ `AUG` = $[T\ h]$, is the "shift" coefficient.
`'S_ftjw'`	shifted $j\omega$-axis, bilinear pole-shifting, continuous-time to continuous-time: $s = \frac{\tilde{s}+p_1}{1+\tilde{s}/p_2}$ `AUG` = $[p_2\ p_1]$.
`'G_Bilin'`	`METHOD = 'G_Bilin'`, general bilinear, continuous-time to continuos-time: $s = \frac{\alpha\tilde{s}+\delta}{\gamma\tilde{s}+\beta}$ `AUG` = $[\alpha\ \ \beta\ \ \gamma\ \ \delta]$.

Example

Example 1. Tustin continuous s-plane to discrete z-plane transforms. Consider the following continuous-time plant (sampled at 20 Hz)

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix};\ T_s = 0.05 .$$

Following is an example of four common "continuous to discrete" `bilin` transformations for the sampled plant:

```
A= [-1 1; 0 -2]; B=[1 0; 1 1];
C= [1 0; 0 1];   D=[0 0; 0 0];
sys = ss(A,B,C,D);                        % ANALOG
Ts=0.05;  % sampling time
```

```
[syst] = c2d(sys,Ts,'tustin');          % Tustin
[sysp] = c2d(sys,Ts,'prewarp',40);        % Pre-warped Tustin
[sysb] = bilin(sys,1,'BwdRec',Ts);      % Backward Rectangular
[sysf] = bilin(sys,1,'FwdRec',Ts);      % Forward Rectangular
w = logspace(-2,3,50); % frequencies to plot
sigma(sys,syst,sysp,sysb,sysf,w);
```

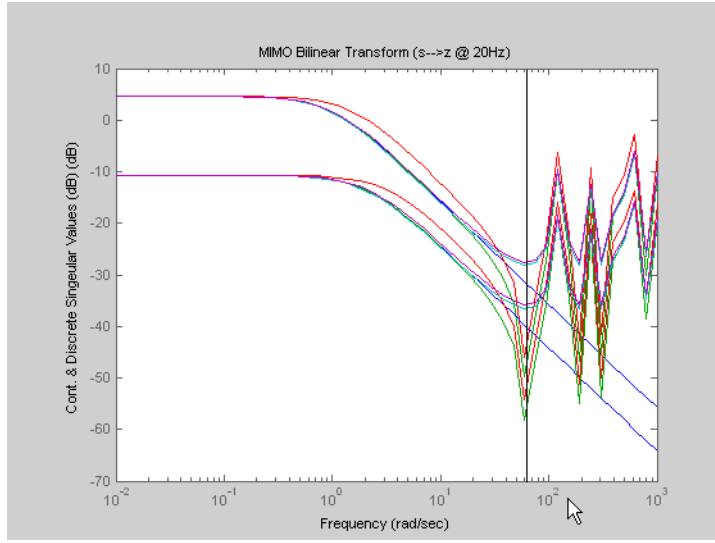


Figure 5-2: . Comparison of 4 Bilinear Transforms from Example 1.

Example 2. Bilinear continuous to continuous pole-shifting `'S_ftjw'`

Design an H mixed-sensitivity controller for the ACC Benchmark plant

$$G(s) = \frac{1}{s^2(s^2+2)}$$

such that all closed-loop poles lie inside a circle in the left half of the s-plane whose diameter lies on between points [p1,p2]=[-12,-2]:

```
p1=-12; p2=-2; s=zpk('s');
G=ss(1/(s^2*(s^2+2)));          % original unshifted plant
Gt=bilin(G,1,'Sft_jw',[p1 p2]); % bilinear pole shifted plant Gt
Kt=mixsyn(Gt,1,[],1);          % bilinear pole shifted controller
```

```
K =bilin(Kt,-1,'Sft_jw',[p1 p2]); % final controller K
```

As shown in Figure 5-3, closed-loop poles are placed in the left circle `[p1 p2]`. The shifted plant, which has its non-stable poles shifted to the inside the right circle, is

$$G_t(s) = 4.765 \times 10^{-5} \frac{(s-12)^4}{(s-2)^2(s^2-4.274s+5.918)}$$

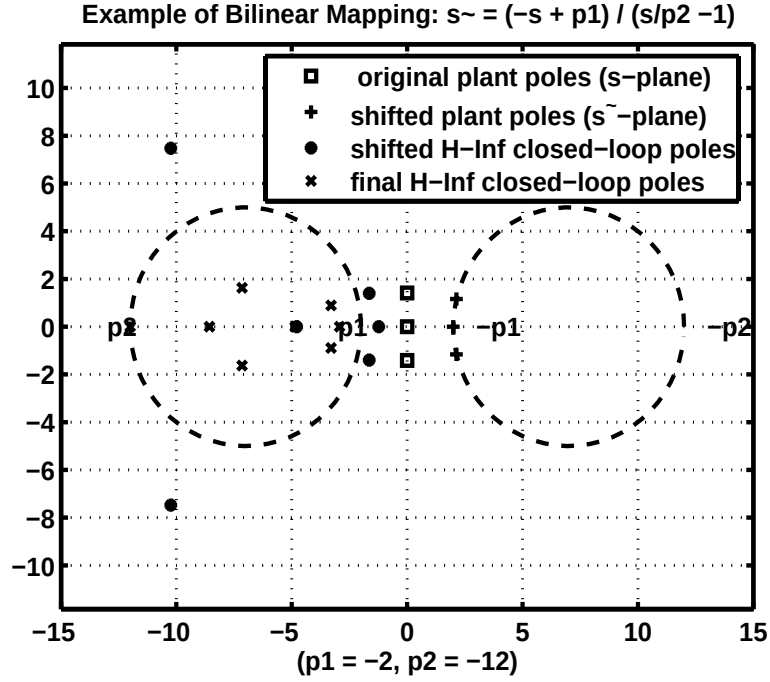


Figure 5-3: `'S_ftjw'` final closed-loop poles are inside the left [p1,p2] circle.

Algorithm

`bilin` employs the state-space formulae in [3]:

$$\left[\begin{array}{c|c} A_b & B_b \\ \hline C_b & D_b \end{array}\right] = \left[\begin{array}{c|c} (\beta A - \delta I)(\alpha I + \gamma A)^{-1} & (\alpha\beta - \gamma\delta)(\alpha I - \gamma A)^{-1}B \\ \hline C(\alpha I - \gamma A)^{-1} & D + \gamma C(\alpha I - \gamma A)^{-1}B \end{array}\right]$$

References

[1] Franklin, G.F., and J.D. Powell, *Digital Control of Dynamics System*, Addison-Wesley, 1980.

[2] Safonov, M.G., R.Y. Chiang, and H. Flashner, "H_∞ Control Synthesis for a Large Space Structure," *AIAA J. Guidance, Control and Dynamics*, 14, 3, p. 513-520, May/June 1991.

[3] Safonov, M.G., "Imaginary-Axis Zeros in Multivariable H_∞ Optimal Control", in R.F. Curtain (editor), *Modelling, Robustness and Sensitivity Reduction in Control Systems*, p. 71-81, Springer-Varlet, Berlin, 1987.

[4] Chiang, R.Y., and M.G. Safonov, "H_∞ Synthesis using a Bilinear Pole Shifting Transform," *AIAA, J. Guidance, Control and Dynamics*, vol. 15, no. 5, p. 1111-1117, September-October 1992.

See Also

`c2d`	Convert from continous- to discrete-time
`d2c`	Convert from continous- to discrete-time
`sectf`	Sector transformation

bstmr

Purpose

Balanced stochastic model truncation (BST) via Schur method

Syntax

```
GRED = bstmr(G)
GRED = bstmr(G,order)
[GRED,redinfo] = bstmr(G,key1,value1,...)
[GRED,redinfo] = bstmr(G,order,key1,value1,...)
```

Description

`bstmr` returns a reduced order model `GRED` of `G` and a struct array `redinfo` containing the error bound of the reduced model and Hankel singular values of the *phase matrix* of the original system [2].

The error bound is computed based on Hankel singular values of the phase matrix of `G`. For a stable system these values indicate the respective state energy of the system. Hence, reduced order can be directly determined by examining these values.

With only one input argument `G`, the function will show a Hankel singular value plot of the phase matrix of `G` and prompt for model order number to reduce.

This method guarantees an error bound on the infinity norm of the *multiplicative* $\| \texttt{GRED}^{-1}\texttt{(G-GRED)} \|_\infty$ or *relative error* $\| \texttt{G}^{-1}\texttt{(G-GRED)} \|_\infty$ for well-conditioned model reduction problems [1]:

$$\left\| G^{-1}(G - Gred) \right\|_\infty \le \prod_{k+1}^{n} (1 + 2\sigma_i(\sqrt{1+\sigma_i^2} + \sigma_i)) - 1$$

This table describes input arguments for `bstmr`.

Argument	Description
`G`	LTI model to be reduced (without any other inputs will plot its Hankel singular values and prompt for reduced order)
`ORDER`	(Optional) an integer for the desired order of the reduced model, or a vector of desired orders for batch runs

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y`, or a vector of integers. By default, all the anti-stable

part of a system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system.

'*MaxError*' can be specified in the same fashion as an alternative for 'ORDER'. In this case, reduced order will be determined when the accumulated product of Hankel SVs shown in the above equation reaches the '*MaxError*'.

Argument	Value	Description
'MaxError'	Real number or vector of different errors	Reduce to achieve H∞ error. When present, '*MaxError*' overides ORDER input.
'Display'	'on' or 'off'	Display Hankel singular plots (default 'off').
'Order'	Integer, vector or cell array	Order of reduced model. Use only if not specified as 2nd argument.

This table describes output arguments.

Argument	Description
GRED	LTI reduced order model. Become multi-dimension array when input is a serial of different model order array.
REDINFO	A STRUCT array with three fields: • REDINFO.ErrorBound (bound on $\| G^{-1}(G\text{-}GRED) \|_\infty$) • REDINFO.StabSV (Hankel SV of stable part of G) • REDINFO.UnstabSV (Hankel SV of unstable part of G)

G can be stable or unstable, continuous or discrete.

Algorithm

Given a state space *(A,B,C,D)* of a system and *k,* the desired reduced order, the following steps will produce a similarity transformation to truncate the original state space system to the k^{th} order reduced model.

1. Find the controllability grammian P and observability grammian Q of the left *spectral factor* $\Phi = \Gamma(\sigma)\Gamma*(-\sigma) = \Omega*(-\sigma)\Omega(\sigma)$ by solving the following Lyapunov and Riccati equations

$$AP + PA^T + BB^T = 0$$
$$B_W = PC^T + BD^T$$
$$QA + A^T Q + (QB_W - C^T)(-DD^T)(QB_W - C^T)^T = 0$$

2. Find the Schur decomposition for PQ in both ascending and descending order, respectively,

$$V_A^T PQ V_A = \begin{bmatrix} \lambda_1 & \cdots & \cdots \\ 0 & \cdots & \cdots \\ 0 & 0 & \lambda_n \end{bmatrix}$$

$$V_D^T PQ V_D = \begin{bmatrix} \lambda_n & \cdots & \cdots \\ 0 & \cdots & \cdots \\ 0 & 0 & \lambda_1 \end{bmatrix}$$

3. Find the left/right orthonormal eigen-bases of PQ associated with the k^{th} big Hankel singular values of the all-pass *phase matrix* $(W^*(s))^{-1}G(s)$.

$$V_A = [V_{R,SMALL}, \overbrace{V_{L,BIG}}^{k}]$$

$$V_D = [\overbrace{V_{R,BIG}}^{k}, V_{L,SMALL}]$$

4. Find the SVD of $(V^T_{L,BIG} V_{R,BIG}) = U \Sigma \varsigma T$
5. Form the left/right transformation for the final k^{th} order reduced model

$$S_{L,BIG} = V_{L,BIG} U\Sigma(1{:}k, 1{:}k)^{-1/2}$$
$$S_{R,BIG} = V_{R,BIG} V\Sigma(1{:}k, 1{:}k)^{-1/2}$$

6. Finally,

$$\left[\begin{array}{c|c} \hat{A} & \hat{B} \\ \hline \hat{C} & \hat{D} \end{array}\right] = \left[\begin{array}{c|c} S^T{}_{L,BIG} A S_{R,BIG} & S^T{}_{L,BIG} B \\ \hline C S_{R,BIG} & D \end{array}\right]$$

The proof of the Schur BST algorithm can be found in [2].

Note The BST model reduction theory requires that the original model D matrix be full rank, for otherwise the Riccati solver fails. For any problem with strictly proper model, you can shift the $j\omega$-axis via `bilin` such that BST/REM approximation can be achieved up to a particular frequency range of interests. Alternatively, you can attach a small but full rank D matrix to the original problem but remove the D matrix of the reduced order model afterwards. As long as the size of D matrix is insignificant inside the control bandwidth, the reduced order model should be fairly close to the true model. By default, the `bstmr` program will assign a full rank D matrix scaled by 0.001 of the minimum eigenvalue of the original model, if its D matrix is not full rank to begin with. This serves the purpose for most problems if user does not want to go through the trouble of model pretransformation.

Example

Given a continuous or discrete, stable or unstable system, `G`, the following commands can get a set of reduced order models based on your selections:

```
rand('state',1234); randn('state',5678);
G = rss(30,5,4); G.d = zeros(5,4);
[g1, redinfo1] = bstmr(G); % display Hankel SV plot
                           % and prompt for order (try 15:20)
[g2, redinfo2] = bstmr(G,20);
[g3, redinfo3] = bstmr(G,[10:2:18]);
[g4, redinfo4] = bstmr(G,'MaxError',[0.01, 0.05]);
for i = 1:4
    figure(i); eval(['sigma(G,g' num2str(i) ');']);
end
```

Reference

[1] Zhou, K., "Frequency weighted L• error bounds," Syst. Contr. Lett., Vol. 21, 115-125, 1993.

[2] Safonov, M.G., and R.Y. Chiang, "Model Reduction for Robust Control: A Schur Relative Error Method," *International J. of Adaptive Control and Signal Processing,* Vol. 2, p. 259-272, 1988.

See Also

`reduce`	Top level model reduction function
`balancmr`	Balanced truncation via square-root method
`hankelmr`	Hankel minimum degree approximation
`schurmr`	Balanced truncation via Schur method
`ncfmr`	Balanced truncation for normalized coprime factors
`hankelsv`	Hankel singular value

Purpose

Replaces ureal atoms by summations of ureal and ucomplex (or ultidyn) atoms.

Syntax

```
MC = complexify(M,alpha)
MC = complexify(M,alpha,'ultidyn')
```

Description

The command complexify replaces ureal atoms with sums of ureal and ucomplex atoms using usubs. Optionally, the sum can consist of a ureal and ultidyn atom.

complexify is used to improve the conditioning of robust stability calculations (robuststab) for situations when there are predominantly ureal uncertain elements.

MC = complexify(M,alpha) results in each ureal atom in MC having the same Name and NominalValue as the corresponding ureal atom in M. If Range is the range of one ureal atom from M, then the range of the corresponding ureal atom in MC is

```
[Range(1)+alpha*diff(Range)/2 Range(2)-alpha*diff(Range)/2]
```

The net effect is that the same real range is covered with a real and complex uncertainty. The real parameter range is reduced by equal amounts at each end, and alpha represents (in a relative sense) the reduction in the total range. The ucomplex atom will add this reduction in range back into MC, but as a ball with real and imaginary parts.

The ucomplex atom has NominalValue of 0, and Radius equal to alpha*diff(Range). Its name is the name of the original ureal atom, appended with the characters '_cmpxfy'.

MC = complexify(M,alpha,'ultidyn') is the same, except that gain-bounded ultidyn atoms are used instead of ucomplex atoms. The ultidyn atom has its Bound equal to alpha*diff(Range).

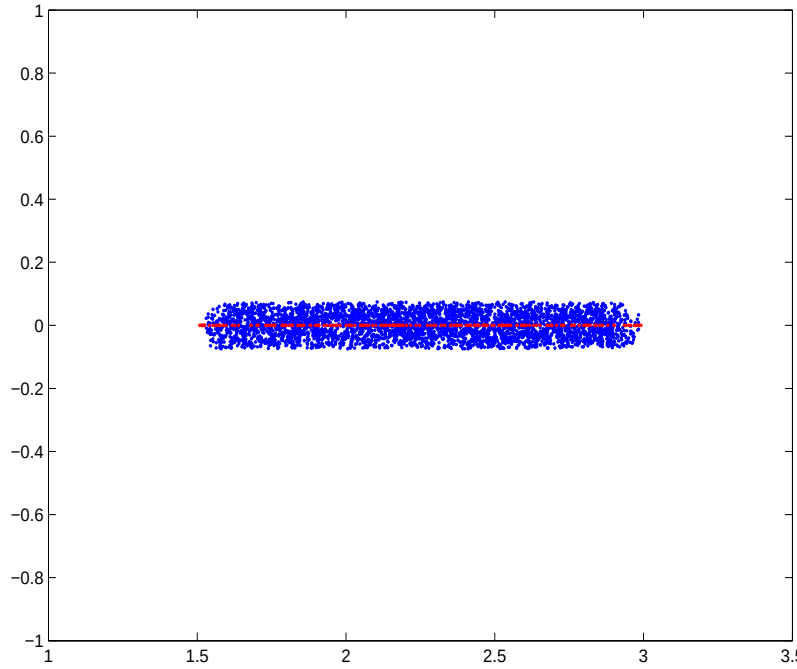
Example

See Robust Control Toolbox™ demo entitled "Getting Reliable Estimates of Robustness Margins" for an example of how complexify is used in robustness analysis.

For illustrative purposes only, create a uncertain real parameter, cast it to a

uncertain matrix, and apply a 10% complexification. Finally, make a scatter plot of the values that the complexified matrix (scalar) can take as well as the values of the original uncertain real parameter.

```
a = umat(ureal('a',2.25,'Range',[1.5 3]));
b = complexify(a,.1);
as = usample(a,200);
bs = usample(b,4000);
plot(real(bs(:)),imag(bs(:)),'.',real(as(:)),imag(as(:)),'r.')
```



See Also

`icomplexify`	Extract real part from `comlpexify` uncertainty
`robuststab`	Calculate robust stability margin

Purpose

Approximately solve constant-matrix, upper bound μ-synthesis problem

Syntax

```
[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure);
[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,opt);
[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,opt,qinit);
[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,opt,'random',N)
```

Description

`cmsclsyn` approximately solves the constant-matrix, upper bound μ-synthesis problem by minimization,

$$\min_{Q \in C^{r \times t}} \mu_{\Delta}(R + UQV)$$

for given matrices $R \in \mathbf{C}^{n \times m}$, $U \in \mathbf{C}^{n \times r}$, $V \in \mathbf{C}^{t \times m}$, and a set $\Delta \subset \mathbf{C}^{m \times n}$. This applies to constant matrix data in *R, U,* and *V*.

`[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure)` minimizes, by choice of Q. `QOPT` is the optimum value of Q, the upper bound of `mussv(R+U*Q*V,BLK)`, `BND`. The matrices `R,U` and `V` are constant matrices of the appropriate dimension. `BlockStructure` is a matrix specifying the perturbation blockstructure as defined for `mussv`.

`[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,OPT)` uses the options specified by `OPT` in the calls to `mussv`. See `mussv` for more information. The default value for `OPT` is `'cUsw'`.

`[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,OPT,QINIT)` initializes the iterative computation from Q = `QINIT`. Because of the nonconvexity of the overall problem, different starting points often yield different final answers. If `QINIT` is an N-D array, then the iterative computation is performed multiple times - the i'th optimization is initialized at Q = `QINIT(:,:,i)`. The output arguments are associated with the best solution obtained in this brute force approach.

`[QOPT,BND] = cmsclsyn(R,U,V,BlockStructure,OPT,'random',N)` initializes the iterative computation from `N` random instances of `QINIT`. If `NCU` is the number of columns of `U`, and `NRV` is the number of rows of `V`, then the approximation to solving the constant matrix μ synthesis problem is two-fold: only the upper bound for μ is minimized, and the minimization is not convex, hence the optimum is generally not found. If `U` is full column rank, or `V` is full row rank, then the problem can (and is) cast as a convex problem, [Packard,

Zhou, Pandey and Becker], and the global optimizer (for the upper bound for μ) is calculated.

Algorithm

The `cmsclsyn` algorithm is iterative, alternatively holding Q fixed, and computing the `mussv` upper bound, followed by holding the upper bound multipliers fixed, and minimizing the bound implied by choice of Q. If `U` or `V` is square and invertible, then the optimization is reformulated (exactly) as an linear matrix inequality, and solved directly, without resorting to the iteration.

Reference

Packard, A.K., K. Zhou, P. Pandey, and G. Becker, "A collection of robust control problems leading to LMI's," *30th IEEE Conference on Decision and Control,* Brighton, UK, 1991, p. 1245–1250.

See Also

`dksyn`	Synthesize a robust controller via D-K iteration
`hinfsyn`	Synthesize a H_∞ controller
`mussv`	Calculate bounds on the Structured Singular Value (μ)
`robuststab`	Calculate stability margins of uncertain systems
`robustperf`	Calculate performance margins of uncertain systems

Purpose

Coprime stability margin of plant-controller feedback loop

Syntax

```
[MARG,FREQ] = cpmargin(P,C)
[MARG,FREQ] = cpmargin(P,C,TOL)
```

Description

`[MARG,FREQ] = cpmargin(P,C)` calculates the normalized coprime factor/gap metric robust stability of the multivariable feedback loop consisting of `C` in negative feedback with `P`. `C` should only be the compensator in the feedback path, not any reference channels, if it is a two degree-of-freedom (2-DOF) architecture. The output `MARG` contains upper and lower bound for the normalized coprime factor/gap metric robust stability margin. `FREQ` is the frequency associated with the upper bound.

`[MARG,FREQ] = cpmargin(P,C,TOL)` specifies a relative accuracy `TOL` for calculating the normalized coprime factor/gap metric robust stability margin. (`TOL=1e-3` by default).

See Also

`loopmargin`	Comprehensive analysis of feedback loops
`gapmetric`	Compute the gap and the Vinnicombe gap metric
`wcmargin`	Worst-case disk gain/phase margins for plant-controller feedback loop

dcgainmr

Purpose

Reduced order model

Syntax

```
[sysr,syse,gain] = dcgainmr(sys,ord)
```

Description

`[sysr,syse,gain] = dcgainmr(sys,ord)` returns a reduced order model of a continuous-time LTI system SYS by truncating modes with least DC gain.

Specify your LTI continuous-time system in sys. The order is specified in `ord`.

This function returns:

- `sysr`—The reduced order models (a multidimensional array if `sys` is an LTI array)
- `syse`—The difference between `sys` and `sysr` (`syse=sys-sysr`)
- `gain`—The g-factors (dc-gains)

The DC gain of a complex mode

```
(1/(s+p))*c*b'
```

is defined as

```
norm(b)*norm(c)/abs(p)
```

See Also

`reduce`

Purpose

Quadratic decay rate of polytopic or affine P-systems

Syntax

```
[drate,P] = decay(ps,options)
```

Description

For affine parameter-dependent systems

$$E(p)\dot{x} = A(p)x,\ p(t) = (p_1(t), \ldots, p_n(t))$$

or polytopic systems

$$E(t)\dot{x} = A(t)x, \qquad (A, E) \in \mathrm{Co}\{(A_1, E_1), \ldots, (A_n, E_n)\},$$

`decay` returns the quadratic decay rate drate, i.e., the smallest $\alpha \in \mathbf{R}$ such that

$$A^{\mathrm{T}}QE + EQA^T < \alpha Q$$

holds for some Lyapunov matrix $Q > 0$ and all possible values of (A, E). Two control parameters can be reset via `options(1)` and `options(2)`:

- If `options(1)=0` (default), `decay` runs in fast mode, using the least expensive sufficient conditions. Set `options(1)=1` to use the least conservative conditions.
- `options(2)` is a bound on the condition number of the Lyapunov matrix P. The default is 10^9.

See Also

`quadstab`	Quadratic stability of polytopic or parameter-dependent systems
`pdlstab`	Robust stability of polytopic or affine parameter-dependent systems (P-system)
`psys`	Specification of uncertain state-space models

decinfo

Purpose

Describe how entries of matrix variable X relate to decision variables

Syntax

```
decinfo(lmisys)
decX = decinfo(lmisys,X)
```

Description

The function `decinfo` expresses the entries of a matrix variable X in terms of the decision variables $x_1, \ldots, x_N$. Recall that the decision variables are the free scalar variables of the problem, or equivalently, the free entries of all matrix variables described in `lmisys`. Each entry of X is either a hard zero, some decision variable x_n, or its opposite $-x_n$.

If `X` is the identifier of X supplied by `lmivar`, the command

```
decX = decinfo(lmisys,X)
```

returns an integer matrix `decX` of the same dimensions as X whose (i, j) entry is

- 0 if $X(i, j)$ is a hard zero
- n if $X(i, j) = x_n$ (the n-th decision variable)
- $-n$ if $X(i, j) = -x_n$

`decX` clarifies the structure of X as well as its entry-wise dependence on $x_1, \ldots, x_N$. This is useful to specify matrix variables with atypical structures (see `lmivar`).

`decinfo` can also be used in interactive mode by invoking it with a single argument. It then prompts the user for a matrix variable and displays in return the decision variable content of this variable.

Example 1

Consider an LMI with two matrix variables X and Y with structure:

- $X = x\, I_3$ with x scalar
- Y rectangular of size 2-by-1

If these variables are defined by

```
setlmis([])
X = lmivar(1,[3 0])
Y = lmivar(2,[2 1])
   :
   :
```

```
lmis = getlmis
```

the decision variables in X and Y are given by

```
dX = decinfo(lmis,X)

dX =
   1   0   0
   0   1   0
   0   0   1

dY = decinfo(lmis,Y)

dY =
   2
   3
```

This indicates a total of three decision variables x_1, x_2, x_3 that are related to the entries of X and Y by

$$X = \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_1 & 0 \\ 0 & 0 & x_1 \end{pmatrix}, \qquad Y = \begin{pmatrix} x_2 \\ x_3 \end{pmatrix}$$

Note that the number of decision variables corresponds to the number of free entries in X and Y when taking structure into account.

Example 2

Suppose that the matrix variable X is symmetric block diagonal with one 2-by-2 full block and one 2-by-2 scalar block, and is declared by

```
setlmis([])
X = lmivar(1,[2 1;2 0])
       :
lmis = getlmis
```

The decision variable distribution in X can be visualized interactively as follows:

```
decinfo(lmis)
```

```
There are 4 decision variables labeled x1 to x4 in this problem.

Matrix variable Xk of interest (enter k between 1 and 1, or 0 to
quit):

?> 1

The decision variables involved in X1 are among {-x1,...,x4}.
Their entry-wise distribution in X1 is as follows
        (0,j>0,-j<0 stand for 0,xj,-xj, respectively):

X1 :

   1   2   0   0
   2   3   0   0
   0   0   4   0
   0   0   0   4

            *********

Matrix variable Xk of interest (enter k between 1 and 1, or 0 to
quit):

?> 0
```

See Also

`lmivar`	Specify the matrix variables in an LMI problem
`mat2dec`	Return the vector of decision variables corresponding to particular values of the matrix variables
`dec2mat`	Given values of the decision variables, derive the corresponding values of the matrix variables `decnbr`

Purpose

Total number of decision variables in system of LMIs

Syntax

```
ndec = decnbr(lmisys)
```

Description

The function `decnbr` returns the number `ndec` of decision variables (free scalar variables) in the LMI problem described in `lmisys`. In other words, `ndec` is the length of the vector of decision variables.

Example

For an LMI system `lmis` with two matrix variables X and Y such that

- X is symmetric block diagonal with one 2-by-2 full block, and one 2-by-2 scalar block
- Y is 2-by-3 rectangular,

the number of decision variables is

```
ndec = decnbr(LMIs)

ndec =
      10
```

This is exactly the number of free entries in X and Y when taking structure into account (see `decinfo` for more details).

See Also

`dec2mat`	Given values of the decision variables, derive the corresponding values of the matrix variables `decnbr`
`decinfo`	Describe how the entries of a matrix variable X relate to the decision variables
`mat2dec`	Return the vector of decision variables corresponding to particular values of the matrix variables

dec2mat

Purpose

Given values of decision variables, derive corresponding values of matrix variables

Syntax

```
valX = dec2mat(lmisys,decvars,X)
```

Description

Given a value `decvars` of the vector of decision variables, `dec2mat` computes the corresponding value `valX` of the matrix variable with identifier `X`. This identifier is returned by `lmivar` when declaring the matrix variable.

Recall that the decision variables are all free scalar variables in the LMI problem and correspond to the free entries of the matrix variables $X_1, \ldots, X_K$. Since LMI solvers return a feasible or optimal value of the vector of decision variables, `dec2mat` is useful to derive the corresponding feasible or optimal values of the matrix variables.

Example

See the description of `feasp`.

See Also

`mat2dec`	Return the vector of decision variables corresponding to particular values of the matrix variables
`decnbr`	Give the total number of decision variables in a system of LMIs
`decinfo`	Describe how the entries of a matrix variable X relate to the decision variables

Purpose

Help specify $c^T x$ objectives for mincx solver

Syntax

```
[V1,...,Vk] = defcx(lmisys,n,X1,...,Xk)
```

Description

`defcx` is useful to derive the c vector needed by `mincx` when the objective is expressed in terms of the matrix variables.

Given the identifiers `X1,...,Xk` of the matrix variables involved in this objective, `defcx` returns the values `V1,...,Vk` of these variables when the n-th decision variable is set to one and all others to zero.

See Also

`mincx`	Minimize a linear objective under LMI constraints
`decinfo`	Describe how the entries of a matrix variable X relate to the decision variables

Purpose

Remove an LMI from system of LMIs

Syntax

```
newsys = dellmi(lmisys,n)
```

Description

`dellmi` deletes the n-th LMI from the system of LMIs described in `lmisys`. The updated system is returned in `newsys`.

The ranking `n` is relative to the order in which the LMIs were declared and corresponds to the identifier returned by `newlmi`. Since this ranking is not modified by deletions, it is safer to refer to the remaining LMIs by their identifiers. Finally, matrix variables that only appeared in the deleted LMI are removed from the problem.

Example

Suppose that the three LMIs

$$A_1^T X_1 + X_1 A_1 + Q_1 < 0$$

$$A_2^T X_2 + X_2 A_2 + Q_2 < 0$$

$$A_3^T X_3 + X_3 A_3 + Q_3 < 0$$

have been declared in this order, labeled `LMI1`, `LMI2`, `LMI3` with `newlmi`, and stored in `lmisys`. To delete the second LMI, type

```
lmis = dellmi(lmisys,LMI2)
```

`lmis` now describes the system of LMIs

$$A_1^T X_1 + X_1 A_1 + Q_1 < 0 \qquad \textbf{(5-3)}$$

$$A_3^T X_3 + X_3 A_3 + Q_3 < 0 \qquad \textbf{(5-4)}$$

and the second variable X_2 has been removed from the problem since it no longer appears in the system (9-4)–(9-5).

To further delete (9-5), type

```
lmis = dellmi(lmis,LMI3)
```

or equivalently

```
lmis = dellmi(lmis,3)
```

Note that (9-5) has retained its original ranking after the first deletion.

See Also

`newlmi`	Attach an identifying tag to LMIs
`lmiedit`	Specify or display systems of LMIs as MATLAB® expressions
`lmiinfo`	Interactively retrieve information about the variables and term content of LMIs

delmvar

Purpose

Remove one matrix variable from LMI problem

Syntax

```
newsys = delmvar(lmisys,X)
```

Description

`delmvar` removes the matrix variable X with identifier `X` from the list of variables defined in `lmisys`. The identifier `X` should be the second argument returned by `lmivar` when declaring X. All terms involving X are automatically removed from the list of LMI terms. The description of the resulting system of LMIs is returned in `newsys`.

Example

Consider the LMI

$$0 < \begin{pmatrix} A^T Y + B^T Y A + Q & CX + D \\ X^T C^T + D^T & -(X + X^T) \end{pmatrix}$$

involving two variables X and Y with identifiers `X` and `Y`. To delete the variable X, type

```
lmisys = delmvar(lmisys,X)
```

Now `lmisys` describes the LMI

$$0 < \begin{pmatrix} A^T Y B + B^T Y A + Q & D \\ D^T & 0 \end{pmatrix}$$

with only one variable Y. Note that Y is still identified by the label `Y`.

See Also

`lmivar`	Specify the matrix variables in an LMI problem
`setmvar`	Instantiate a matrix variable and evaluate all LMI terms involving this matrix variable
`lmiinfo`	Interactively retrieve information about the variables and term content of LMIs

Purpose

Diagonalize vector of uncertain matrices and systems

Syntax

```
v = diag(x)
```

Description

If x is a vector of uncertain system models or matrices, `diag(x)` puts x on the main diagonal. If x is a matrix of uncertain system models or matrices, `diag(x)` is the main diagonal of `x. diag(diag(x))` is a diagonal matrix of uncertain system models or matrices.

Example

The statement produces a diagonal system `mxg` of size 4-by-4. Given multivariable system `xx`, a vector of the diagonal elements of `xxg` is found using `diag`.

```
x = rss(3,4,1);
xg = frd(x,logspace(-2,2,80));
size(xg)

FRD model with 4 output(s) and 1 input(s), at 80 frequency
point(s).

mxg = diag(xg);
size(mxg)
FRD model with 4 output(s) and 4 input(s), at 80 frequency
point(s).

xxg = [xg(1:2,1) xg(3:4,1)];
m = diag(xxg);
size(m)
FRD model with 2 output(s) and 1 input(s), at 80 frequency
point(s).
```

See Also

`append` Group models by appending their inputs and outputs

dkitopt

Purpose

Create options object for use with `dksyn`

Syntax

```
dkitopt
opt = dkitopt
opt = dkitopt('name1',value1,'name2',value2,...)
```

Description

`opt=dkitopt` creates an object of class `dkitopt`, used to define user-specified options in the D-K iteration procedure performed with `dksyn`. All properties of `opt` are set to their default values.

`options = dkitopt('name1',value1,'name2',value2,...)` uses the trailing Property/Value pairs to set user-specified values of individual option properties. a `dkitopt` object called `options` with specific values assigned to certain properties. Property names specification is not case-insensitive, and only enough charaacers to uniquely specify the property name are required.

If the `AutoIter` property is set to `'off'`, the D-K iteration procedure is interactive. You are prompted to fit the D-Scale data and provide input on the control design process.

This table lists the `dkitopt` object properties.

Object Property	Description
`FrequencyVector`	Frequency vector used for analysis. Default is an empty matrix ([]) which results in the frequency range and number of points chosen automatically.
`InitialController`	Controller used to initiate first iteration. Default is an empty SS object.
`AutoIter`	Automated μ-synthesis mode. Default is `'on'`.
`DisplayWhileAutoIter`	Displays iteration progress in `AutoIter` mode. Default is `'off'`.
`StartingIterationNumber`	Starting iteration number. Default is 1.

Object Property	Description
NumberOfAutoIterations	Number of D-K iterations to perform. Default is 10.
AutoScalingOrder	Maximum state order for fitting D-scaling data. Default is 5.
AutoIterSmartTerminate	Automatic termination of iteration procedure based on progress of design iteration. Default is 'on'.
AutoIterSmartTerminateTol	Tolerance used by AutoIterSmartTerminate. Default is 0.005.
Default	Structure of property default values.
Meaning	Structure text description of each property.

If the AutoIterSmartTerminate property is on, and a stopping criteria (described below) is satisfied, the iteration (performed by dksyn) will terminate before reaching the specified number of automated iterations (value of NumberOfAutoIterations). The stopping criteria involves the objective value (peak value, across frequency, of the upper bound for μ) in the current iteration, denoted v_0, as well as the previous two iterations, (denoted v_{-1} and v_{-2}) and the value of AutoIterSmartTerminateTol. If

$$\left|v_0 - v_{-1}\right| < AutoIterSmartTerminateTol * v_0$$

and

$$\left|v_{-1} - v_{-2}\right| < AutoIterSmartTerminateTol * v_0$$

then the stopping criteria is satisfied (for lack of progress). The stopping criteria is also satisfied if

$$v_0 > v_{-1} + 20 * AutoIterSmartTerminateTol * v_0$$

which captures a significant increase (undesirable) in the objective.

Example

This statement creates a `dkitopt` options object called `opt` with default property values.

```
opt = dkitopt
Property Object Values:
              FrequencyVector: []
            InitialController: [0x0 ss]
                     AutoIter: 'on'
          DisplayWhileAutoIter: 'off'
        StartingIterationNumber: 1
         NumberOfAutoIterations: 10
               AutoScalingOrder: 5
         AutoIterSmartTerminate: 'on'
      AutoIterSmartTerminateTol: 0.0050
                        Default: [1x1 struct]
                        Meaning: [1x1 struct]
```

The properties can be modified directly with assignment statements: here user-specified values for the frequency vector, the number of iterations, and the maximum state dimension of the D-scale fittings are set.

```
opt.FrequencyVector = logspace(-2,3,80);
opt.NumberOfAutoIterations = 16;
opt.AutoScalingOrder = 16;
opt
Property Object Values:
              FrequencyVector: [1x80 double]
            InitialController: [0x0 ss]
                     AutoIter: 'on'
          DisplayWhileAutoIter: 'off'
        StartingIterationNumber: 1
         NumberOfAutoIterations: 16
               AutoScalingOrder: 16
         AutoIterSmartTerminate: 'on'
      AutoIterSmartTerminateTol: 0.0050
                        Default: [1x1 struct]
                        Meaning: [1x1 struct]
```

The same property/value pairs may be set with a single call to `dkitopt`.

```
opt = dkitopt('FrequencyVector',logspace(-2,3,80),...
                'NumberOfAutoIterations',16,...
                'AutoScalingOrder',9);
```

Algorithm

The `dksyn` command stops iterating before the total number of automated iterations (`'NumberOfAutoIterations'`) if `'AutoIterSmartTerminate'` is set to `'on'` and a stopping criterion is satisfied. The stopping criterion involves the m(i) value of the current ith iteration, `m(`i`-1)` and `m(`i`-2)`, the previous two iterations and the options property `'AutoIterSmartTerminateTol'`. The D–K iteration procedure automatically terminates if the difference between each of the three μ values is less than the relative tolerance of `AutoIterSmartTerminateTol` ×μ(i) or the current μ value μ(i) has increased relative to the μ value of the previous iteration μ(i–1) by 20×`AutoIterSmartTerminateTol`.

See Also

`dksyn`	Synthesize a robust controller via D-K iteration
`h2syn`	Synthesize a H2 optimal controller
`hinfsyn`	Synthesize a H_∞ optimal controller
`mussv`	Calculate bounds on the Structured Singular Value (μ)
`robopt`	Create a `robustperf/robuststab` option object
`robuststab`	Calculate stability margins of uncertain systems
`robustperf`	Calculate performance margins of uncertain systems
`wcgopt`	Create a wcgain option object

dksyn

Purpose

Synthesis of robust controller via μ-synthesis D-K iteration

Syntax

```
k = dksyn(p,nmeas,ncont)
[k,clp,bnd] = dksyn(p,nmeas,ncont)
[k,clp,bnd] = dksyn(p,nmeas,ncont,opt)
[k,clp,bnd,dkinfo] = dksyn(p,nmeas,ncont,prevdkinfo,opt)
[k,clp,bnd,dkinfo] = dksyn(p)
```

Description

`dksyn` synthesizes a robust controller via D-K iteration. The D-K iteration procedure is an approximation to μ-synthesis control design. The objective of μ-synthesis is to minimize the structure singular value μ of the corresponding robust performance problem associated with the uncertain system `p`. The uncertain system `p` is an open-loop interconnection containing known components including the nominal plant model, uncertain parameters, `ucomplex`, and unmodeled LTI dynamics, `ultidyn`, and performance and uncertainty weighting functions. You use weighting functions to include magnitude and frequency shaping information in the optimization. The control objective is to synthesize a stabilizing controller `k` that minimizes the robust performance μ value, which corresponds to `bnd`.

The D-K iteration procedure involves a sequence of minimizations, first over the controller variable K (holding the D variable associated with the scaled μ upper bound fixed), and then over the D variable (holding the controller K variable fixed). The D-K iteration procedure is not guaranteed to converge to the minimum μ value, but often works well in practice. `dksyn` automates the D-K iteration procedure and the options object `dkitopt` allows you to customize its behavior.

`[k,clp,bnd] = dksyn(p,nmeas,ncont)` synthesizes a robust controller `k` for the uncertain open-loop plant model `p` via the D-K iteration. `p` is a `uss` and has `nmeas` measurement outputs and `ncont` control inputs. It is assumed that the measurement outputs and control inputs correspond to the last `nmeas` outputs and `ncont` inputs. `k` is the controller, `clp` is the closed-loop interconnection and `bnd` is the robust performance bound for the uncertain closed-loop system `clp`. `p`, `k`, `clp`, and `bnd` are related as follows:

```
clp = lft(p,k);
bnd = robustperf(clp);
```

`[k,clp,bnd] = dksyn(p,nmeas,ncont,opt)` specifies options described in `dkitopt`. `opt` is created via the command `opt=dkitopt`. See `dkitopt` for more details.

`[k,clp,bnd,dkinfo] = dksyn(p,nmeas,ncont,prevdkinfo,opt)` allows you to use information from a previous `dksyn` iteration. `prevdkinfo` is a structure from a previous attempt at designing a robust controller using `dksyn`. `prevdkinfo` is used when the `dksyn` starting iteration is not 1 (`opt.StartingIterationNumber = 1`) to determine the correct D-scalings to initiate the iteration procedure.

`dkinfo` is an N-by-1 cell array where N is the total number of iterations performed. The `i`th cell contains a structure with the following fields:.

Field	Description
`K`	Controller at `i`th iteration, a `ss` object
`Bnds`	Robust performance bound on the closed-loop system (`double`)
`DL`	Left D-scale, a `ss` object
`DR`	Right D-scale, a `ss` object
`MussvBnds`	Upper and lower μ bounds, a `frd` object
`MussvInfo`	Structure returned from `mussv` at each iteration.

`k = dksyn(p)` is a valid calling argument provided p is a `uss` object and has two-input/two-output partitioning as defined by `mktito`.

Example

The following statements create a robust performance control design for an unstable, uncertain single-input/single-output plant model. The nominal plant model, G, is an unstable first order system $\frac{s}{s-1}$.

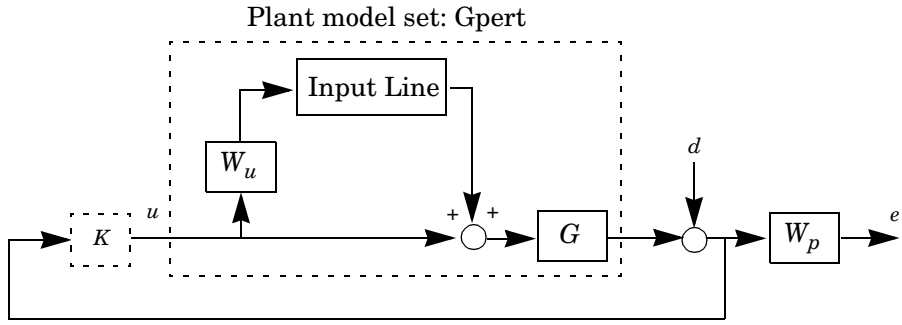
```
G = tf(1,[1 -1]);
```

The model itself is uncertain. At low frequency, below 2 rad/s, it can vary up to 25% from its nominal value. Around 2 rad/s the percentage variation starts to

increase and reaches 400% at approximately 32 rad/s. The percentage model uncertainty is represented by the weight `Wu` which corresponds to the frequency variation of the model uncertainty and the uncertain LTI dynamic object `InputUnc`.

```
Wu = 0.25*tf([1/2 1],[1/32 1]);
InputUnc = ultidyn('InputUnc',[1 1]);T
```

The uncertain plant model `Gpert` represents the model of the physical system to be controlled.




```
Gpert = G*(1+InputUnc*Wu);
```

The robust stability objective is to synthesize a stabilizing LTI controller for all the plant models parameterized by the uncertain plant model, `Gpert`. The control interconnection structure is shown in the following figure.

The goal is to synthesize a controller that stabilizes and achieves the closed-loop performance objectives for all possible plant models in `Gpert`. The performance objective is defined as a weighted sensitivity minimization problem. See the preceding figure.

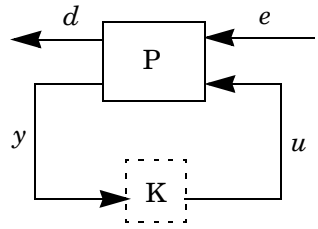
The sensitivity function, S, is defined as $S = \frac{1}{1+PK}$ where `P` is the plant model and `K` is the controller. A weighted sensitivity minimization problem selects a weight `Wp`, which corresponds to the *inverse* of the desired sensitivity function of the closed-loop system as a function of frequency. Hence the product of the sensitivity weight `Wp` and actual closed-loop sensitivity function is less than 1 across all frequencies. The sensitivity weight `Wp` has a gain of 100 at low

frequency, begins to decrease at 0.006 rad/s, and reaches a minimum magnitude of 0.25 after 2.4 rad/s.

```
Wp = tf([1/4 0.6],[1 0.006]);
```

The defined sensitivity weight `Wp` implies that the desired disturbance rejection should be at least 100:1 disturbance rejection at DC, rise slowly between 0.006 and 2.4 rad/s, and allow the disturbance rejection to increase above the open-loop level, 0.25, at high frequency.

When the plant model is uncertain, the closed-loop performance objective is to achieve the desired sensitivity function for all plant models defined by the uncertain plant model, `Gpert`. The performance objective for an uncertain system is a robust performance objective. A block diagram of this uncertain closed-loop system illustrating the performance objective (closed-loop transfer function from $d \rightarrow e$) is shown.



From the definition of the robust performance control objective, the weighted, uncertain control design interconnection model, which includes the robustness and performance objectives, can be constructed and is denoted by `P`. The robustness and performance weights are selected such that if the robust performance structure singular value, `bnd`, of the closed-loop uncertain system, `clp`, is less than 1 then the performance objectives have been achieved for all the plant models in the model set.

You can form the uncertain transfer matrix `P` from `[d; u]` to `[e; y]` using the following commands.

```
P = [Wp; 1 ]*[1 Gpert];
[K,clp,bnd] = dksyn(P,1,1);
bnd
bnd =
```

```
    0.6860
P = [Wp; 1 ]*[1 Gpert];
[K,clp,bnd] = dksyn(P,1,1);
bnd
bnd =
    0.6860
```

The controller `K` achieves a robust performance μ value `bnd` of 0.685. Therefore you have achieved the robust performance objectives for the given problem.

You can use the `robustperf` command to analyze the closed-loop robust performance of `clp`.

```
[rpnorm,wcf,wcu,report] = robustperf(clp);
disp(report{1})
```

```
Uncertain system, clp, achieves robust performance. The analysis
showed clp can tolerate 146% of the model uncertainty and achieve
the performance and stability objectives. A model uncertainty
exists of size 146% that results in a peak gain performance of
0.686 at 0.569 rad/s.
```

Algorithm

The *D-K* iteration procedure is an approximation to μ-synthesis control design. It involves a sequence of minimizations, first over the controller variable *K* (holding the *D* variable associated with the scaled μ upper bound fixed), and then over the *D* variable (holding the controller *K* variable fixed). The *D-K* iteration procedure is not guaranteed to converge to the minimum μ value, but often works well in practice.

`dksyn` automates the *D-K* iteration procedure. Internally, the algorithm works with the generalized scaled plant model `P`, which is extracted from a `uss` object using the command `lftdata`.

The following is a list of what occurs during a single, complete step of the *D-K* iteration.

1. (In the first iteration, this step is skipped.) The μ calculation (from the previous step) provides a frequency-dependent scaling matrix, D_f. The fitting procedure fits these scalings with rational, stable transfer function matrices. After fitting, plots of

$$\bar{\sigma}(D_f(j\omega)F_L(P,K)(j\omega)D_f^{-1}(j\omega))$$

and

$$\bar{\sigma}(\hat{D}_f(j\omega)F_L(P,K)(j\omega)\hat{D}_f^{-1}(j\omega))$$

are shown for comparison.

(In the first iteration, this step is skipped.) The rational $\hat{D}$ is absorbed into the open-loop interconnection for the next controller synthesis. Using either the previous frequency-dependent D's or the just-fit rational $\hat{D}$, an estimate of an appropriate value for the H_∞ norm is made. This is simply a conservative value of the scaled closed-loop H_∞ norm, using the most recent controller and either a frequency sweep (using the frequency-dependent D's) or a state-space calculation (with the rational D's).

2. (The first iteration begins at this point.) A controller is designed using H_∞ synthesis on the scaled open-loop interconnection. If you set the `DisplayWhileAutoIter` field in `dkitopt` to `'on'`, the following information is displayed:
   a. The progress of the γ-iteration is displayed.
   b. The singular values of the closed-loop frequency response are plotted.
   c. You are given the option to change the frequency range. If you change it, all relevant frequency responses are automatically recomputed.
   d. You are given the option to rerun the H_∞ synthesis with a set of modified parameters if you set the `AutoIter` field in `dkitopt` to `'off'`. This is convenient if, for instance, the bisection tolerance was too large, or if `maximum gamma value` was too small.
3. The structured singular value of the closed-loop system is calculated and plotted.
4. An iteration summary is displayed, showing all the controller order, as well as the peak value of μ of the closed-loop frequency responses.
5. The choice of stopping or performing another iteration is given.

Subsequent iterations proceed along the same lines without the need to reenter the iteration number. A summary at the end of each iteration is updated to reflect data from all previous iterations. This often provides valuable information about the progress of the robust controller synthesis procedure.

Interactive Fitting of *D*-Scalings

Setting the `AutoIter` field in `dkitopt` to `'off'` requires that you interactively fit the *D*-scales each iteration. During step 2 of the *D-K* iteration procedure, you are prompted to enter your choice of options for fitting the *D*-scaling data. You press return after, the following is a list of your options.

```
Enter Choice (return for list):
Choices:
nd          Move to Next D-Scaling
nb          Move to Next D-Block

i           Increment Fit Order
d           Decrement Fit Order
apf         Auto-PreFit
mx 3        Change Max-Order to 3
at 1.01     Change Auto-PreFit tol to 1.01
0           Fit with zeroth order
2           Fit with second order
n           Fit with n'th order
e           Exit with Current Fittings
s           See Status
```

- `nd` and `nb` allow you to move from one *D*-scale data to another. `nd` moves to the next scaling, whereas `nb` moves to the next scaling block. For scalar *D*-scalings, these are identical operations, but for problems with full *D*-scalings, (perturbations of the form δI) they are different. In the (1,2) subplot window, the title displays the *D*-scaling block number, the row/column of the scaling that is currently being fitted, and the order of the current fit (with `d` for data when no fit exists).
- You can increment or decrement the order of the current fit (by 1) using `i` and `d`.
- `apf` automatically fits each *D*-scaling data. The default maximum state order of individual *D*-scaling is 5. The `mx` variable allows you to change the maximum *D*-scaling state order used in the automatic prefitting routine. `mx` must be a positive, nonzero integer. `at` allows you to define how close the rational, scaled μ upper bound is to approximate the actual μ upper bound in a norm sense. Setting `at` to `1` would require an exact fit of the *D*-scale data, and is not allowed. Allowable values for `at` are greater than 1. This setting

plays a role (mildly unpredictable, unfortunately) in determining where in the (*D*,*K*) space the *D-K* iteration converges.

- Entering a positive integer at the prompt will fit the current *D*-scale data with that state order rational transfer function.
- e exits the *D*-scale fitting to continue the *D-K* iteration.
- The variable s displays a status of the current and fits.

Limitations

There are two shortcomings of the *D-K* iteration control design procedure:

- Calculation of the structured singular value $\mu_\Delta(\cdot)$ is approximated by its upper bound. This is not a serious problem because the value of μ and its upper bound are often close.
- The *D-K* iteration is not guaranteed to converge to a global, or even local minimum [SteD]. This is a serious problem, and represents the biggest limitation of the design procedure.

In spite of these drawbacks, the *D-K* iteration control design technique appears to work well on many engineering problems. It has been applied to a number of real-world applications with success. These applications include vibration suppression for flexible structures, flight control, chemical process control problems, and acoustic reverberation suppression in enclosures.

Reference

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- Balas, G.J., A.K. Packard, and J.T. Harduvel, "Application of μ-synthesis techniques to momentum management and attitude control of the space station," *AIAA Guidance, Navigation and Control Conference,* New Orleans, August 1991.
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- Packard, A., J. Doyle, and G. Balas, "Linear, multivariable robust control with a μ perspective," *ASME Journal of Dynamic Systems, Measurement and Control,* 50th Anniversary Issue, Vol. 115, no. 2b, June 1993, p. 310-319.

- [SteD:] Stein, G., and J. Doyle, "Beyond singular values and loopshapes," *AIAA Journal of Guidance and Control*, Vol. 14, No. 1, January, 1991, p. 5-16.

See Also

`dkitopt`	Create a dksyn options object
`h2syn`	Synthesize a H_2 optimal controller
`hinfsyn`	Synthesize a H_∞ optimal controller
`loopmargin`	Comprehensive analysis of feedback loop
`mktito`	Create a two-input/two-output LTI partition
`mussv`	Calculate bounds on the Structured Singular Value (μ)
`robuststab`	Calculate stability margins of uncertain systems
`robustperf`	Calculate performance margins of uncertain systems
`wcgain`	Calculate worst-case gain of a system
`wcsens`	Calculate worst-case sensitivities for feedback loop
`wcmargin`	Calculate worst-case margins for feedback loop

Purpose

Interpret disk gain and phase margins

Syntax

```
dmplot
dmplot(diskgm)
[dgm,dpm] = dmplot
```

Description

`dmplot` plots disk gain margin (`dgm`) and disk phase margin (`dpm`). Both margins are derived from the largest disk that

- Contains the critical point (-1,0)
- Does not intersect the Nyquist plot of the open-loop response L

`diskgm` is the radius of this disk and a lower bound on the classical gain margin.

`dmplot(diskgm)` plots the maximum allowable phase variation as a function of the actual gain variation for a given disk gain margin `diskgm` (the maximum gain variation being `diskgm`). The closed-loop system is guaranteed to remain stable for all combined gain/phase variations inside the plotted ellipse.

`[dgm,dpm] = dmplot` returns the data used to plot the gain/phase variation ellipse.

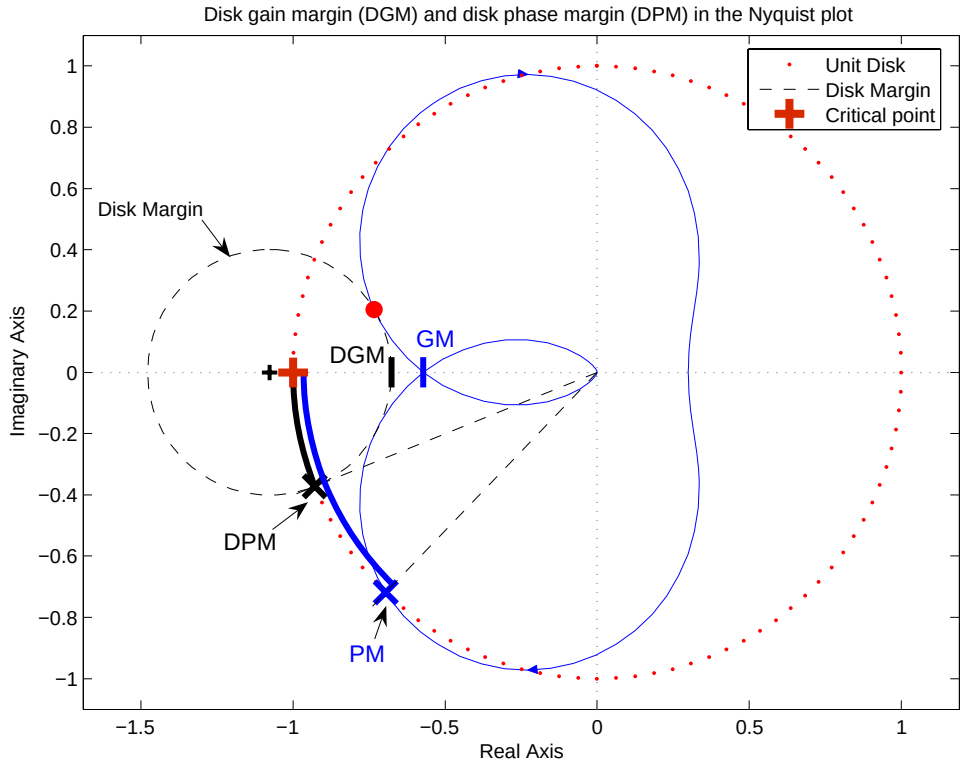
Example

When you call `dmplot` (without an argument), the resulting plot shows a comparison of a disk margin analysis with the classical notations of gain and phase margins. The Nyquist plot is of the loop transfer function L(s)

$$L(s) = \frac{\frac{s}{30} + 1}{(s+1)(s^2 + 1.6s + 16)}$$

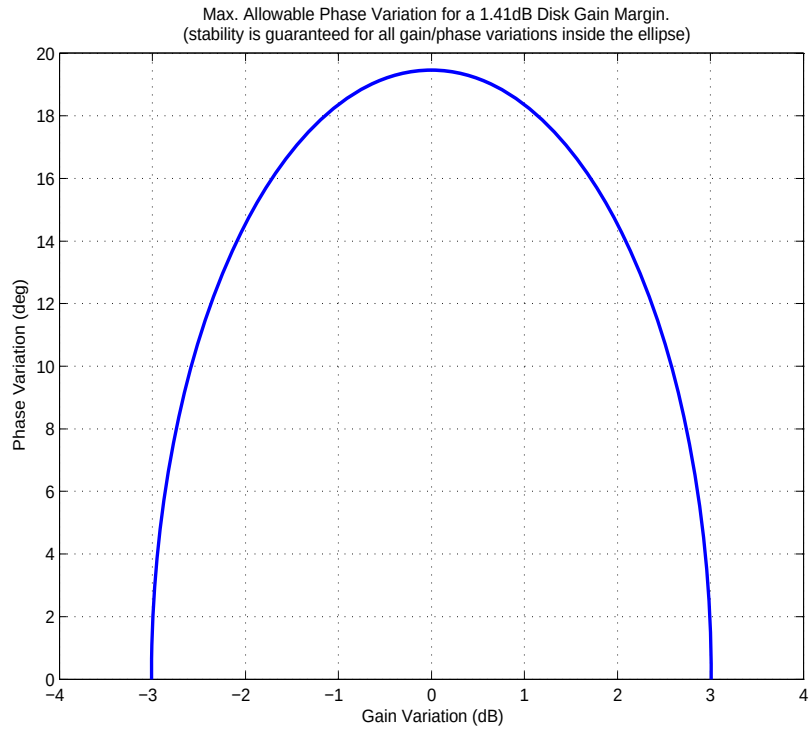
- The Nyquist plot of L corresponds to the blue line.
- The unit disk corresponds to the dotted red line.
- GM and PM indicate the location of the classical gain and phase margins for the system L.
- DGM and DPM correspond to the disk gain and phase margins, respectively. The disk margins provide a lower bound on classical gain and phase margins.

- The disk margin circle, represented by the dashed black line, corresponds to the largest disk centered at `(DGM + 1/DGM)/2` that just touches the loop transfer function L. This location is indicated by the red dot.



The x-axis corresponds to the gain variation, in dB, and the y-axis corresponds to the phase variation allowable, in degrees. For a disk gain margin corresponding to 3 dB (1.414), the closed-loop system is stable for all phase and gain variations inside the blue ellipse. For example, the closed-loop system can simultaneously tolerate +/- 2 dB gain variation and +/- 14 deg phase variations.

```
dmplot(1.414)
```



Reference

Barrett, M.F., Conservatism with robustness tests for linear feedback control systems, Ph.D. Thesis, Control Science and Dynamical Systems, University of Minnesota, 1980.

Blight, J.D., R.L. Dailey, and Gangsass, D., "Practical control law design for aircraft using multivariable techniques," *International Journal of Control*, Vol. 59, No. 1, 1994, 93-137.

Bates, D., and I. Postlethwaite, Robust Multivariable Control of Aerospace Systems, Delft University Press, Delft, The Netherlands, ISBN: 90-407-2317-6, 2002.

See Also

`loopmargin`	Comprehensive stability analysis of feedback loops
`wcmargin`	Calculate worst-case margins for feedback loop

drawmag

Purpose

Mouse-based tool for sketching and fitting

Syntax

```
[sysout,pts] = drawmag(data)
[sysout,pts] = drawmag(data,init_pts)
```

Description

`drawmag` interactively uses the mouse in the plot window to create `pts` (the `frd` object) and `sysout` (a stable minimum-phase `ss` object), which approximately fits the frequency response (magnitude) in `pts`.

Input arguments:

`data`	Either a frequency response object that is plotted as a reference, or a constant matrix of the form $[x_{min}\ x_{max}\ y_{min}\ y_{max}]$ specifying the plot window on the data.
`init_pts`	Optional `frd` objects of initial set of points

Output arguments:

`sysout`	Stable, minimum-phase `ss` object that approximately fits, in magnitude, the `pts data`.
`pts`	Frequency response of points.

While `drawmag` is running, all interaction with the program is through the mouse and/or the keyboard. The mouse, if there is one, must be in the plot window. The program recognizes several commands:

- Clicking the mouse button adds a point at the crosshairs. If the crosshairs are outside the plotting window, the points are plotted when the fitting, windowing, or replotting mode is invoked. Typing `a` is the same as clicking the mouse button.
- Typing `r` removes the point with frequency nearest that of the crosshairs.
- Typing any integer between `0` and `9` fits the existing points with a transfer function of that order. The fitting routine approximately minimizes the maximum error in a log sense. The new fit is displayed along with the points, and the most recent previous fit, if it exists.
- Typing `w` uses the crosshair location as the initial point in creating a window. Moving the crosshairs and clicking the mouse or pressing any key then gives

a second point at the new crosshair location. These two points define a new window on the data, which is immediately replotted. This is useful in fine tuning parts of the data. You can call windowing repeatedly.

- Typing `p` simply replots the data using a window that covers all the current data points as well as whatever was specified in `in`. Typically used after windowing to view all the data.
- Typing `k` invokes the keyboard using the `keyboard` command. You should exercise caution when using this option, as it can wreak havoc on the program if variables are changed.

See Also

`ginput`	Graphically input information from mouse
`loglog`	Plotting frequency responses on log-log scale

evallmi

Purpose

Given particular instance of decision variables, evaluate all variable terms in system of LMIs

Syntax

```
evalsys = evallmi(lmisys,decvars)
```

Description

`evallmi` evaluates all LMI constraints for a particular instance `decvars` of the vector of decision variables. Recall that `decvars` fully determines the values of the matrix variables $X_1, \ldots, X_K$. The "evaluation" consists of replacing all terms involving $X_1, \ldots, X_K$ by their matrix value. The output `evalsys` is an LMI system containing only constant terms.

The function `evallmi` is useful for validation of the LMI solvers' output. The vector returned by these solvers can be fed directly to `evallmi` to evaluate all variable terms. The matrix values of the left- and right-hand sides of each LMI are then returned by `showlmi`.

Observation

`evallmi` is meant to operate on the output of the LMI solvers. To evaluate all LMIs for particular instances of the matrix variables $X_1, \ldots, X_K$, first form the corresponding decision vector x with `mat2dec` and then call `evallmi` with x as input.

Example

Consider the feasibility problem of finding $X > 0$ such that

$$A^T X A - X + I < 0$$

where $A = \begin{pmatrix} 0.5 & -0.2 \\ 0.1 & -0.7 \end{pmatrix}$. This LMI system is defined by:

```
setlmis([])
X = lmivar(1,[2 1]) % full symmetric X

lmiterm([1 1 1 X],A',A) % LMI #1: A'*X*A
lmiterm([1 1 1 X],-1,1) % LMI #1: -X
lmiterm([1 1 1 0],1) % LMI #1: I
lmiterm([-2 1 1 X],1,1) % LMI #2: X
lmis = getlmis
```

To compute a solution `xfeas`, call `feasp` by

```
[tmin,xfeas] = feasp(lmis)
```

The result is

```
tmin =
   -4.7117e+00

xfeas' =
   1.1029e+02  -1.1519e+01  1.1942e+02
```

The LMI constraints are therefore feasible since `tmin` < 0. The solution X corresponding to the feasible decision vector `xfeas` would be given by `X = dec2mat(lmis,xfeas,X)`.

To check that `xfeas` is indeed feasible, evaluate all LMI constraints by typing

```
evals = evallmi(lmis,xfeas)
```

The left- and right-hand sides of the first and second LMIs are then given by

```
[lhs1,rhs1] = showlmi(evals,1)
[lhs2,rhs2] = showlmi(evals,2)
```

and the test

```
eig(lhs1-rhs1)
ans =
   -8.2229e+01
   -5.8163e+01
```

confirms that the first LMI constraint is satisfied by `xfeas`.

See Also

`showlmi`	Return the left- and right-hand sides of an LMI after evaluation of all variable terms
`setmvar`	Instantiate a matrix variable and evaluate all LMI terms involving this matrix variable
`dec2mat`	Given values of the decision variables, derive the corresponding values of the matrix variables
`mat2dec`	Return the vector of decision variables corresponding to particular values of the matrix variables

Purpose

Find a solution to a given system of LMIs

Syntax

```
[tmin,xfeas] = feasp(lmisys,options,target)
```

Description

The function `feasp` computes a solution `xfeas` (if any) of the system of LMIs described by `lmisys`. The vector `xfeas` is a particular value of the decision variables for which all LMIs are satisfied.

Given the LMI system

$$N^T LxN \le M^T R(x)M, \qquad \textbf{(5-5)}$$

`xfeas` is computed by solving the auxiliary convex program:

Minimize t subject to $N^T L(x)N - M^T R(x)M \le tI$.

The global minimum of this program is the scalar value `tmin` returned as first output argument by `feasp`. The LMI constraints are feasible if `tmin` ð 0 and strictly feasible if `tmin` < 0. If the problem is feasible but not strictly feasible, `tmin` is positive and very small. Some post-analysis may then be required to decide whether `xfeas` is close enough to feasible.

The optional argument `target` sets a target value for `tmin`. The optimization code terminates as soon as a value of t below this target is reached. The default value is `target` = 0.

Note that `xfeas` is a solution in terms of the decision variables and not in terms of the matrix variables of the problem. Use `dec2mat` to derive feasible values of the matrix variables from `xfeas`.

Control Parameters

The optional argument `options` gives access to certain control parameters for the optimization algorithm. This five-entry vector is organized as follows:

- `options(1)` is not used
- `options(2)` sets the maximum number of iterations allowed to be performed by the optimization procedure (100 by default)
- `options(3)` resets the *feasibility radius*. Setting options(3) to a value $R > 0$ further constrains the decision vector $x = (x_1, \ldots, x_N)$ to lie within the ball

$$\sum_{i=1}^{N} x_i^2 < R^2$$

In other words, the Euclidean norm of `xfeas` should not exceed R. The feasibility radius is a simple means of controlling the magnitude of solutions. Upon termination, `feasp` displays the *f-radius saturation*, that is, the norm of the solution as a percentage of the feasibility radius R.

The default value is $R = 10^9$. Setting options(3) to a negative value activates the "flexible bound" mode. In this mode, the feasibility radius is initially set to 10^8, and increased if necessary during the course of optimization

- `options(4)` helps speed up termination. When set to an integer value $J > 0$, the code terminates if t did not decrease by more than one percent in relative terms during the last J iterations. The default value is 10. This parameter trades off speed vs. accuracy. If set to a small value (< 10), the code terminates quickly but without guarantee of accuracy. On the contrary, a large value results in natural convergence at the expense of a possibly large number of iterations.
- `options(5) = 1` turns off the trace of execution of the optimization procedure. Resetting `options(5)` to zero (default value) turns it back on.

Setting `option(i)` to zero is equivalent to setting the corresponding control parameter to its default value. Consequently, there is no need to redefine the entire vector when changing just one control parameter. To set the maximum number of iterations to 10, for instance, it suffices to type

```
options=zeros(1,5)   % default value for all parameters
options(2)=10
```

Memory Problems

When the least-squares problem solved at each iteration becomes ill conditioned, the `feasp` solver switches from Cholesky-based to QR-based linear algebra (see "Memory Problems" on page 5-203 for details). Since the QR mode typically requires much more memory, MATLAB® may run out of memory and display the message

```
??? Error using ==> feaslv
Out of memory. Type HELP MEMORY for your options.
```

You should then ask your system manager to increase your swap space or, if no additional swap space is available, set `options(4) = 1`. This will prevent switching to QR and `feasp` will terminate when Cholesky fails due to numerical instabilities.

Example

Consider the problem of finding $P > I$ such that

$$A_1^T P + P A_1 < 0 \qquad \textbf{(5-6)}$$

$$A_2^T P + P A_2 < 0 \qquad \textbf{(5-7)}$$

$$A_3^T P + P A_3 < 0 \qquad \textbf{(5-8)}$$

with data

$$A_1 = \begin{pmatrix} -1 & 2 \\ 1 & -3 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -0.8 & 1.5 \\ 1.3 & -2.7 \end{pmatrix}, \quad A_3 = \begin{pmatrix} -1.4 & 0.9 \\ 0.7 & -2.0 \end{pmatrix}$$

This problem arises when studying the quadratic stability of the polytope of matrices $\mathrm{Co}\{A_1, A_2, A_3\}$.

To assess feasibility with `feasp`, first enter the LMIs (9-8)–(9-10) by:

```
setlmis([])
p = lmivar(1,[2 1])

lmiterm([1 1 1 p],1,a1,'s') % LMI #1
lmiterm([2 1 1 p],1,a2,'s') % LMI #2
lmiterm([3 1 1 p],1,a3,'s') % LMI #3
lmiterm([-4 1 1 p],1,1) % LMI #4: P
lmiterm([4 1 1 0],1) % LMI #4: I
lmis = getlmis
```

Then call `feasp` to find a feasible decision vector:

```
[tmin,xfeas] = feasp(lmis)
```

This returns `tmin = -3.1363`. Hence (9-8)–(9-10) is feasible and the dynamical system $\dot{x} = A(t)x$ is quadratically stable for $A(t) \in \mathrm{Co}\{A_1, A_2, A_3\}$.

To obtain a Lyapunov matrix P proving the quadratic stability, type

```
P = dec2mat(lmis,xfeas,p)
```

This returns

$$P = \begin{pmatrix} 270.8 & 126.4 \\ 126.4 & 155.1 \end{pmatrix}$$

It is possible to add further constraints on this feasibility problem. For instance, you can bound the Frobenius norm of P by 10 while asking `tmin` to be less than or equal to –1. This is done by

```
[tmin,xfeas] = feasp(lmis,[0,0,10,0,0],-1)
```

The third entry `10` of `options` sets the feasibility radius to 10 while the third argument `-1` sets the target value for `tmin`. This yields `tmin = -1.1745` and a matrix `P` with largest eigenvalue $\lambda_{\max}(P) = 9.6912$.

Reference

The feasibility solver `feasp` is based on Nesterov and Nemirovski's Projective Method described in

Nesterov, Y., and A. Nemirovski, *Interior Point Polynomial Methods in Convex Programming: Theory and Applications*, SIAM, Philadelphia, 1994.

Nemirovski, A., and P. Gahinet, "The Projective Method for Solving Linear Matrix Inequalities," *Proc. Amer. Contr. Conf.*, 1994, Baltimore, Maryland, p. 840–844.

The optimization is performed by the C-MEX file `feaslv.mex`.

See Also

`mincx`	Minimize a linear objective under LMI constraints
`gevp`	Generalized eigenvalue minimization under LMI constraints
`dec2mat`	Given values of the decision variables, derive the corresponding values of the matrix variables

fitfrd

Purpose

Fit frequency response data with state-space model

Syntax

```
B = fitfrd(A,N)
B = fitfrd(A,N,RD)
B = fitfrd(A,N,RD,WT)
```

Description

`B = fitfrd(A,N)` is a state-space object with state dimension N, where `A` is an `frd` object and N is a nonnegative integer. The frequency response of `B` closely matches the D-scale frequency response data in `A`.

`A` must have either 1 row or 1 column, although it need not be 1-by-1. `B` will be the same size as `A`. In all cases, `N` should be a nonnegative scalar.

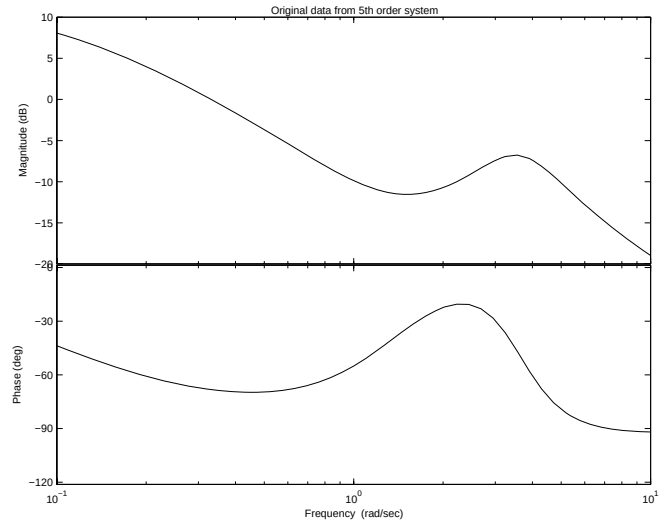
`B = fitfrd(A,N,RD)` forces the relative degree of `B` to be `RD`. `RD` must be a nonnegative integer. The default value for `RD` is 0. If `A` is a row (or column) then `RD` can be a vector of the same size as well, specifying the relative degree of each entry of `B`. If `RD` is a scalar, then it specifies the relative degree for all entries of `B`. You can specify the default value for `RD` by setting `RD` to an empty matrix.

`B = fitfrd(A,N,RD,WT)` uses the magnitude of `WT` to weight the optimization fit criteria. `WT` can be a `double, ss` or `frd`. If `WT` is a scalar, then it is used to weight all entries of the error criteria (`A-B`). If `WT` is a vector, it must be the same size as `A`, and each individual entry of `WT` acts as a weighting function on the corresponding entry of (`A-B`).

Example

You can use the `fitfrd` command to fit D-scale data. For example, create D-scale frequency response data from a fifth-order system.

```
sys = tf([1 2 2],[1 2.5 1.5])*tf(1,[1 0.1]);
sys = sys*tf([1 3.75 3.5],[1 2.5 13]);
omeg = logspace(-1,1);
sysg = frd(sys,omeg);
bode(sysg,'r-');
```

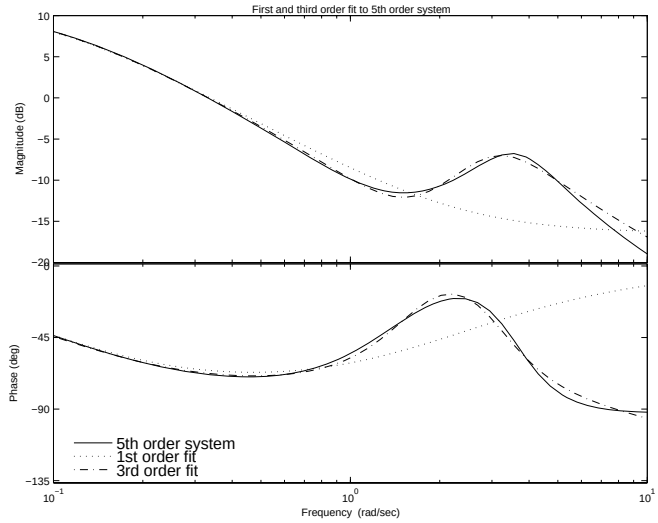


You can try to fit the frequency response D-scale data `sysg` with a first-order system, `b1`. Similarly, you can fit the D-scale data with a third-order system, `b3`.

```
b1 = fitfrd(sysg,1);
b3 = fitfrd(sysg,3);
```

Compare the original D-scale data `sysg` with the frequency responses of the first and third-order models calculated by `fitfrd`:

```
b1g = frd(b1,omeg);
b3g = frd(b3,omeg);

bode(sysg,'r-',b1g,'k:',b3g,'b-.')
```



Limitations

Numerical conditioning problems arise if the state order of the fit `N` is selected to be higher than required by the dynamics of `A`.

See Also

`fitmagfrd` Fit magnitude data with stable LTI model

Purpose

Fit frequency response magnitude data with stable, minimum-phase state-space model

Syntax

```
B = fitmagfrd(A,N)
B = fitmagfrd(A,N,RD)
B = fitmagfrd(A,N,RD,WT)
B = fitmagfrd(A,N,RD,WT,C)
```

Description

`B = fitmagfrd(A,N)` is a stable, minimum-phase `ss` object, with state-dimension `N`, whose frequency response magnitude closely matches the magnitude data in `A`. `A` is a 1-by-1 `frd` object, and `N` is a nonnegative integer.

`B = fitmagfrd(A,N,RD)` forces the relative degree of `B` to be `RD`. `RD` must be a nonnegative integer whose default value is `0`. You can specify the default value for `RD` by setting `RD` to an empty matrix.

`B = fitmagfrd(A,N,RD,WT)` uses the magnitude of `WT` to weight the optimization fit criteria. `WT` can be a `double`, `ss` or `frd`. If `WT` is a scalar, then it is used to weight all entries of the error criteria `(A-B)`. If `WT` is a vector, it must be the same size as `A`, and each individual entry of `WT` acts as a weighting function on the corresponding entry of `(A-B)`. The default value for `WT` is 1, and you can specify it by setting `WT` to an empty matrix.

`B = fitmagfrd(A,N,RD,WT,C)` enforces additional magnitude constraints on `B`, relative to `A` as:

- if C(w) == -1, then enforce abs(B(w)) <= abs(A(w)
- if C(w) == 1, then enforce abs(B(w)) >= abs(A(w))
- if C(w) == 0, then no additional constraint

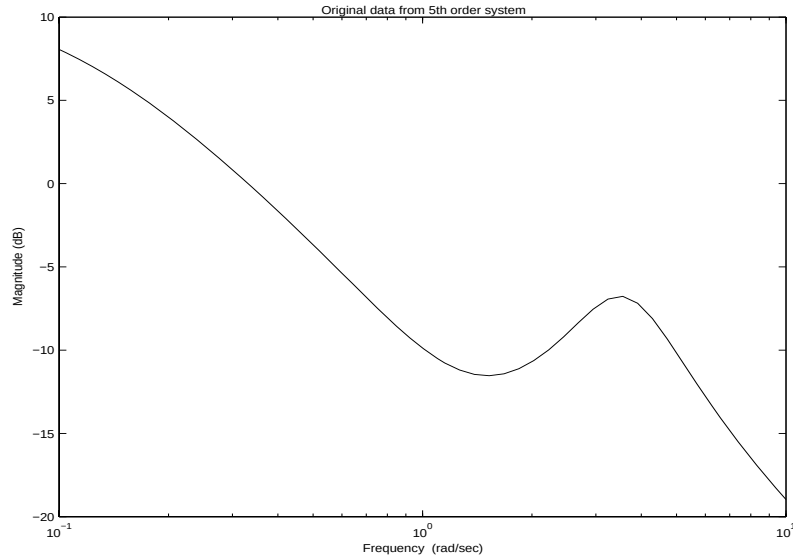
where w denotes the frequency. `C` should be a scalar, `double`, or `frd`, with `C.Frequency` equal to `A.Frequency`.

Example

You can use the `fitmagfrd` command to fit frequency response magnitude data. Create frequency response data from a fifth-order system.

```
sys = tf([1 2 2],[1 2.5 1.5])*tf(1,[1 0.1]);
sys = sys*tf([1 3.75 3.5],[1 2.5 13]);
omeg = logspace(-1,1);
```

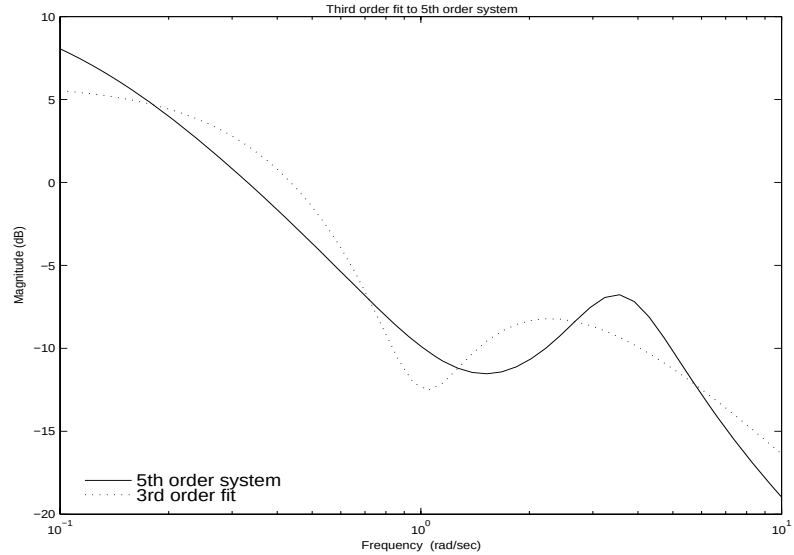
```
sysg = abs(frd(sys,omeg));
bodemag(sysg,'r');
```



Fit the magnitude data with a minimum-phase, stable third-order system:

```
ord = 3;
b1 = fitmagfrd(sysg,ord);
b1g = frd(b1,omeg);
```

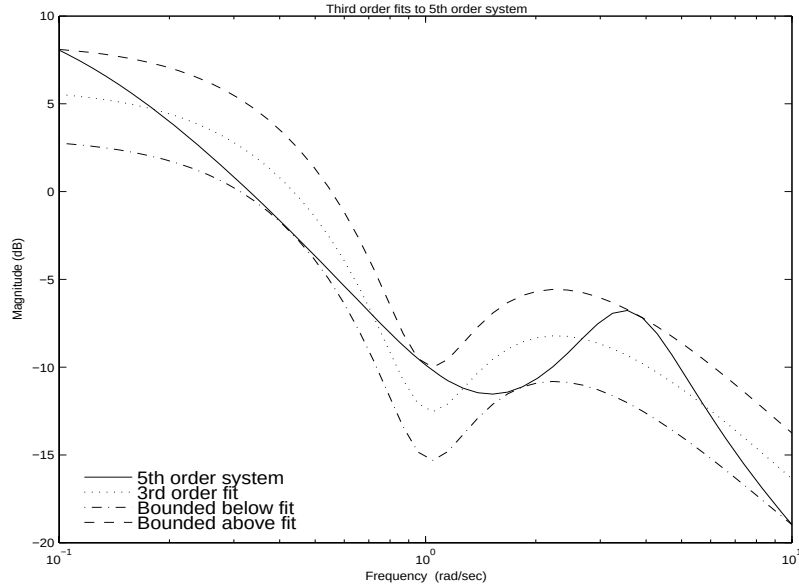
```
bodemag(sysg,'r',b1g,'k:');
```



Fit the magnitude data with a third order system constrained to lie below and above the given data.

```
b2 = fitmagfrd(sysg,ord,[],[],-1);
b2g = frd(b2,omeg);
b3 = fitmagfrd(sysg,ord,[],[],1);
b3g = frd(b3,omeg);
```

```
bodemag(sysg,'r',b1g,'k:',b2g,'b-.',b3g,'m--')
```



Algorithm

`fitmagfrd` uses a version of log-Chebychev magnitude design, solving

```
min f     subject to (at every frequency point in A):
        |d|^2 /(1+ f/WT) < |n|^2/A^2 < |d|^2*(1 + f/WT)
```

plus additional constraints imposed with `C`. `n, d` denote the numerator and denominator, respectively, and `B = n/d`. `n` and `d` have orders (`N-RD`) and `N`, respectively. The problem is solved using bisection on `f` and linear programming for fixed `f`. An alternate approximate method, which cannot enforce the constraints defined by `C`, is `B = fitfrd(genphase(A),N,RD,WT)`.

Limitations

This input `frd` object must be either a scalar 1-by-1 object or, a row, or column vector.

Reference

Oppenheim, A.V., and R.W. Schaffer, *Digital Signal Processing,* Prentice Hall, New Jersey, 1975, p. 513.

See Also

`fitfrd`	Fits frequency response data with state-space model

Purpose

Compute bounds on the gap and Vinnicombe gap distances (metric) between two systems

Syntax

```
[gap,nugap] = gapmetric(p0,p1)
[gap,nugap] = gapmetric(p0,p1,tol)
```

Description

`[gap,nugap] = gapmetric(p0,p1)` calculates upper bounds on the gap and nugap (Vinnicombe) metric between systems p0 and p1. The gap and nugap values lie between 0 and 1. A small value (relative to 1) implies that any controller that stabilizes p0 will likely stabilize p1, and, moreover, that the closed-loop gains of the two closed-loop systems will be similar. A gap or nugap of 0 implies that p0 equals p1, and a value of 1 implies that the plants are far apart. The input and output dimensions of p0 and p1 must be the same.

`[gap,nugap] = gapmetric(p0,p1,tol)` specifies a relative accuracy for calculating the gap metric and nugap metric. The default value for tol is 0.001. The computed answers are guaranteed to satisfy

```
gap-tol < gapexact(p0,p1) <=  gap
```

Example

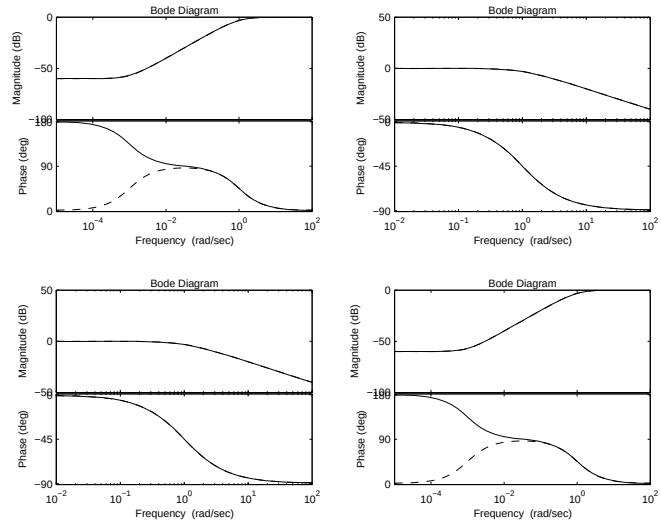
Consider two plant models. One plant is an unstable, first-order, with transfer function 1/(s-0.001) and the other plant is stable first-order with transfer function 1/(s+0.001).

```
p1 = tf(1,[1 -0.001]);
p2 = tf(1,[1 0.001]);
```

Despite the fact that one plant is unstable and the other is stable, these plants are *close* in the gap and nugap metrics. Intuitively, this is obvious, because, for instance, the feedback controller K=1 stabilizes both plants and renders the closed-loop systems nearly identical.

```
[g,ng] = gapmetric(p1,p2)
g =
    0.0029
ng =
    0.0020
K = 1;
H1 = loopsens(p1,K);
H2 = loopsens(p2,K);
subplot(2,2,1); bode(H1.Si,'-',H2.Si,'--');
```

```
subplot(2,2,2); bode(H1.Ti,'-',H2.Ti,'--');
subplot(2,2,3); bode(H1.PSi,'-',H2.PSi,'--');
subplot(2,2,4); bode(H1.CSo,'-',H2.CSo,'--');
```



Next, consider two stable plant models that differ by a first-order system. One plant is the transfer function 50/(s+50) and the other plant is the transfer function 50/(s+50) * 8/(s+8).

```
p3 = tf([50],[1 50]);
p4 = tf([8],[1 8])*p3;
```

Although the two systems have similar high-frequency dynamics and the same unity gain at low frequency, the plants are modestly far apart in the `gap` and `nugap` metrics.

```
[g,ng] = gapmetric(p3,p4)
g =
    0.6156
ng =
    0.6147
```

Algorithm

gap and nugap compute the gap and ν gap metrics between two LTI objects. Both quantities give a numerical value δ(p0,p1) between 0 and 1 for the distance between a nominal system p0 (G_0) and a perturbed system p1 (G_1). The gap metric was introduced into the control literature by Zames and El-Sakkary 1980, and exploited by Georgiou and Smith 1990. The ν gap metric was derived by Vinnicombe 1993. For both of these metrics the following robust performance result holds from Qui and Davidson 1992, and Vinnicombe 1993

$$\arcsin b(G_1,K_1) \geq \arcsin b(G_0,K_0) - \arcsin \delta(G_0,G_1) - \arcsin \delta(K_0,K_1)$$

where

$$b(G,K) = \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I - GK)^{-1} \begin{bmatrix} G & I \end{bmatrix} \right\|_\infty^{-1}$$

The interpretation of this result is that if a nominal plant G_0 is stabilized by controller K_0, with "stability margin" $b(G_0,K_0)$, then the stability margin when G_0 is perturbed to G_1 and K_0 is perturbed to K_1 is degraded by no more than the above formula. Note that $1/b(G,K)$ is also the signal gain from disturbances on the plant input and output to the input and output of the controller. The ν gap is always less than or equal to the gap, so its predictions using the above robustness result are tighter.

To make use of the gap metrics in robust design, weighting functions need to be introduced. In the above robustness result, G needs to be replaced by W_2GW_1 and K by $W_1^{-1}KW_2^{-1}$ (similarly for G_0, G_1, K_0 and K_1). This makes the weighting functions compatible with the weighting structure in the H_∞ loop shaping control design procedure (see loopsyn and ncfsyn for more details).

The computation of the gap amounts to solving 2-block H_∞ problems (Georgiou, Smith 1988). The particular method used here for solving the H_∞ problems is based on Green et al., 1990. The computation of the nugap uses the method of Vinnicombe, 1993.

Reference

Georgiou, T.T., "On the computation of the gap metric, " *Systems Control Letters,* Vol. 11, 1988, p. 253-257

Georgiou, T.T., and M. Smith, "Optimal robustness in the gap metric," *IEEE Transactions on Automatic Control,* Vol. 35, 1990, p. 673-686

Green, M., K. Glover, D. Limebeer, and J.C. Doyle, "A J-spectral factorization approach to H_∞ control," *SIAM J. of Control and Opt.,* 28(6), 1990, p. 1350-1371

Qiu, L., and E.J. Davison, "Feedback stability under simultaneous gap metric uncertainties in plant and controller," *Systems Control Letters,* Vol. 18-1, 1992 p. 9-22

Vinnicombe, G., "Measuring Robustness of Feedback Systems," PhD Dissertation, Department of Engineering, University of Cambridge, 1993.

Zames, G., and El-Sakkary, "Unstable systems and feedback: The gap metric," *Proceedings of the Allerton Conference,* October 1980, p. 380-385

See Also

`loopmargin`	Comprehensive analysis of feedback loop
`loopsyn`	H_∞ - loop shaping controller synthesis
`ncfsyn`	H_∞ - normalized coprime controller synthesis
`robuststab`	Calculate stability margins of uncertain systems
`wcsens`	Calculate worst-case sensitivities for feedback loop
`wcmargin`	Calculate worst-case margins for feedback loop

genphase

Purpose

Generate the phase associated with stable, minimum phase system using only its frequency response magnitude

Syntax

```
resp = genphase(d)
```

Description

`genphase` uses the complex-cepstrum algorithm to generate a complex frequency response `resp` whose magnitude is equal to the real, positive response `d`, but whose phase corresponds to a stable, minimum-phase function. The input, `d`, and output, `resp`, are `frd` objects.

Reference

Oppenheim, A.V., and R.W. Schaffer, *Digital Signal Processing,* Prentice Hall, New Jersey, 1975, p. 513.

See Also

`fitfrd`	Fit frequency response data with LTI model
`fitmagfrd`	Fitting magnitude data with stable LTI model

Purpose

Internal description of LMI system

Syntax

```
lmisys = getlmis
```

Description

After completing the description of a given LMI system with `lmivar` and `lmiterm`, its internal representation lmisys is obtained with the command

```
lmisys = getlmis
```

This MATLAB® representation of the LMI system can be forwarded to the LMI solvers or any other LMI-Lab function for subsequent processing.

See Also

`setlmis`	Initialize the description of an LMI system
`lmivar`	Specify the matrix variables in an LMI problem
`lmiterm`	Specify the term content of LMIs
`newlmi`	Attach an identifying tag to LMIs

Purpose

Generalized eigenvalue minimization under LMI constraints

Syntax

```
[lopt,xopt] = gevp(lmisys,nlfc,options,linit,xinit,target)
```

Description

`gevp` solves the generalized eigenvalue minimization problem

Minimize λ subject to:

$$C(x) < D(x) \quad \textbf{(5-9)}$$

$$0 < B(x) \quad \textbf{(5-10)}$$

$$A(x) < \lambda B(x) \quad \textbf{(5-11)}$$

where $C(x) < D(x)$ and $A(x) < \lambda B(x)$ denote systems of LMIs. Provided that (9-11)–(9-12) are jointly feasible, `gevp` returns the global minimum `lopt` and the minimizing value `xopt` of the vector of decision variables x. The corresponding optimal values of the matrix variables are obtained with `dec2mat`.

The argument `lmisys` describes the system of LMIs (9-11)–(9-13) for $\lambda = 1$. The LMIs involving λ are called the *linear-fractional constraints* while (9-11)–(9-12) are regular LMI constraints. The number of linear-fractional constraints (9-13) is specified by `nlfc`. All other input arguments are optional. If an initial feasible pair (λ_0, x_0) is available, it can be passed to `gevp` by setting `linit` to λ_0 and `xinit` to x_0. Note that `xinit` should be of length `decnbr(lmisys)` (the number of decision variables). The initial point is ignored when infeasible. Finally, the last argument `target` sets some target value for λ. The code terminates as soon as it has found a feasible pair (λ, x) with λ ð target.

Caution

When setting up your `gevp` problem, be cautious to

- Always specify the linear-fractional constraints (9-13) *last* in the LMI system. `gevp` systematically assumes that the last `nlfc` LMI constraints are linear fractional
- Add the constraint $B(x) > 0$ or any other LMI constraint that enforces it (see Remark below). This positivity constraint is required for regularity and good formulation of the optimization problem.

Control Parameters

The optional argument `options` gives access to certain control parameters of the optimization code. In `gevp`, this is a five-entry vector organized as follows:

- `options(1)` sets the desired relative accuracy on the optimal value `lopt` (default = 10^{-2}).
- `options(2)` sets the maximum number of iterations allowed to be performed by the optimization procedure (100 by default).
- `options(3)` sets the feasibility radius. Its purpose and usage are as for `feasp`.
- `options(4)` helps speed up termination. If set to an integer value $J > 0$, the code terminates when the progress in λ over the last J iterations falls below the desired relative accuracy. By progress, we mean the amount by which λ decreases. The default value is 5 iterations.
- `options(5) = 1` turns off the trace of execution of the optimization procedure. Resetting `options(5)` to zero (default value) turns it back on.

Setting `option(i)` to zero is equivalent to setting the corresponding control parameter to its default value.

Example

Given

$$A_1 = \begin{pmatrix} -1 & 2 \\ 1 & -3 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -0.8 & 1.5 \\ 1.3 & -2.7 \end{pmatrix}, \quad A_3 = \begin{pmatrix} -1.4 & 0.9 \\ 0.7 & -2.0 \end{pmatrix},$$

consider the problem of finding a single Lyapunov function $V(x) = x^T P x$ that proves stability of

$$\dot{x} = A_i x \quad (i = 1, 2, 3)$$

and maximizes the decay rate $-\frac{dV(x)}{dt}$. This is equivalent to minimizing α subject to

$$I < P \tag{5-12}$$

$$A_1^T P + P A_1 < \alpha P \tag{5-13}$$

$$A_2^T P + P A_2 < \alpha P \tag{5-14}$$

$$A_3^T P + P A_3 < \alpha P \qquad \textbf{(5-15)}$$

To set up this problem for `gevp`, first specify the LMIs (9-15)–(9-17) with $\alpha = 1$:

```
setlmis([]);
p = lmivar(1,[2 1])

lemiterm([1 1 1 0],1) % P > I : I
lemiterm([ 1 1 1 p],1,1) % P > I : P
lemiterm([2 1 1 p],1,a1,'s') % LFC # 1 (lhs)
lemiterm([ 2 1 1 p],1,1) % LFC # 1 (rhs)
lemiterm([3 1 1 p],1,a2,'s') % LFC # 2 (lhs)
lemiterm([ 3 1 1 p],1,1) % LFC # 2 (rhs)
lemiterm([4 1 1 p],1,a3,'s') % LFC # 3 (lhs)
lemiterm([ 4 1 1 p],1,1) % LFC # 3 (rhs)
lmis = getlmis
```

Note that the linear fractional constraints are defined last as required. To minimize α subject to (9-15)–(9-17), call `gevp` by

```
[alpha,popt]=gevp(lmis,3)
```

This returns `alpha = -0.122` as optimal value (the largest decay rate is therefore 0.122). This value is achieved for

$$P = \begin{pmatrix} 5.58 & -8.35 \\ -8.35 & 18.64 \end{pmatrix}$$

Remark

Generalized eigenvalue minimization problems involve standard LMI constraints (9-11) and linear fractional constraints (9-13). For well-posedness, the positive definiteness of $B(x)$ must be enforced by adding the constraint $B(x) > 0$ to the problem. Although this could be done automatically from inside the code, this is not desirable for efficiency reasons. For instance, the set of constraints (9-12) may reduce to a single constraint as in the example above. In this case, the single extra LMI "$P > I$" is enough to enforce positivity of *all* linear-fractional right-hand sides. It is therefore left to the user to devise the least costly way of enforcing this positivity requirement.

Reference

The solver `gevp` is based on Nesterov and Nemirovski's Projective Method described in

Nesterov, Y., and A. Nemirovski, *Interior Point Polynomial Methods in Convex Programming: Theory and Applications*, SIAM, Philadelphia, 1994.

The optimization is performed by the CMEX file `fpds.mex`.

See Also

`dec2mat`	Given values of the decision variables, derive the corresponding values of the matrix variables
`decnbr`	Give the total number of decision variables in a system of LMIs
`feasp`	Find a solution to a given system of LMIs
`mincx`	Minimize a linear objective under LMI constraints

gridureal

Purpose

Grid ureal parameters uniformly over their range

Syntax

```
B = gridreal(A,N)
[B,SampleValues] = gridreal(A,N)
[B,SampleValues] = gridreal(A,Names,N)
[B,SampleValues] = gridreal(A,Names1,N1,Names2,N2,...)
```

Description

`B = gridureal(A,N)` substitutes `N` uniformly-spaced samples of the uncertain real parameters in `A`. The samples are chosen to cut "diagonally" across the cube of real parameter uncertainty space. The array `B` has size equal to `[size(A) N]`. For example, suppose `A` has 3 uncertain real parameters, say `X`, `Y` and `Z`. Let `(x1, x2 , , and xN)` denote `N` uniform samples of `X` across its range. Similar for `Y` and `Z`. Then sample `A` at the points `(x1, y1, z1)`, `(x2, y2, z2)`, and `(xN, yN, zN)` to obtain the result `B`.

If `A` depends on additional uncertain objects, then `B` will be an uncertain object.

`[B,SampleValues] = gridureal(A,N)` additionally returns the specific sampled values (as a `structure` whose fieldnames are the names of `A`'s uncertain elements) of the uncertain reals. Hence, `B` is the same as `usubs(A,SampleValues)`.

`[B,SampleValues] = gridureal(A,NAMES,N)` samples only the uncertain reals listed in the `NAMES` variable (`cell`, or `char` array). Any entries of `NAMES` that are not elements of `A` are simply ignored. Note that `gridureal(A, fieldnames(A.Uncertainty),N) is the same as gridureal(A,N).`

`[B,SampleValues] = gridureal(A,NAMES1,N1,NAMES2,N2,...)` takes `N1` samples of the uncertain real parameters listed in `NAMES1`, and `N2` samples of the uncertain real parameters listed in `NAMES2` and so on. `size(B)` will equal `[size(A) N1 N2 ...]`.

Example

Create two uncertain real parameters `gamma` and `tau`. The nominal value of `gamma` is 4 and its range is 3 to 5. The nominal value of `tau` is 0.5 and its value can change by +/- 30 percent.

```
gamma = ureal('gamma',4);
tau = ureal('tau',.5,'Percentage',30);
```

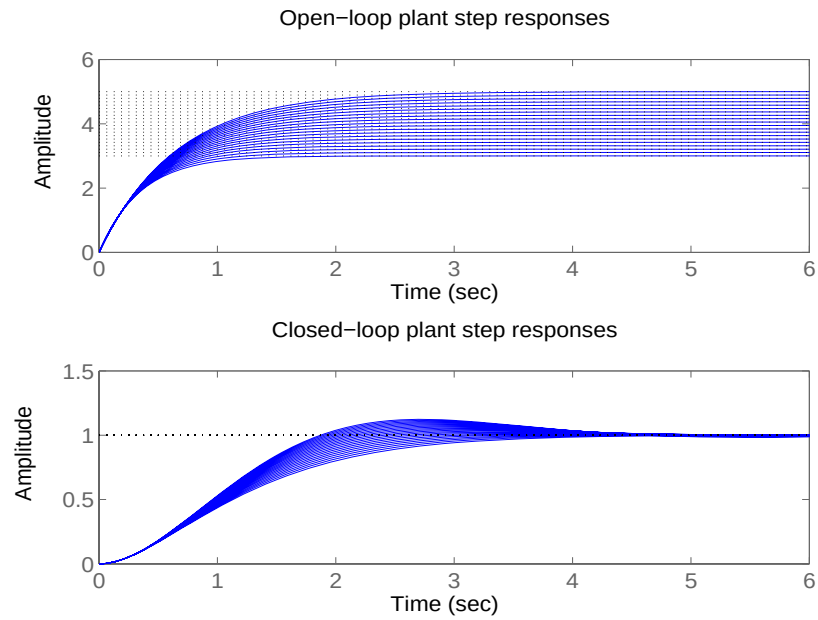
These uncertain parameters are used to construct an uncertain transfer function `p`. An integral controller, `c`, is synthesized for the plant `p` based on the

nominal values of `gamma` and `tau`. The uncertain closed-loop system `clp` is formed.

```
p = tf(gamma,[tau 1]);
KI = 1/(2*tau.Nominal*gamma.Nominal);
c = tf(KI,[1 0]);
clp = feedback(p*c,1);
```

The figure below shows the open-loop unit step response (top plot) and closed-loop response (bottom plot) for a grid of 20 values of `gamma` and `tau`.

```
subplot(2,1,1); step(gridureal(p,20),6)
title('Open-loop plant step responses')
subplot(2,1,2); step(gridureal(clp,20),6)
```



It clearly illustrates the low-frequency closed-loop insensitivity achieved by the PI control system.

Multi-Parameter Example

The next example illustrates the different options in gridding high-dimensional (e.g., `n` greater than 2) parameter spaces. An uncertain matrix, `m`, is constructed from four uncertain real parameters, `a`, `b`, `c` and `d`, each making up the individual entries.

```
a=ureal('a',1); b=ureal('b',2); c=ureal('c',3); d=ureal('d',4);
m = [a b;c d];
```

In the first case, the `(a,b)` space is gridded at five places, and the `(c,d)` space at 3 places. The uncertain matrix `m` is evaluated at these 15 grid-points resulting in the matrix `m1`.

```
m1 = gridureal(m,{'a';'b'},5,{'c';'d'},3);
```

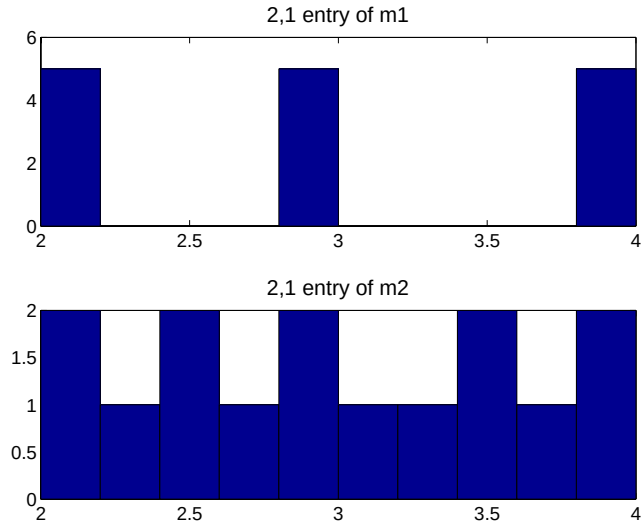
In the second case, the `(a,b,c,d)` space is gridded at 15 places, and the uncertain matrix `m` is sampled at these 15 points. The resulting matrix is `m2`.

```
m2 = gridureal(m,{'a';'b';'c';'d'},15);
```

The (2,1) entry of `m` is just the uncertain real parameter `c`. Below, you see the histogram plots of the (2,1) entry of both `m1` and `m2`. The (2,1) entry of `m1` only takes on 3 distinct values, while the (2,1) entry of `m2` (which is also `c`) takes on 15 distinct values uniformly through its range.

```
subplot(2,1,1)
hist(m1(2,1,:))
title('2,1 entry of m1')
subplot(2,1,2)
hist(m2(2,1,:))
```

```
title('2,1 entry of m2')
```



See Also

`usample`	Generates random samples of an atom
`usubs`	Substitutes values for atoms

Purpose

Mixed H_2/H_∞ synthesis with pole placement constraints

Syntax

```
[gopt,h2opt,K,R,S] = hinfmix(P,r,obj,region,dkbnd,tol)
```

Description

`h2hinfyn` performs multi-objective output-feedback synthesis. The control problem is sketched in this figure.

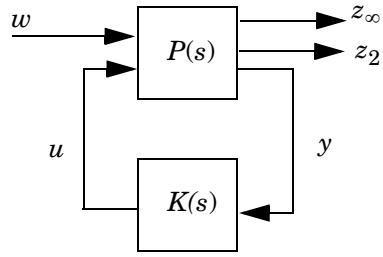


Figure 5-4: Mixed H_2/H_∞ synthesis

If $T_\infty(s)$ and $T_2(s)$ denote the closed-loop transfer functions from w to z_∞ and z_2, respectively, `hinfmix` computes a suboptimal solution of the following synthesis problem:

Design an LTI controller $K(s)$ that minimizes the mixed H_2/H_∞ criterion

$$\alpha\|T_\infty\|_\infty^2 + \beta\|T_2\|_2^2$$

subject to

- $\|T_\infty\|_\infty < \gamma_0$
- $\|T_2\|_2 < \nu_0$
- The closed-loop poles lie in some prescribed LMI region D.

Recall that $\|\cdot\|_\infty$ and $\|\cdot\|_2$ denote the H_∞ norm (RMS gain) and H_2 norm of transfer functions.

`P` is any SS, TF, or ZPK LTI representation of the plant $P(s)$, and `r` is a three-entry vector listing the lengths of z_2, y, and u. Note that z_∞ and/or z_2 can be empty. The four-entry vector `obj` = $[\gamma_0, \nu_0, \alpha, \beta]$ specifies the H_2/H_∞

constraints and trade-off criterion, and the remaining input arguments are optional:

- `region` specifies the LMI region for pole placement (the default `region = []` is the open left-half plane). Use `lmireg` to interactively build the LMI region description `region`
- `dkbnd` is a user-specified bound on the norm of the controller feedthrough matrix D_K. The default value is 100. To make the controller $K(s)$ strictly proper, set `dkbnd = 0`.
- `tol` is the required relative accuracy on the optimal value of the trade-off criterion (the default is 10^{-2}).

The function `h2hinfsyn` returns guaranteed H_∞ and H_2 performances `gopt` and `h2opt` as well as the `SYSTEM` matrix `K` of the LMI-optimal controller. You can also access the optimal values of the LMI variables R, S via the extra output arguments `R` and `S`.

A variety of mixed and unmixed problems can be solved with `hinfmix`. In particular, you can use `hinfmix` to perform pure pole placement by setting `obj = [0 0 0 0]`. Note that both z_∞ and z_2 can be empty in such case.

Reference

Chilali, M., and P. Gahinet, "H_∞ Design with Pole Placement Constraints: An LMI Approach," to appear in *IEEE Trans. Aut. Contr.*, 1995.

Scherer, C., "Mixed H2 H-infinity Control," to appear in *Trends in Control: A European Perspective*, volume of the special contributions to the ECC 1995.

See Also

`lmireg`	Specify LMI regions for pole placement purposes
`msfsyn`	Multi-model/multi-objective state-feedback synthesis

Purpose

H_2 control synthesis for LTI plant

Syntax

```
[K,CL,GAM,INFO]=H2SYN(P,NMEAS,NCON)
```

Description

`h2syn` computes a stabilizing H_2 optimal `lti/ss` controller `K` for a partitioned LTI plant `P`. The controller, `K`, stabilizes the plant `P` and has the same number

$$\mathtt{P} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$

of states as `P`. The LTI system `P` is partitioned where inputs to B_1 are the disturbances, inputs to B_2 are the control inputs, output of C_1 are the errors to be kept small, and outputs of C_2 are the output measurements provided to the controller. B_2 has column size (`NCON`) and C_2 has row size (`NMEAS`).

If `P` is constructed with `mktito`, you can omit `NMEAS` and `NCON` from the arguments.

The closed-loop system is returned in `CL` and the achieved H_2 cost γ in `GAM`. — see Figure 2. `INFO` is a `struct` array that returns additional information about the design.

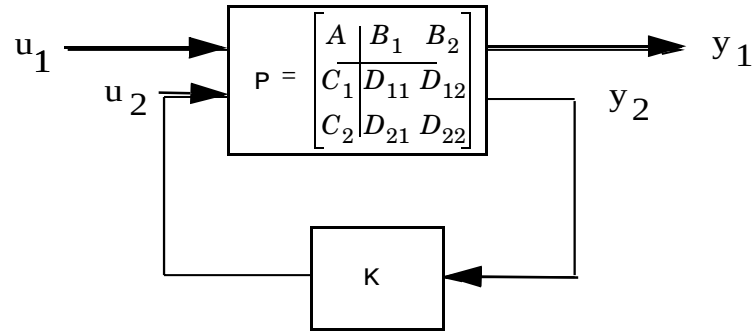


Figure 5-5: $\mathbf{H_2}$ control system `CL= lft(P,K)`$=T_{y_1u_1}$.

Output Arguments:	
K	LTI controller
CL= lft(P,K)	LTI closed-loop system $T_{y_1u_1}$
GAM = norm(CL)	the H_2 optimal cost $\gamma = \|T_{y_1u_1}\|_2$
INFO	additional output information

Additional output — structure array INFO containing possible additional information depending on METHOD):

INFO.NORMS	Norms of 4 different quantities, full information control cost (FI), output estimation cost (OEF), direct feedback cost (DFL) and full control cost (FC). NORMS = [FI OEF DFL FC];
INFO.KFI	Full-information gain matrix (constant feedback) $u_2(t) = K_{FI}x(t)$
INFO.GFI	Full-information closed-loop system GFI=ss(A-B2*KFI,B1,C1-D12*KFI,D11)
INFO.HAMX	X Hamiltonian matrix (state-feedback)
INFO.HAMY	Y Hamiltonian matrix (Kalman filter)

Examples

Example 1: Stabilize 4-by-5 unstable plant with 3-states, NMEAS=2, NCON=2.

```
rand('seed',0);randn('seed',0);
P=rss(3,4,5)';
[K,CL,GAM]=h2syn(P,2,1);
open_loop_poles=pole(P)
closed_loop_poles=pole(CL)
```

Example 2: Mixed-Sensitivity H_2 loop-shaping. Here the goal is to shape the `sigma` plots of sensitivity $S:=(I+GK)^{-1}$ and complementary sensitivity $T:=GK(I+GK)^{-1}$, by choosing a stabilizing K the minimizes the H_2 norm of

$$T_{y_1 u_1} \stackrel{\Delta}{=} \begin{bmatrix} W_1 S \\ (W_2/G)T \\ W_3 T \end{bmatrix}$$

where $G(s) = \frac{s-1}{s-2}$, $W_1 = \frac{0.1(s+1000)}{100s+1}$, $W_2 = 0.1$, no W_3.

```
s=zpk('s');
G=10*(s-1)/(s+1)^2;
W1=0.1*(s+100)/(100*s+1); W2=0.1; W3=[];
P=(G,W1,W2,W3);
[K,CL,GAM]=h2syn(P);
L=G*K; S=inv(1+L); T=1-S;
sigma(L,'k-.',S,'r',T,'g')
```

Algorithm

The H_2 optimal control theory has its roots in the frequency domain interpretation the cost function associated with time-domain state-space LQG control theory [1]. The equations and corresponding nomenclature used here are taken from the Doyle *et al.*, 1989 [2]-[3].

`h2syn` solves the H_2 optimal control problem by observing that it is equivalent to a conventional Linear-Quadratic Gaussian (LQG) optimal control problem. For simplicity, we shall describe the details of algorithm only for the continuous-time case, in which case the cost function J_{LQG} satisfies

$$J_{LQG} = \lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T y_1^T y_1 dt\right\}$$

$$= \lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T [x^T \;\; u_2^T]\begin{bmatrix} Q & N_c \\ N_c^T & R\end{bmatrix}\begin{bmatrix} x \\ u_2\end{bmatrix} dt\right\}$$

$$= \lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T [x^T \;\; u_2^T]\begin{bmatrix} C_1^T \\ D_{12}^T\end{bmatrix}[C_1 \;\; D_{12}]\begin{bmatrix} x \\ u_2\end{bmatrix} dt\right\}$$

with plant noise u_1 channel of intensity I, passing through the matrix [B1;0;D12] to produce equivalent white correlated with plant ξ and white measurement noise θ having joint correlation function

$$E\left\{\begin{bmatrix}\xi(t)\\ \theta(t)\end{bmatrix}[\xi(\tau) \;\; \theta(\tau)]^T\right\} = \begin{bmatrix} \Xi & N_f \\ N_f^T & \Theta\end{bmatrix}\delta(t-\tau)$$

$$= \begin{bmatrix} B_1 \\ D_{21}\end{bmatrix}\begin{bmatrix} B_1^T & D_{21}^T\end{bmatrix}\delta(t-\tau)$$

The H_2 optimal controller $K(s)$ is thus realizable in the usual LQG manner as a full-state feedback K_{FI} and a Kalman filter with residual gain matrix K_{FC}.

1 Kalman Filter

$$\hat{\dot{x}} = A\hat{x} + B_2 u_2 + K_{FC}(y_2 - C_2\hat{x} - D_{22}u_2)$$

$$K_{FC} = (YC_2^T + N_f)\Theta^{-1} = (YC_2^T + B_1 D_{21}^T)(D_{21}D_{21}^T)^{-1}$$

where $Y = Y^T \geq 0$ solves the Kalman filter Riccati equation

$$YA^T + AY - (YC_2^T + N_f)\Theta^{-1}(C_2 Y + N_f^T) + \Xi = 0$$

2 Full-State Feedback

$$u_2 = K_{FI}\hat{x}$$

$$K_{FI} = R^{-1}(B_2^T X + N_c^T) = (D_{12}^T D_{12})^{-1}(B_2^T X + D_{12}^T C_1)$$

where $X = X^T \geq 0$ solves the state-feedback Riccati equation

$$A^T X + XA - (XB_2 + N_c)R^{-1}(B_2^T X + N_c^T) + Q = 0$$

The final *positive*-feedback H_2 optimal controller $u_2 = K(s)y_2$ has a familiar closed-form

$$K(s) := \left[\begin{array}{c|c} A - K_{FC}C_2 - B_2K_{FI} + K_{FC}D_{22}K_{FI} & K_f \\ \hline -K_{FI} & 0 \end{array}\right]$$

`h2syn` implements the continuos optimal H_2 control design computations using the formulae described in the Doyle, *et al.*[2]; for discrete-time plants, h2syn uses the same controller formula, except that the corresponding discrete time Riccati solutions (dare) are substituted for X and Y. A Hamiltonian is formed and solved via a Riccati equation. In the continuous-time case, the optimal H_2-norm is infinite when the plant D_{11} matrix associated with the input disturbances and output errors is *non*-zero; in this case, the optimal H_2 controller returned by `h2syn` is computed by first setting D_{11} to zero.

3 Optimal Cost GAM.

The full information (FI) cost is given by the equation $(\text{trace}(B_1' X_2 B_1))^{\frac{1}{2}}$. The output estimation cost (OEF) is given by $(\text{trace}(F_2 Y_2 F_2'))^{\frac{1}{2}}$, where $F_2 =: -(B_2' X_2 + D_{12}' C_1)$. The disturbance feedforward cost (DFL) is $(\text{trace}(L_2' X_2 L_2))^{\frac{1}{2}}$, where L_2 is defined by $-(Y_2 C_2' + B_1 D_{21}')$ and the full control cost (FC) is given by $(\text{trace}(C_1 Y_2 C_1'))^{\frac{1}{2}}$. X_2 and Y_2 are the solutions to the X and Y Riccati equations, respectively. For for continuous-time plants with zero feedthrough term (`D11 = 0`), and for all discrete-time plants, the optimal H_2 cost $\gamma = \|T_{y_1 u_1}\|_2$ is

```
GAM =sqrt(FI^2 + OEF^2+ trace(D11*D11'));
```

otherwise, `GAM = Inf`.

Limitations

- (A, B_2, C_2) must be *stabilizable and detectable*.
- D_{12} must have full column rank and D_{21} must have full row rank

References

[1] Safonov, M.G., A.J. Laub, and G. Hartmann, "Feedback Properties of Multivariable Systems: The Role and Use of Return Difference Matrix," *IEEE Trans. of Automat. Contr*., AC-26, pp. 47-65, 1981.

[2] Doyle, J.C., K. Glover, P. Khargonekar, and B. Francis, "State-space solutions to standard H_2 and H_∞ control problems," *IEEE Transactions on Automatic Control,* vol. 34, no. 8, pp. 831–847, August 1989.

[3] Glover, K., and J.C. Doyle, "State-space formulae for all stabilizing controllers that satisfy an H_∞ norm bound and relations to risk sensitivity," *Systems and Control Letters,* 1988. vol. 11, pp. 167–172, August 1989.

See Also

`augw`	Augment plant weights for control design
`hinfsyn`	H_∞ synthesis controller

hankelmr

Purpose

Hankel minimum degree approximation (MDA) without balancing

Syntax

```
GRED = hankelmr(G)
GRED = hankelmr(G,order)
[GRED,redinfo] = hankelmr(G,key1,value1,...)
[GRED,redinfo] = hankelmr(G,order,key1,value1,...)
```

Description

`hankelmr` returns a reduced order model `GRED` of `G` and a struct array `redinfo` containing the error bound of the reduced model and Hankel singular values of the original system.

The error bound is computed based on Hankel singular values of `G`. For a stable system Hankel singular values indicate the respective state energy of the system. Hence, reduced order can be directly determined by examining the system Hankel SV's, σ_i.

With only one input argument `G`, the function will show a Hankel singular value plot of the original model and prompt for model order number to reduce.

This method guarantees an error bound on the infinity norm of the *additive error* $\| \texttt{G-GRED} \|_\infty$ for well-conditioned model reduced problems [1]:

$$\|G - Gred\|_\infty \le 2 \sum_{k+1}^{n} \sigma_i$$

Note It seems this method is similar to the additive model reduction routines `balancmr` and `schurmr`, but actually it can produce more reliable reduced order model when the desired reduced model has nearly controllable and/or observable states (has Hankel singular values close to machine accuracy). `hankelmr` will then select an optimal reduced system to satisfy the error bound criterion regardless the order one might naively select at the beginning.

This table describes input arguments for `hankelmr`.

Argument	Description
`G`	LTI model to be reduced (without any other inputs will plot its Hankel singular values and prompt for reduced order)
`ORDER`	(Optional) an integer for the desired order of the reduced model, or optionally a vector packed with desired orders for batch runs

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y, or a vector of integers`. By default, all the anti-stable part of a system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system.

`'MaxError'` can be specified in the same fashion as an alternative for `'ORDER'`. In this case, reduced order will be determined when the sum of the tails of the Hankel sv's reaches the `'MaxError'`.

Argument	Value	Description
`'MaxError'`	Real number or vector of different errors	Reduce to achieve H∞ error. When present, `'MaxError'` overides `ORDER` input.
`'Weights'`	`{Wout,Win}` cell array	Optimal 1x2 cell array of LTI weights `Wout` (output) and `Win` (input). Default for both is identity. Weights must be invertible.
`'Display'`	`'on'` or `'off'`	Display Hankel singular plots (default `'off'`).
`'Order'`	Integer, vector or cell array	Order of reduced model. Use only if not specified as 2nd argument.

Weights on the original model input and/or output can make the model reduction algorithm focus on some frequency range of interests. But weights have to be stable, minimum phase and invertible.

This table describes output arguments.

Argument	Description
`GRED`	LTI reduced order model. Become multi-dimensional array when input is a serial of different model order array.
`REDINFO`	A STRUCT array with 4 fields: • `REDINFO.ErrorBound` (bound on $\|$ G-GRED $\|\infty$) • `REDINFO.StabSV` (Hankel SV of stable part of G) • `REDINFO.UnstabSV` (Hankel SV of unstable part of G) • `REDINFO.Ganticausal` (Anti-causal part of Hankel MDA)

`G` can be stable or unstable, continuous or discrete.

Note If `size(GRED)` is not equal to the order you specified. The optimal Hankel MDA algorithm has selected the best Minimum Degree Approximate it can find within the allowable machine accuracy.

Algorithm

Given a state-space *(A,B,C,D)* of a system and *k,* the desired reduced order, the following steps will produce a similarity transformation to truncate the original state space system to the k^{th} order reduced model.

1. Find the controllability and observability grammians P and Q.
2. Form the descriptor

   $$E = QP - \rho^2 I$$

   where $\sigma_k > \rho \geq \sigma_{k+1}$, and descriptor state-space

Take SVD of descriptor E and partition the result into k^{th} order truncation form

$$\left[\begin{array}{c|c} Es-\overline{A} & \overline{B} \\ \hline \overline{C} & \overline{D} \end{array}\right] = \left[\begin{array}{c|c} \rho^2 A^T + QAP & QB \\ \hline CP & D \end{array}\right]$$

$$E = [U_{E1}, U_{E2}] \begin{bmatrix} \Sigma_E & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{E1}^T \\ V_{E2}^T \end{bmatrix}$$

3 Apply the transformation to the descriptor state-space system above we have

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} U_{E1}^T \\ U_{E2}^T \end{bmatrix} (\rho^2 A^T + QAP) \begin{bmatrix} V_{E1} & V_{E2} \end{bmatrix}$$

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} U_{E1}^T \\ U_{E2}^T \end{bmatrix} \begin{bmatrix} QB & -C^T \end{bmatrix}$$

$$\begin{bmatrix} C_1 & C_2 \end{bmatrix} = \begin{bmatrix} CP \\ -\rho B^T \end{bmatrix} \begin{bmatrix} V_{E1} & V_{E2} \end{bmatrix}$$

$$D_1 = D$$

4 Form the equivalent state-space model.

$$\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} = \begin{bmatrix} \Sigma_E^{-1}(A_{11} - A_{12}A_{22}^{\dagger}A_{21}) & \Sigma_E^{-1}(B_1 - A_{12}A_{22}^{\dagger}B_2) \\ C_1 - C_2A_{22}^{\dagger}A_{21} & D_1 - C_2A_{22}^{\dagger}B_2 \end{bmatrix}$$

The final k^{th} order Hankel MDA is the stable part of the above state-space realization. Its anticausal part is stored in `redinfo.Ganticausal`.

The proof of the Hankel MDA algorithm can be found in [2]. The error system between the original system G and the *Zeroth Order Hankel MDA* G_0 is an all-pass function [1].

Example

Given a continuous or discrete, stable or unstable system, `G`, the following commands can get a set of reduced order models based on your selections:

```
rand('state',1234); randn('state',5678);G = rss(30,5,4);
[g1, redinfo1] = hankelmr(G); % display Hankel SV plot
                              % and prompt for order (try 15:20)
[g2, redinfo2] = hankelmr(G,20);
[g3, redinfo3] = hankelmr(G,[10:2:18]);
[g4, redinfo4] = hankelmr(G,'MaxError',[0.01, 0.05]);
rand('state',12345); randn('state',6789);
wt1 = rss(6,5,5); wt1.d = eye(5)*2;
wt2 = rss(6,4,4); wt2.d = 2*eye(4);
[g5, redinfo5] = hankelmr(G, [10:2:18], 'weight',{wt1,wt2});
for i = 1:5
   figure(i); eval(['sigma(G,g' num2str(i) ');']);
end
```

Figure 5-6, Singular Value Bode Plot of G (30-state, 5 outputs, 4 inputs), on page 5-113 shows a singular value Bode plot of a random system `G` with 20 states, 5 output and 4 inputs. The error system between `G` and its *Zeroth order Hankel MDA* has it infinity norm equals to an all pass function, as shown in Figure 5-7, All-Pass Error System Between G and Zeroth Order G Anticausal, on page 5-113 (ref.: [5]).

The *Zeroth order Hankel MDA* and its error system sigma plot are obtained via commands

```
[g0,redinfo0] = hankelmr(G,0);
sigma(G-redinfo0.Ganticausal)
```

This interesting all-pass property is unique in Hankel MDA model reduction.

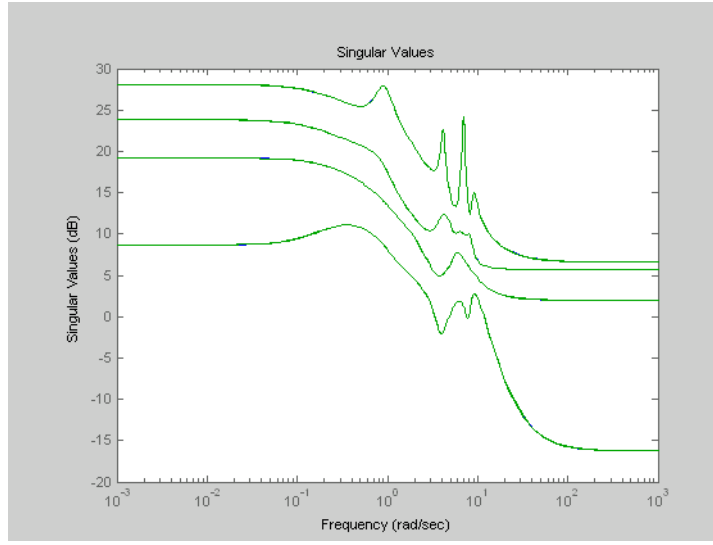


Figure 5-6: Singular Value Bode Plot of G (30-state, 5 outputs, 4 inputs)

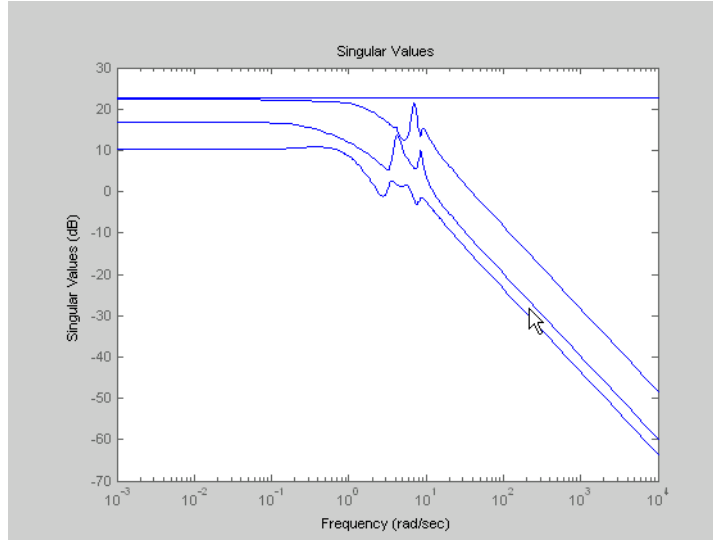


Figure 5-7: All-Pass Error System Between G and Zeroth Order G Anticausal

Reference

[1] Glover, K., "All Optimal Hankel Norm Approximation of Linear Multivariable Systems, and Their $\mathbf{L}_\infty$ – error Bounds," *Int. J. Control*, vol. 39, no. 6, pp. 1145-1193, 1984.

[2] Safonov, M.G., R.Y. Chiang, and D.J.N. Limebeer, "Optimal Hankel Model Reduction for Nonminimal Systems," *IEEE Trans. on Automat. Contr.*, vol. 35, no. 4, April 1990, pp. 496-502.

See Also

`reduce`	Top level model reduction routines
`balancmr`	Balanced truncation via square-root method
`schurmr`	Balanced truncation via Schur method
`bstmr`	Balanced stochastic truncation via Schur method
`ncfmr`	Balanced truncation for normalized coprime factors
`hankelsv`	Hankel singular value

Purpose

Compute Hankel singular values for stable/unstable or continuous/discrete system

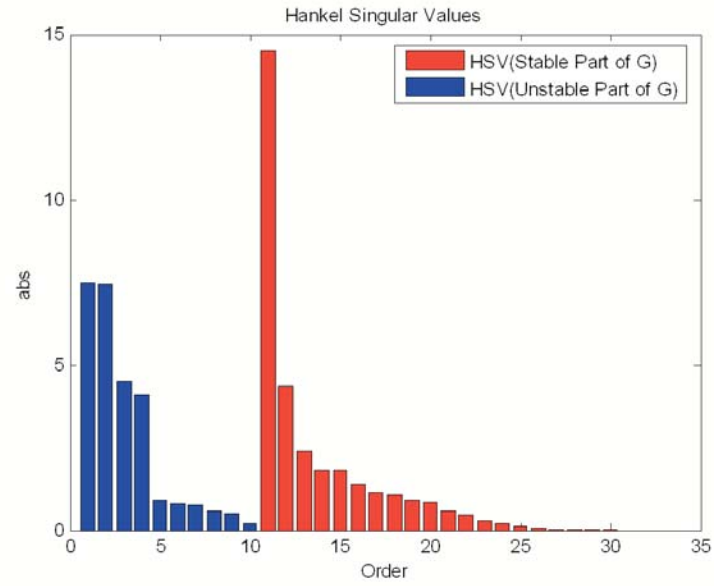
Syntax

```
hankelsv(G)
hankelsv(G,ErrorType,style)
[sv_stab,sv_unstab]=hankelsv(G,ErrorType,style)
```

Description

`[sv_stab,sv_unstab]=hankelsv(G,ErrorType,style)` returns a column vector `SV_STAB` containing the Hankel singular values of the stable part of `G` and `SV_UNSTAB` of anti-stable part (if it exists). The Hankel SV's of anti-stable part `ss(a,b,c,d)` is computed internally via `ss(-a,-b,c,d)`. Discrete model is converted to continuous one via the bilinear transform.

`hankelsv(G)` with no output arguments draws a bar graph of the Hankel singular values such as the following:



This table describes optional input arguments for `hankelsvd`.

Argument	Value	Description

`ERRORTYPE`	`'add'`
	`'mult'`
	`'ncf'`
		Regular Hankel SV's of G
		Hankel SV's of phase matrix
		Hankel SV's of coprime factors

`STYLE`	`'abs'` `'log'`	Absolute value logarithm scale

Algorithm

For `ErrorType = 'add'`, `hankelsv` implements the numerically robust square root method to compute the Hankel singular values [1]. Its algorithm goes as follows:

Given a stable model `G`, with controllability and observability grammians `P` and `Q`, compute the SVD of `P` and `Q`:

```
[Up,Sp,Vp] = svd(P);
[Uq,Sq,Vq] = svd(Q);
```

Then form the square roots of the grammians:

```
Lr = Up*diag(sqrt(diag(Sp)));
Lo = Uq*diag(sqrt(diag(Sq)));
```

The Hankel singular values are simply:

σ_H `= svd(Lo'*Lr);`

This method not only takes the advantages of robust SVD algorithm, but also ensure the computations stay well within the "square root" of the machine accuracy.

For `ErrorType = 'mult'`, `hankelsv` computes the Hankel singular value of the phase matrix of `G` [2].

For `ErrorType = 'ncf'`, `hankelsv` computes the Hankel singular value of the normalized coprime factor pair of the model [3].

Reference

[1] Safonov, M.G., and R.Y. Chiang, "A Schur Method for Balanced Model Reduction," *IEEE Trans. on Automat. Contr.*, vol. AC-2, no. 7, July 1989, pp. 729-733.

[2] Safonov, M.G., and R.Y. Chiang, "Model Reduction for Robust Control: A Schur Relative Error Method," *International J. of Adaptive Control and Signal Processing, Vol. 2, pp. 259-272, 1988.*

[3] Vidyasagar, M., *Control System Synthesis - A Factorization Approach.* London: The MIT Press, 1985.

See Also

`reduce`	Top level model reduction routines
`balancmr`	Balanced truncation via square-root method
`schurmr`	Balanced truncation via Schur method
`bstmr`	Balanced stochastic truncation via Schur method
`ncfmr`	Balanced truncation for normalized coprime factors
`hankelmr`	Hankel minimum degree approximation

hinfgs

Purpose

Synthesis of gain-scheduled H_∞ controllers

Syntax

```
[gopt,pdK,R,S] = hinfgs(pdP,r,gmin,tol,tolred)
```

Description

Given an affine parameter-dependent plant

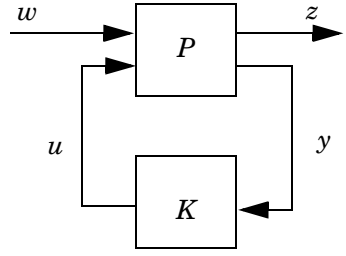
$$P\begin{cases} \dot{x} &= A(p)x + B_1(p)w + B_2u \\ z &= C_1(p)x + D_{11}(p)w + D_{12}u \\ y &= C_2x + D_{21}w + D_{22}u \end{cases}$$

where the time-varying parameter vector $p(t)$ ranges in a box and is measured in real time, `hinfgs` seeks an affine parameter-dependent controller

$$K\begin{cases} \dot{\zeta} &= A_K(p)\zeta + B_K(p)y \\ u &= C_K(p)\zeta + D_K(p)y \end{cases}$$

scheduled by the measurements of $p(t)$ and such that

- K stabilizes the closed-loop system



  for all admissible parameter trajectories $p(t)$
- K minimizes the closed-loop quadratic H_∞ performance from w to z.

The description `pdP` of the parameter-dependent plant P is specified with `psys` and the vector `r` gives the number of controller inputs and outputs (set `r=[`p_2`,m2]` if $y \in \mathbf{R}^{p2}$ and $u \in \mathbf{R}^{m_2}$). Note that `hinfgs` also accepts the polytopic model of P returned, e.g., by `aff2pol`.

`hinfgs` returns the optimal closed-loop quadratic performance `gopt` and a polytopic description of the gain-scheduled controller `pdK`. To test if a closed-loop quadratic performance γ is achievable, set the third input `gmin` to γ. The arguments `tol` and `tolred` control the required relative accuracy on `gopt` and the threshold for order reduction. Finally, `hinfgs` also returns solutions R, S of the characteristic LMI system.

Controller Implementation

The gain-scheduled controller `pdK` is parametrized by $p(t)$ and characterized by the values K_{Π_j} of $\begin{pmatrix} A_K(p) & B_K(p) \\ C_K(p) & D_K(p) \end{pmatrix}$ at the corners ${}^3_j$ of the parameter box. The command

```
Kj = psinfo(pdK,'sys',j)
```

returns the j-th vertex controller K_{Π_j} while

```
pv = psinfo(pdP,'par')
vertx = polydec(pv)
Pj = vertx(:,j)
```

gives the corresponding corner ${}^3_j$ of the parameter box (`pv` is the parameter vector description).

The controller scheduling should be performed as follows. Given the measurements $p(t)$ of the parameters at time t,

1. Express $p(t)$ as a convex combination of the ${}^3_j$:

   $$p(t) = \alpha_1 {}^3_1 + \ldots + \alpha_N {}^3_N, \qquad \alpha_j \check{S} 0, \qquad \sum_{i=1}^{N} \alpha_j = 1$$

   This convex decomposition is computed by `polydec`.

2. Compute the controller state-space matrices at time t as the convex combination of the vertex controllers K_{Π_j}:

   $$\begin{pmatrix} A_K(t) & B_K(t) \\ C_K(t) & D_K(t) \end{pmatrix} = \sum_{i=1}^{N} \alpha_j K_{\Pi_j}.$$

3 Use $A_K(t)$, $B_K(t)$, $C_K(t)$, $D_K(t)$ to update the controller state-space equations.

Reference

Apkarian, P., P. Gahinet, and G. Becker, "Self-Scheduled H_∞ Control of Linear Parameter-Varying Systems," submitted to *Automatica*, October 1995.

Becker, G., Packard, P., "Robust Performance of Linear-Parametrically Varying Systems Using Parametrically-Dependent Linear Feedback," *Systems and Control Letters*, 23 (1994), pp. 205-215.

Packard, A., "Gain Scheduling via Linear Fractional Transformations," *Syst. Contr. Letters*, 22 (1994), pp. 79-92.

See Also

`psys`	Specification of uncertain state-space models
`pvec`	Quantification of uncertainty on physical parameters
`pdsimul`	Time response of a parameter-dependent system along a given parameter trajectory
`polydec`	Compute polytopic coordinates wrt. box corners

Purpose

Compute $\mathbf{H}_\infty$ optimal controller for LTI plant

Syntax

```
[K,CL,GAM,INFO] = hinfsyn(P)
[K,CL,GAM,INFO] = hinfsyn(P,NMEAS,NCON)
[K,CL,GAM,INFO] = hinfsyn(P,NMEAS,NCON,KEY1,VALUE1,KEY2,VALUE2,...)
```

Description

`hinfsyn` computes a stabilizing $\mathbf{H}_\infty$ optimal `lti/ss` controller `K` for a partitioned `lti` plant `P`.

$$\mathtt{P} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$

The controller, `K`, stabilizes the `P` and has the same number of states as `P`. The system `P` is partitioned where inputs to B_1 are the disturbances, inputs to B_2 are the control inputs, output of C_1 are the errors to be kept small, and outputs of C_2 are the output measurements provided to the controller. B_2 has column size (`NCON`) and C_2 has row size (`NMEAS`). The optional `KEY` and `VALUE` inputs determine tolerance, solution method and so forth—see Figure 11-8 for details.

The closed-loop system is returned in `CL` and the achieved $\mathbf{H}_\infty$ cost γ in `GAM`. `INFO` is a `struct` array that returns additional information about the design—see Figure 11-9.

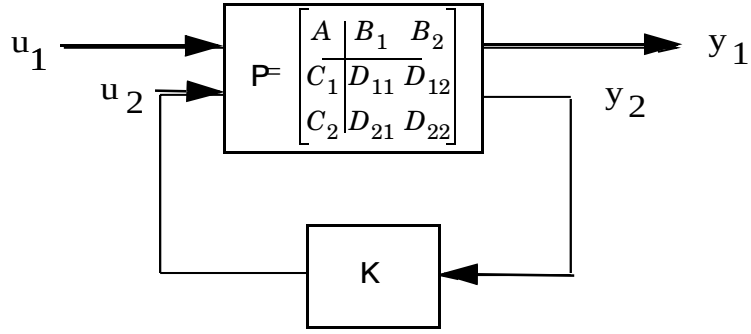


Figure 5-8: $\mathbf{H}_\infty$ control system `CL= lft(P,K)` **=** $T_{y_1 u_1}$.

Key	Value	Meaning
'GMAX'	real	Initial upper bound on GAM (default=Inf)
'GMIN'	real	Initial lower bound on GAM (default=0)
'TOLGAM'	real	Relative error tolerance for GAM (default=.01)
'SO'	real	Frequency SO at which entropy is evaluated, only applies to METHOD 'maxe' (default=Inf)
'METHOD'	*'ric'*	Standard 2-Riccati solution (default)
	'lmi'	LMI solution
	'maxe'	Maximum entropy solution
'DISPLAY'	*'off'* *'on'*	No command window display, or command window displays synthesis progress information (default)

Figure 5-9: Optional input arguments (KEY, VALUE) pairs

When DISPLAY='*on*', the hinfsyn program displays several variables indicating the progress of the algorithm. For each γ value being tested, the minimum magnitude, real part of the eigenvalues of the X and Y Hamiltonian matrices are displayed along with the minimum eigenvalue of X_∞ and Y_∞, which are the solutions to the X and Y Riccati equations, respectively. The maximum eigenvalue of $X_\infty Y_\infty$, scaled by γ^{-2},is also displayed. A # sign is placed to the right of the condition that failed in the printout.

Output arguments:	
K	lti controller
CL= lft(P,K)	lti closed-loop system $T_{y_1 u_1}$
GAM = norm(CL,Inf)	$\mathbf{H}_\infty$ cost $\gamma = \left\| T_{y_1 u_1} \right\|_\infty$
INFO	Additional output information

Additional output — structure array `INFO` containing possible additional information depending on `METHOD`):

`INFO.AS`	All solutions controller, `lti` two-port LFT
`INFO.KFI`	Full information gain matrix (constant feedback $u_2(t) = K_{FI}\begin{bmatrix} x(t) \\ u_1(t) \end{bmatrix}$
`INFO.KFC`	Full control gain matrix (constant output-injection; K_{FC} is the dual of K_{FI})
`INFO.GAMFI`	$\mathbf{H}_\infty$ cost for full information K_{FI}
`INFO.GAMFC`	$\mathbf{H}_\infty$ cost for full control K_{FC}

Algorithm

The default `'ric'` method uses the two-Riccati formulae ([4],[5]) with loopshifting [6]. In the case of the `'lmi'` method, `hinfsyn` employs the LMI technique ([7],[8],[9]). With `'METHOD' 'maxe', K` returns the max entropy $\mathbf{H}_\infty$ controller that minimize an entropy integral relating to the point `s0`; i.e.,

$$\text{Entropy} = -\frac{\gamma^2}{2\pi}\int_{-\infty}^{\infty} \ln\left|\det I - \gamma^{-2} T_{y_1u_1}(j\omega)'\, T_{y_1u_1}(j\omega)\right| \left[\frac{{s_0}^2}{{s_0}^2 + \omega^2}\right] d\omega$$

where $T_{y_1u_1}$ is the closed-loop transfer function `CL`. With all methods, `hinfsyn` uses a standard γ-iteration technique to determine the optimal value of γ. Starting with high and low estimates of γ. The γ-iteration is a b*isection algorithm* that iterates on the value of γ in an effort to approach the optimal H_∞ control design. The stopping criterion for the bisection algorithm requires the relative difference between the last γ value that failed and the last γ value that passed be less than `TOLGAM` (default=.01)

At each value of γ, the algorithm employed requires tests to determine whether a solution exists for a given γ value. In the case of the `'ric'` method, the conditions checked for the existence of a solution are:

- H and J Hamiltonian matrices (which are formed from the state-space data of P and the γ level) must have no imaginary-axis eigenvalues.

- the stabilizing Riccati solutions X_∞ and Y_∞ associated with the Hamiltonian matrices must exist and be positive, semi-definite.
- spectral radius of (X_∞, Y_∞) must be less than or equal to γ^2.

When, `DISPLAY` is `'on'`, the `hinfsyn` program displays several variables, which indicate which of the above conditions are satisfied for each γ value being tested. In the case of the default `'ric'` method, the display includes the current value of γ being tested, real part of the eigenvalues of the X and Y Hamiltonian matrices along with the minimum eigenvalue of X_∞ and Y_∞, which are the solutions to the X and Y Riccati equations, respectively. The maximum eigenvalue of $X_\infty Y_\infty$, scaled by γ^{-2}, is also displayed. A # sign is placed to the right of the condition that failed in the printout. A similar `display` is produced with method `'lmi'`

The algorithm works best when the following conditions are satisfied by the plant:

D_{12} and D_{21} have full rank.

$\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for all $\omega \in \mathbf{R}$.

$\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank for all $\omega \in \mathbf{R}$.

When the above rank conditions do not hold, the controller may have undesirable properties: If D_{12} and D_{21} are not full rank, the $\mathbf{H}_\infty$ controller `K` may have large high-frequency gain. If either of the latter two rank conditions does not hold at some frequency ω, the controller may have very lightly damped poles near that frequency ω.

In general, the solution to the infinity-norm optimal control problem is non-unique. Whereas the `K` returned by `hinfsyn` is only a particular $F(s)$, when the `'ric'` method is selected, the `INFO.AS` field of `INFO` give you in addition the all- solution controller parameterization $K_{AS}(s)$ such that all solutions to the infinity-norm control problem are parameterized by a free stable contraction

map $U(s)$ constrained by $(\|U(s)\|_\infty < 1)$ (see Figure 5-9); that is, every stabilizing controller $K(s)$ that makes $\|T_{y_1u_1}\|_\infty \stackrel{\Delta}{=} \sup_\omega \sigma_{max}(T_{y_1u_1}(j\omega)) < \gamma$

```
K=lft(INFO.AS,U)
```

where `U` is a stable LTI system satisfying `norm(U,Inf) <1`

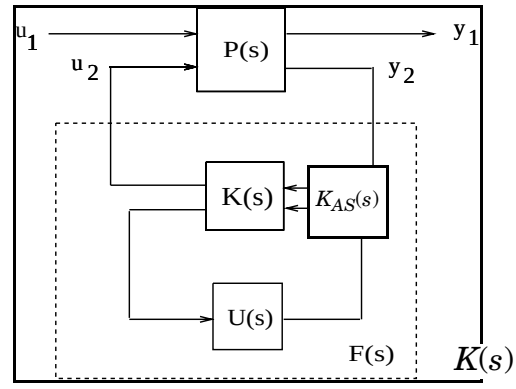


Figure 5-10: All-solution $K_{AS}(s)$ returned by `INFO.AS`

An important use of the infinity-norm control theory is for direct shaping of closed-loop singular value Bode plots of control systems. In such cases, the system $P(s)$ will typically be the plant augmented with suitable loop-shaping filters — see and `mixsyn`.

Examples

Following are three simple problems solved via `hinfsyn`.

Example 1: A random 4-by-5 plant with 3-states, `NMEAS=2`, `NCON=2`

```
rand('seed',0);randn('seed',0);
P=rss(3,4,5);
[K,CL,GAM]=hinfsyn(P,2,2);
```

The optimal $\mathbf{H}_\infty$ cost in this case is `GAM`=0.2641. You verify that $\|T_{y_1u_1}\|_\infty \stackrel{\Delta}{=} \sup_\omega \sigma_{max}(T_{y_1u_1}(j\omega)) < \gamma$ with a sigma plot

```
sigma(CL,ss(GAM));
```

Example 2: Mixed-Sensitivity $G(s) = \frac{s-1}{s-2}$, $W_1 = \frac{0.1(s+1000)}{100s+1}$, $W_2 = 0.1$, no W_3.

```
s=zpk('s');
G=(s-1)/(s+1);
W1=0.1*(s+100)/(100*s+1); W2=0.1; W3=[];
P=augw(G,W1,W2,W3);
[K,CL,GAM]=hinfsyn(P);
sigma(CL,ss(GAM));
```

In this case, `GAM` = 0.1854 = -14.6386 db

Example 3: Mixed sensitivity with W_1 removed.

```
s=zpk('s');
G=(s-1)/(s+1);
W1=[]; W2=0.1; W3=[];
P=augw(G,W1,W2,W3)
[K,CL,GAM]=hinfsyn(P)
```

In this case, `GAM=0`, `K=0`, and `CL=K*(1+G*K)=0`.

Limitation

The plant must be stabilizable from the control inputs u_2 and detectable from the measurement output y_2:

- (A,B_2) must be stabilizable and (C_2,A) must be detectable.

Otherwise, an the `hinfsyn` returns an error.

References

[4] Glover, K., and J.C. Doyle, "State-space formulae for all stabilizing controllers that satisfy an H_∞norm bound and relations to risk sensitivity," *Systems and Control Letters,* vol. 11, pp. 167–172, 1988.

[5] Doyle, J.C., K. Glover, P. Khargonekar, and B. Francis, "State-space solutions to standard H_2 and H_∞ control problems," *IEEE Transactions on Automatic Control,* vol. 34, no. 8, pp. 831–847, August 1989

[6] Safonov, M.G., D.J.N. Limebeer, and R.Y. Chiang, "Simplifying the $\mathbf{H}^\infty$ Theory via Loop Shifting, Matrix Pencil and Descriptor Concepts", *Int. J. Contr.*, vol. 50, no. 6, pp. 2467-2488, 1989.

[7] Packard, A., K. Zhou, P. Pandey, J. Leonhardson, and G. Balas, "Optimal, constant I/O similarity scaling for full-information and state-feedback problems," *Systems and Control Letters*, 19:271–280, 1992.

[8] Gahinet, P., and P. Apkarian, "A linear matrix inequality approach to H_1-control," *Int J. Robust and Nonlinear Control*, 4(4):421–448, July–August 1994.

[9] Iwasaki, T., and R.E. Skelton, "All controllers for the general H_∞-control problem: LMI existence conditions and state space formulas," *Automatica*, 30(8):1307–1317, August 1994.

See Also

`augw`	Augments plant weights for control design
`h2syn`	H_2 synthesis controller
`loopsyn`	H_∞ - loop shaping controller synthesis
`mktito`	2-input 2-output partition of a USS object
`ncfsyn`	H_∞ - normailized coprime controller synthesis

icomplexify

Purpose

Helper function for complexify.

Syntax

```
DeltaR = icomplexify(DeltaCR)
```

Description

`icomplexify` works on structures to extract a real value from a pair of related fields.

`DeltaR = icomplexify(DeltaCR)` affects field pairs of `DeltaCR` named `'foo'` and `'foo_cmpxfy'` where `'foo'` can be any field name. `DeltaR` is the same as `DeltaCR` except that the fields `'foo_cmpxfy'` are removed. `complexify`, by default, complexifies the real uncertainty with ucomplex atoms, though optionally `ultidyn` atoms can be used. If a `ucomplex` uncertainty was used to complexify the uncertain system, the real parts of `'foo_cmpxfy'` are added to the real parts of `'foo'`. If a `ultidyn` uncertainty was used to complexify the uncertain system, only the real parts of `'foo'` are returned.

See Also

`complexify`	Replace `ureal` atoms with `ureal` and `ucomplex` atoms
`robuststab`	Calculate robust stability margin

Purpose

Create empty `iconnect` (interconnection) objects

Syntax

```
H = iconnect
```

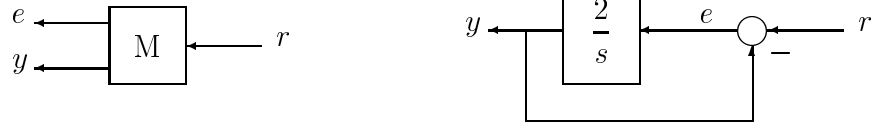
Description

Interconnection objects (class `iconnect`) are an alternative to `sysic`, and are used to build complex interconnections of uncertain matrices and systems.

An `iconnect` object has 3 fields to be set by the user, `Input`, `Output` and `Equation`. `Input` and `Output` are `icsignal` objects, while `Equation`.is a cell-array of equality constraints (using `equate`) on `icsignal` objects. Once these are specified, then the `System` property is the input/output model, implied by the constraints in `Equation`. relating the variables defined in `Input` and `Output`.

Example

`iconnect` can be used to create the transfer matrix `M` as described in the following figure.



Create three scalar `icsignal`: `r`, `e` and `y`. Create an empty `iconnect` object, `M`. Define the output of the interconnection to be `[e; y]`, and the input to be `r`. Define two constraints among the variables: `e = r-y`, and `y = (2/s) e`. Get the transfer function representation of the relationship between the input (`r`) and the output `[e; y]`.

```
r = icsignal(1);
e = icsignal(1);
y = icsignal(1);
M = iconnect;
M.Input = r;
M.Output = [e;y];
M.Equation{1} = equate(e,r-y);
M.Equation{2} = equate(y,tf(2,[1 0])*e);
tf(M.System)
```

The transfer functions from input to outputs are

```
        s
#1:  -----
     s + 2

        2
#2:  -----
     s + 2
```

By not explicitly introducing `e`, this can be done more concisely with only one equality constraint.

```
r = icsignal(1);
y = icsignal(1);
N = iconnect;
N.Input = r;
N.Output = [r-y;y];
N.Equation{1} = equate(y,tf(2,[1 0])*(r-y));
tf(N.System)
```

You have created the same transfer functions from input to outputs.

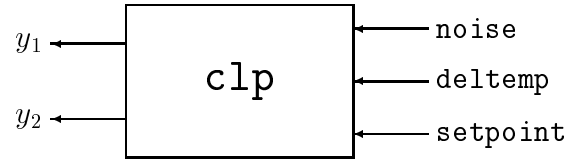
```
        s
#1:  -----
     s + 2

        2
#2:  -----
     s + 2
```

You can also specify uncertain, multivariable interconnections using `iconnect`. Consider two uncertain motor/generator constraints among 4 variables `[V;I;T;W]`, `V-R*I-K*W=0`, and `T=K*I`. Find the uncertain `2x2` matrix `B` so that `[V;T] = B*[W;I]`.

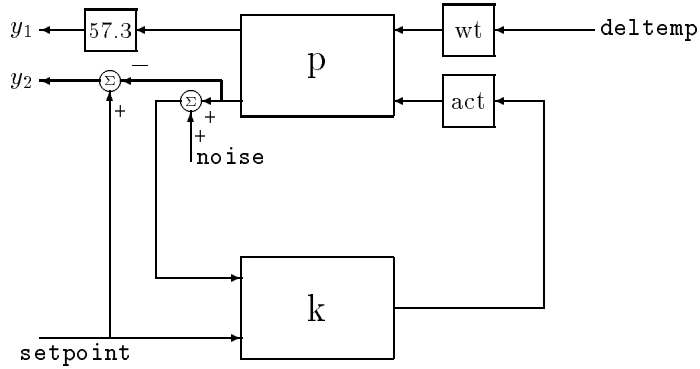
```
R = ureal('R',1,'Percentage',[-10 40]);
K = ureal('K',2e-3,'Percentage',[-30 30]);
V = icsignal(1);
I = icsignal(1);
T = icsignal(1);
W = icsignal(1);
```

```
M = iconnect;
M.Input = [W;I];
M.Output = [V;T];
M.Equation{1} = equate(V-R*I-K*W,iczero(1));
M.Equation{2} = equate(T,K*I);
B = M.System
UMAT: 2 Rows, 2 Columns
  K: real, nominal = 0.002, variability = [-30  30]%, 2 occurrences
  R: real, nominal = 1, variability = [-10  40]%, 1 occurrence
B.NominalValue
ans =
    0.0020    1.0000
         0    0.0020
```

A simple system interconnection, identical to the system illustrated in the `sysic` reference pages. Consider a three-input, two-output state space matrix *T*,



which has internal structure



```
P = rss(3,2,2);
K = rss(1,1,2);
A = rss(1,1,1);
W = rss(1,1,1);
M = iconnect;
noise = icsignal(1);
deltemp = icsignal(1);
setpoint = icsignal(1);
yp = icsignal(2);
rad2deg = 57.3
rad2deg =
   57.3000
M.Equation{1} =
equate(yp,P*[W*deltemp;A*K*[noise+yp(2);setpoint]]);
M.Input = [noise;deltemp;setpoint];
M.Output = [rad2deg*yp(1);setpoint-yp(2)];
T = M.System;
size(T)
State-space model with 2 outputs, 3 inputs, and 6 states.
```

Algorithm

Each equation represents an equality constraint among the variables. You choose the input and output variables, and the `imp2exp` function makes the implicit relationship between them explicit.

Limitations

The syntax for `iconnect` objects and `icsignals` is very flexible. Without care, you can build inefficient (i.e., nonminimal) representations where the state dimension of the interconnection is greater than the sum of the state dimensions of the components. This is in contrast to `sysic`. In `sysic`, the syntax used to specify inputs to systems (the `input_to_ListedSubSystemName` variable) forces you to include each subsystem of the interconnection only once in the equations. Hence, interconnections formed with `sysic` are componentwise minimal. That is, the state dimension of the interconnection equals the sum of the state dimensions of the components.

See Also

`equate`	Equates expressions for `icsignal` objects
`icsignal`	Constructs an `icsignal` object
`sysic`	Constructs system interconnection

Purpose

Create an `icsignal` object of specified dimension

Syntax

```
v = icsignal(n);
v = icsignal(n,'name');
```

Description

`icsignal` creates an `icsignal` object, which is a symbolic column vector. The `icsignal` object is used with `iconnect` objects to specify signal constraints described by the interconnection of components.

`v = icsignal(n)` creates an `icsignal` object of vector length `n`. The value of `n` must be a nonnegative integer. `icsignal` objects are symbolic column vectors, used in conjunction with `iconnect` (interconnection) objects to specify the signal constraints described by an interconnection of components.

`v = icsignal(n,name)` creates an `icsignal` object of dimension `n`, with internal name identifier given by the character string argument `name`.

See Also

`iconnect`	Equates expressions for `icsignal` objects
`sysic`	Constructs system interconnections

imp2exp

Purpose

Convert implicit linear relationship to explicit input-output relation

Syntax

```
B = imp2exp(A,yidx,uidx)
```

Description

`B = imp2exp(A,yidx,uidx)` transforms a linear constraint between variables `Y` and `U` of the form `A(:,[yidx;uidx])*[Y;U] = 0` into an explicit input/output relationship `Y = B*U`. The vectors `yidx` and `uidx` refer to the columns (inputs) of `A` as referenced by the explicit relationship for `B`.

The constraint matrix `A` can be a `double, ss, tf, zpk` and `frd` object as well as an uncertain object, including `umat`, `uss` and `ufrd`. The result `B` will be of the same class.

Example

Scalar Algebraic Constraint

Consider the constraint `4y + 7u = 0`. Solving for `y` gives `y = 1.75u`. You form the equation using `imp2exp`:

```
A = [4 7];
Yidx = 1;
Uidx = 2;
```

and then

```
B = imp2exp(A,Yidx,Uidx)
B =
   -1.7500
```

yields `B` equal to `-1.75`.

Matrix Algebraic Constraint

Consider two motor/generator constraints among 4 variables `[V;I;T;W]`, namely `[1 -1 0 -2e-3;0 -2e-3 1 0]*[V;I;T;W] = 0`. You can find the 2-by-2 matrix `B` so that `[V;T] = B*[W;I]` using `imp2exp`.

```
A = [1 -1 0 -2e-3;0 -2e-3 1 0];
Yidx = [1 3];
Uidx = [4 2];
B = imp2exp(A,Yidx,Uidx)
B =
    0.0020    1.0000
         0    0.0020
```

You can find the 2-by-2 matrix `C` so that `[I;W] = C*[T;V]`

```
Yidx = [2 4];
Uidx = [3 1];
C = imp2exp(A,Yidx,Uidx)
C =
         500           0
     -250000         500
```

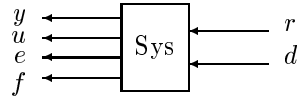
Uncertain Matrix Algebraic Constraint

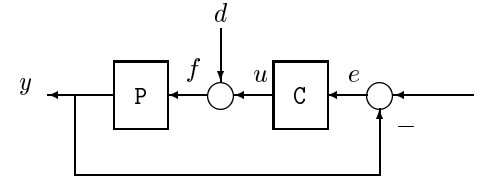
Consider two uncertain motor/generator constraints among 4 variables `[V;I;T;W]`, namely `[1 -R 0 -K;0 -K 1 0]*[V;I;T;W] = 0`. You can find the uncertain 2-by-2 matrix `B` so that `[V;T] = B*[W;I]`.

```
R = ureal('R',1,'Percentage',[-10 40]);
K = ureal('K',2e-3,'Percentage',[-30 30]);
A = [1 -R 0 -K;0 -K 1 0];
Yidx = [1 3];
Uidx = [4 2];
B = imp2exp(A,Yidx,Uidx)
UMAT: 2 Rows, 2 Columns
  K: real, nominal = 0.002, variability = [-30  30]%, 2 occurrences
  R: real, nominal = 1, variability = [-10  40]%, 1 occurrence
```

Scalar Dynamic System Constraint

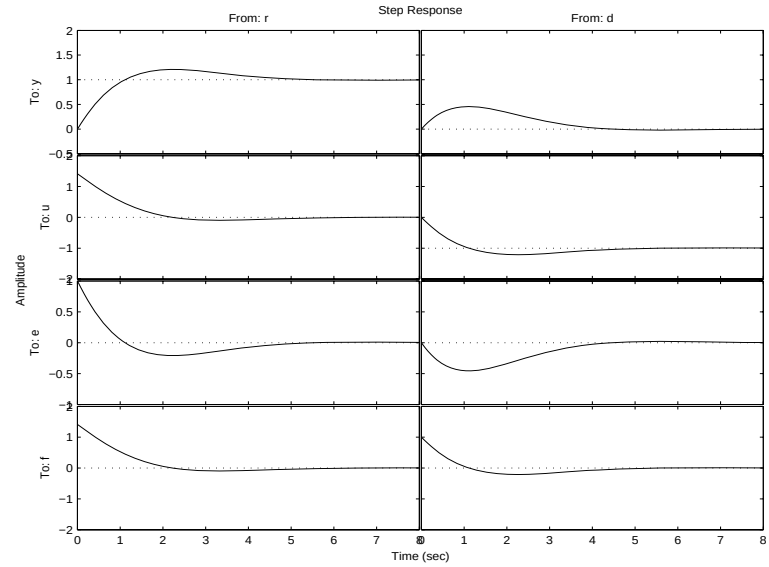
Consider a standard single-loop feedback connection of controller `C` and an uncertain plant `P`, described by the equations `e=r-y, u=Ce; f=d+u; y=Pf`.






```
P = tf([1],[1 0]);
C = tf([2*.707*1 1^2],[1 0]);
A = [1 -1 0 0 0 -1;0 -C 1 0 0 0;0 0 -1 -1 1 0;0 0 0 0 -P 1];
OutputIndex = [6;3;2;5];  % [y;u;e;f]
InputIndex = [1;4];       % [r;d]
Sys = imp2exp(A,OutputIndex,InputIndex);
```

```
Sys.InputName = {'r';'d'};
Sys.OutputName = {'y';'u';'e';'f'};
pole(Sys)
ans =
  -0.7070 + 0.7072i
  -0.7070 - 0.7072i
step(Sys)
```



Algorithm

The number of rows of `A` must equal the length of `yidx`.

See Also

`iconnect`	Equates expressions for `icsignal` objects
`inv`	Forms the system inverse

Purpose

System realization via Hankel singular value decomposition

Syntax

```
[a,b,c,d,totbnd,hsv] = imp2ss(y)
[a,b,c,d,totbnd,hsv] = imp2ss(y,ts,nu,ny,tol)
[ss,totbnd,hsv] = imp2ss(imp)
[ss,totbnd,hsv] = imp2ss(imp,tol)
```

Description

The function `imp2ss` produces an approximate state-space realization of a given impulse response

```
imp=mksys(y,t,nu,ny,'imp');
```

using the Hankel SVD method proposed by S. Kung [2]. A continuous-time realization is computed via the inverse Tustin transform (using `bilin`) if t is positive; otherwise a discrete-time realization is returned. In the SISO case the variable y is the impulse response vector; in the MIMO case y is an N+1-column matrix containing $N + 1$ time samples of the matrix-valued impulse response $H_0, \ldots, H_N$ of an `nu`-input, `ny`-output system stored row-wise:

$$y = [H_0(:)';H_2(:)';H_3(:)';\ldots;H_N(:)']$$

The variable *tol* bounds the $\mathbf{H}^\infty$ norm of the error between the approximate realization *(a, b, c, d)* and an exact realization of y; the order, say n, of the realization *(a, b, c, d)* is determined by the infinity norm error bound specified by the input variable *to*l. The inputs `ts, nu, ny, tol` are optional; if not present they default to the values `ts = 0, nu = 1, ny =` (number of rows of y)/`nu, tol =` $0.01\bar{\sigma}_1$. The output `hsv =` $[\bar{\sigma}_1, \bar{\sigma}_2, \ldots]'$ returns the singular values (arranged in descending order of magnitude) of the Hankel matrix:

$$\Gamma = \begin{bmatrix} H_1 & H_2 & H_3 & \ldots & H_N \\ H_2 & H_3 & H_4 & \ldots & 0 \\ H_3 & H_4 & H_5 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_N & 0 & \ldots & \ldots & 0s \end{bmatrix}$$

Denoting by G_N a high-order exact realization of y, the low-order approximate model G enjoys the $\mathbf{H}^\infty$ norm bound

$$\left\| G - G_N \right\|_\infty \le totbnd$$

where

$$totbnd = 2 \sum_{i = n+1}^{N} \bar{\sigma}_i$$

Algorithm

The realization *(a, b, c, d)* is computed using the Hankel SVD procedure proposed by Kung [2] as a method for approximately implementing the classical Hankel factorization realization algorithm. Kung's SVD realization procedure was subsequently shown to be equivalent to doing balanced truncation (`balmr`) on an exact state space realization of the finite impulse response {*y*(1),…*y*(*N*)} [3]. The infinity norm error bound for discrete balanced truncation was later derived by Al-Saggaf and Franklin [1]. The algorithm is as follows:

1. Form the Hankel matrix Γ from the data *y*.
2. Perform SVD on the Hankel matrix

   $$\Gamma = U\Sigma V^* = [U_1 U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V^*_1 \\ V^*_2 \end{bmatrix} = U_1 \Sigma_1 V^*_1$$

   where Σ_1 has dimension $n \times n$ and the entries of Σ_2 are nearly zero. U_1 and V_1 have *ny* and *nu* columns, respectively.
3. Partition the matrices U_1 and V_1 into three matrix blocks:

   $$U_1 = \begin{bmatrix} U_{11} \\ U_{12} \\ U_{13} \end{bmatrix}; \begin{bmatrix} V_{11} \\ V_{12} \\ V_{13} \end{bmatrix}$$

   where $U_{11},\ U_{13} \in C^{ny \times n}$ and $V_{11},\ V_{13} \in C^{nu \times n}$.
4. A discrete state-space realization is computed as

$$A = \Sigma_1^{-1/2}\overline{U}\Sigma_1^{1/2}$$

$$B = \Sigma_1^{-1/2}V^*{}_{11}$$

$$C = U_{11}\Sigma_1^{-1/2}$$

$$D = H_0$$

where

$$\overline{U} = \begin{bmatrix} U_{11} \\ U_{12} \end{bmatrix}' \begin{bmatrix} U_{12} \\ U_{13} \end{bmatrix}$$

5 If the sampling time t is greater than zero, then the realization is converted to continuous time via the inverse of the Tustin transform

$$s = \frac{2}{t}\frac{z-1}{z+1};$$

Otherwise, this step is omitted and the discrete-time realization calculated in Step 4 is returned.

References

[1] Al-Saggaf, U.M., and G.F. Franklin, "An Error Bound for a Discrete Reduced Order Model of a Linear Multivariable System," *IEEE Trans. on Autom. Contr.*, AC-32, 1987, p. 815-819.

[2] Kung, S.Y., "A New Identification and Model Reduction Algorithm via Singular Value Decompositions," *Proc.Twelth Asilomar Conf. on Circuits, Systems and Computers*, November 6-8, 1978, p. 705-714.

[3] Silverman, L.M., and M. Bettayeb, "Optimal Approximation of Linear Systems," *Proc. American Control Conf.*, San Francisco, CA, 1980.

ispsys

Purpose

True for parameter-dependent systems

Syntax

```
bool = ispsys(sys)
```

Description

`bool = ispsys(sys)` returns `1` if `sys` is a polytopic or parameter-dependent system.

See Also

`psys`	Specification of uncertain state-space models
`psinfo`	Inquire about polytopic or parameter-dependent systems created with `psys`

Purpose

Check whether argument is uncertain class type

Syntax

```
B = isuncertain(A)
```

Description

Returns `true` if input argument is uncertain, `false` otherwise. Uncertain classes are `umat`, `ufrd`, `uss`, `ureal`, `ultidyn`, `ucomplex`, `ucomplexm`, and `udyn`.

Example

In this example, you verify the correct operation of `isuncertain` on `double`, `ureal`, `ss`, and `uss` `objects`.

```
isuncertain(rand(3,4))
ans =
     0
isuncertain(ureal('p',4))
ans =
     1
isuncertain(rss(4,3,2))
ans =
     0
isuncertain(rss(4,3,2)*[ureal('p1',4) 6;0 1])
ans =
     1
```

Limitations

`isuncertain` only checks the class of the input argument, and does not actually verify that the input argument is truly uncertain. Create a `umat` by *lifting* a constant (i.e., not-uncertain) matrix to the `umat` class.

```
A = umat([2 3;4 5;6 7]);
```

Note that although `A` is in class `umat`, it is not actually uncertain. Nevertheless, based on class, the result of `isuncertain(A)` is `true`.

```
isuncertain(A)
ans =
     1
```

The result of `simplify(A)` is a `double`, and hence not uncertain.

```
isuncertain(simplify(A))
ans =
     0
```

lftdata

Purpose

Decompose uncertain objects into fixed normalized and fixed uncertain parts

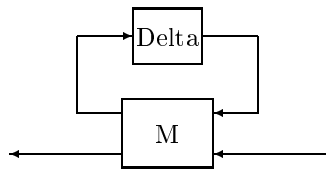
Syntax

```
[M,Delta] = lftdata(A);
[M,Delta] = lftdata(A,List);
[M,Delta,Blkstruct] = lftdata(A);
[M,Delta,Blkstruct,Normunc] = lftdata(A);
```

Description

`lftdata` decomposes an uncertain object into a fixed certain part and a normalized uncertain part. `lftdata` can also partially decompose an uncertain object into an uncertain part and a normalized uncertain part. Uncertain objects (`umat, ufrd, uss`) are represented as certain (i.e., not-uncertain) objects in feedback with block-diagonal concatenations of uncertain elements.

`[M,Delta] = lftdata(A)` separates the uncertain object `A` into a certain object `M` and a normalized uncertain matrix `Delta` such that `A` is equal to `lft(Delta,M)`, as shown below.



If `A` is a `umat`, then `M` will be `double`; if `A` is a `uss`, then `M` will be `ss`; if `A` is a `ufrd`, then `M` will be `frd`. In all cases, `Delta` is a `umat`.

`[M,Delta] = lftdata(A,List)` separates the uncertain object `A` into an uncertain object `M`, in feedback with a normalized uncertain matrix `Delta`. `List` is a cell (or char) array of names of uncertain elements of `A` that make up `Delta`. All other uncertainty in `A` remains in `M`.

`lftdata(A,fieldnames(A.Uncertainty))` is the same as `lftdata(A)`.

`[M,DELTA,BLKSTRUCT] = lftdata(A)` returns an `N`-by-1 structure array `BLKSTRUCT`, where `BLKSTRUCT(i)` describes the `i`-th normalized uncertain element. This uncertainty description can be passed directly to the low-level structured singular value analysis function `mussv`.

`[M,DELTA,BLKSTRUCT,NORMUNC] = lftdata(A)` returns the cell array `NORMUNC` of normalized uncertain elements. Each normalized element has the string `'Normalized'` appended to its original name to avoid confusion. Note that `lft(blkdiag(NORMUNC{:}),M)` is equivalent to `A`.

Example

Create an uncertain matrix `A` with 3 uncertain parameters `p1`, `p2` and `p3`. You can decompose `A` into its certain, `M`, and normalized uncertain parts, `Delta`.

```
p1 = ureal('p1',-3,'perc',40);
p2 = ucomplex('p2',2);
A = [p1 p1+p2;1 p2];
[M,Delta] = lftdata(A);
```

You can inspect the difference between the original uncertain matrix, `A`, and the result formed by combining the two results from the decomposition.

```
simplify(A-lft(Delta,M))
ans =
     0     0
     0     0
M
M =
          0         0    1.0954    1.0954
          0         0         0    1.0000
     1.0954    1.0000   -3.0000   -1.0000
          0    1.0000    1.0000    2.0000
```

You can check the worst-case norm of the uncertain part using `wcnorm`. Compare samples of the uncertain part `A` with the uncertain matrix `A`.

```
wcn = wcnorm(Delta)
wcn =
    lbound: 1.0000
    ubound: 1.0001
usample(Delta,5)
ans(:,:,1) =
   0.8012                  0
        0            0.2499 + 0.6946i
ans(:,:,2) =
   0.4919                  0
        0            0.2863 + 0.6033i
```

```
ans(:,:,3) =
  -0.1040                  0
        0            0.7322 - 0.3752i
ans(:,:,4) =
   0.8296                  0
        0            0.6831 + 0.1124i
ans(:,:,5) =
   0.6886                  0
        0            0.0838 + 0.3562i
```

Uncertain Systems

Create an uncertain matrix `A` with 2 uncertain real parameters `v1` and `v2` and create an uncertain system `G` using `A` as the dynamic matrix and simple matrices for the input and output.

```
A = [ureal('p1',-3,'perc',40) 1;1 ureal('p2',-2)];
sys = ss(A,[1;0],[0 1],0);
sys.InputGroup.ActualIn = 1;
sys.OutputGroup.ActualOut = 1;
```

You can decompose `G` into a certain system, `Msys`, and a normalized uncertain matrix, `Delta`. You can see from `Msys` that it is certain and that the input and output groups have been adjusted.

```
[Msys,Delta] = lftdata(sys);
Msys

a =
       x1  x2
   x1  -3   1
   x2   1  -2


b =
          u1     u2     u3
   x1  1.095      0      1
   x2      0      1      0


c =
          x1     x2
```

```
   y1  1.095      0
   y2      0      1
   y3      0      1


d =
       u1  u2  u3
   y1   0   0   0
   y2   0   0   0
   y3   0   0   0

Input groups:
      Name      Channels
    ActualIn       3
     p1_NC         1
     p2_NC         2

Output groups:
      Name       Channels
    ActualOut       3
      p1_NC         1
      p2_NC         2

Continuous-time model.
```

You can compute the norm on samples of the difference between the original uncertain matrix and the result formed by combining `Msys` and `Delta`.

```
norm(usample(sys-lft(Delta,Msys),'p1',4,'p2',3),'inf')
ans =
     0     0     0
     0     0     0
     0     0     0
     0     0     0
```

Partial Decomposition

Create an uncertain matrix `A` and derive an uncertain matrix `B` using an implicit-to-explicit conversion, `imp2exp`. Note that `B` has 2 uncertain parameters `R` and `K`. You can decompose `B` into certain, `M`, and normalized uncertain parts, `Delta`.

```
R = ureal('R',1,'Percentage',[-10 40]);
K = ureal('K',2e-3,'Percentage',[-30 30]);
A = [1 -R 0 -K;0 -K 1 0];
Yidx = [1 3];
Uidx = [4 2];
B = imp2exp(A,Yidx,Uidx);
[M,Delta] = lftdata(B);
```

The same operation can be performed by defining the uncertain parameters, `K` and `R`, to be extracted.

```
[MK,DeltaR] = lftdata(B,'R');
MK
UMAT: 3 Rows, 3 Columns
  K: real, nominal = 0.002, variability = [-30  30]%, 2 occurrences
[MR,DeltaK] = lftdata(B,'K');
MR
UMAT: 4 Rows, 4 Columns
  R: real, nominal = 1, variability = [-10  40]%, 1 occurrence

simplify(B-lft(Delta,M))
ans =
     0     0
     0     0
simplify(B-lft(DeltaR,MK))
ans =
     0     0
     0     0
simplify(B-lft(DeltaK,MR))
ans =
     0     0
     0     0
```

Sample and inspect the uncertain part as well as the difference between the original uncertain matrix and the sampled matrix. You can see the result formed by combining the two results from the decomposition.

```
[Mall,Deltaall] = lftdata(B,{'K';'R'});
simplify(Mall)-M
ans =
     0     0     0     0     0
```

```
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
```

See Also

`lft`	Forms Redheffer star product of systems
`ssdata`	Returns uncertain state-space data

ltiarray2uss

Purpose

Computes uncertain system bounding given LTI ss array

Syntax

```
usys = ltiarray2uss(P,Parray,ord)
[usys,wt] = ltiarray2uss(P,Parray,ord)
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord)
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,'InputMult')
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,'OutputMult')
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,'Additive')
```

Description

`ltiarray2uss` calculates an uncertain system `usys` with nominal value `P`, and whose range of behavior includes the given array of systems, `Parray`.

`usys = ltiarray2uss(P,Parray,ord)`, `usys` is formulated as an input multiplicative uncertainty model,
`usys = P*(I + wt*ultidyn('IMult',[size(P,2) size(P,2)]))`, where `wt` is a stable scalar system, whose magnitude overbounds the relative difference, `(P - Parray)/P`. The state order of the weighting function used to bound the multiplicative difference between `P` and `Parray` is `ord`. Both `P` and `Parray` must be in the classes `ss/tf/zpk/frd`. If `P` is an `frd` then `usys` will be a `ufrd` object, otherwise `usys` will be a `uss` object. The `ultidyn` atom is named based on the variable name of `Parray` in the calling workspace.

`[usys,wt] = ltiarray2uss(P,Parray,ord)`, returns the weight `wt` used to bound the infinity norm of `((P - Parray)/P)`.

`[usys,wt] = ltiarray2uss(P,Parray,ord,'OutputMult')`, uses multiplicative uncertainty at the plant output (as opposed to input multiplicative uncertainty). The formula for `usys` is
`usys = (I + wt*ultidyn('Name',[size(P,1) size(P,1)])*P)`.

`[usys,wt] = ltiarray2uss(P,Parray,ord,'Additive')`, uses additive uncertainty. `usys = P + wt*ultidyn('Name',[size(P,1) size(P,2)])`. `wt` is a frequency domain overbound of the infinity norm of `(Parray - P)`.

`[usys,wt] = ltiarray2uss(P,Parray,ord,'InputMult')`, uses multiplicative uncertainty at the plant input (this is the default). The formula for `usys` is
`usys = P*(I + wt*ultidyn('Name',[size(P,2) size(P,2)]))`.

`[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,type)` returns the norm of the difference (absolute difference for additive, and relative difference

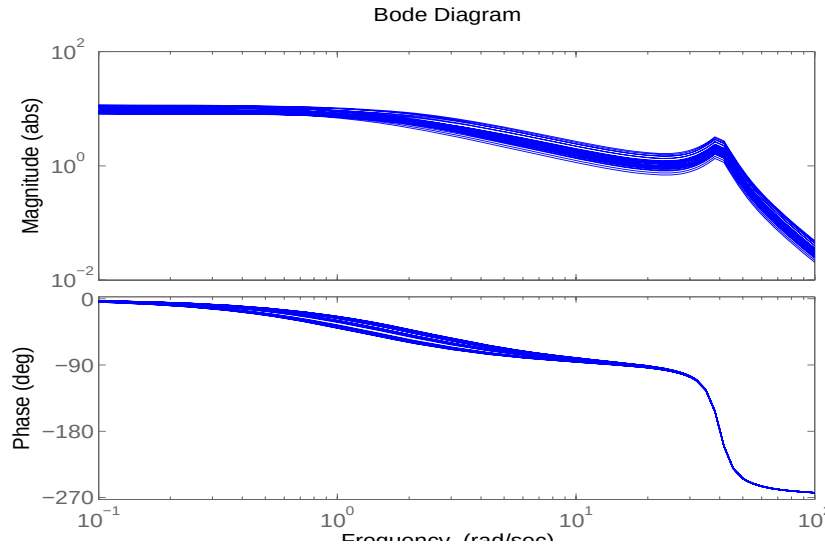
for multiplicative uncertainty) between the nominal model `P` and `Parray`. `wt` satisfies `diffdata(w_i) < |wt(w_i)|` at all frequency points.

Example

See the Robust Control Toolbox™ demo entitled “First-cut Robust Design” for a more detailed example of how to use `ltiarray2uss`.

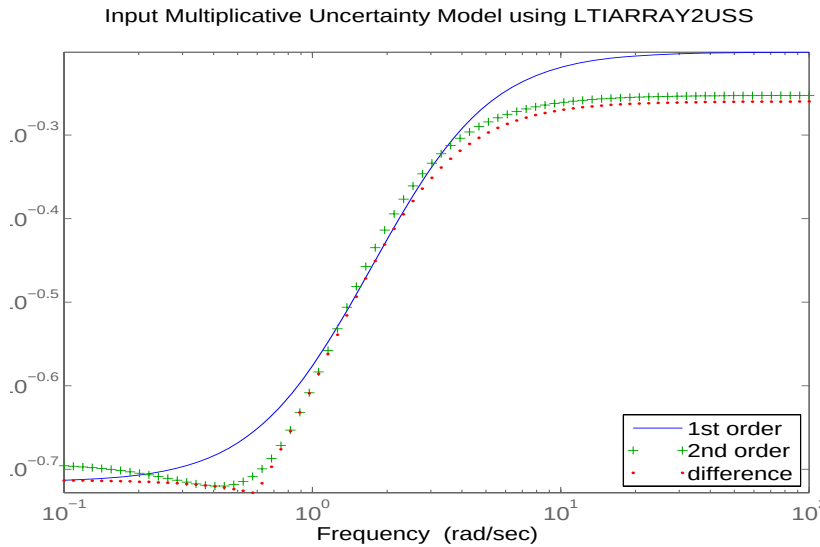
Consider a third order transfer function with an uncertain gain, filter time constant and a lightly damped flexible mode. This model is used to represent a physical system from frequency response data is acquired.

```
gain = ureal('gain',10,'Perc',20);
tau = ureal('tau',.6,'Range',[.42 .9]);
wn = 40;
zeta = 0.1;
usys = tf(gain,[tau 1])*tf(wn^2,[1 2*zeta*wn wn^2]);
sysnom = usys.NominalValue;
parray = usample(usys,30);
om = logspace(-1,2,80);
parrayg = frd(parray,om);
bode(parrayg)
```



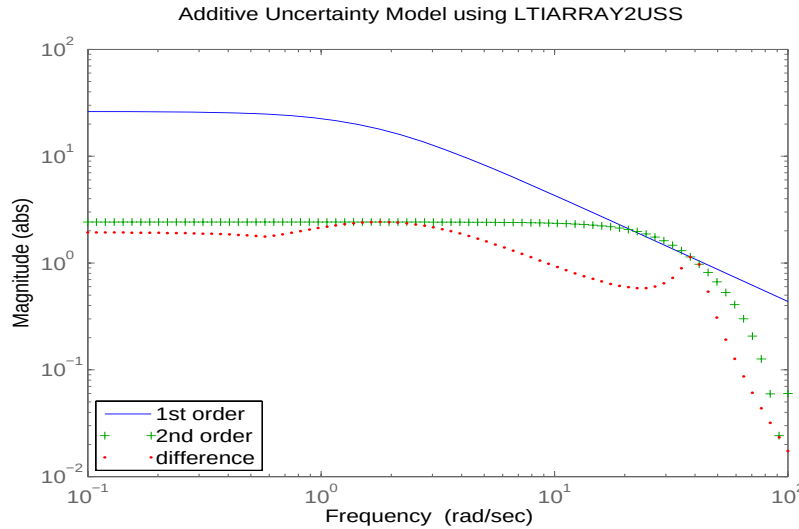
The frequency response data in `parray` represents 30 experiments performed on the system. The command `ltiarray2uss` is used to generate an uncertain model, `umod`, based on the frequency response data. Initially an input multiplicative uncertain model is used to characterize the collection of 30 frequency responses. First and second order input multiplicative uncertainty weight are calculated from the data.

```
[umodIn1,wtIn1,diffdataIn] = ltiarray2uss(sysnom,parrayg,1);
[umodIn2,wtIn2,diffdataIn] = ltiarray2uss(sysnom,parrayg,2);
bodemag(wtIn1,'b-',wtIn2,'g+',diffdataIn,'r.',om)
```



Alternatively, an additive uncertain model is used to characterize the collection of 30 frequency responses.

```
[umodAdd1,wtAdd1,diffdataAdd] = ...
ltiarray2uss(sysnom,parrayg,1,'Additive');
[umodAdd2,wtAdd2,diffdataAdd] = ...
ltiarray2uss(sysnom,parrayg,2,'Additive');
bodemag(wtAdd1,'b-',wtAdd2,'g+',diffdataAdd,'r.',om)
```



See Also

`fitmagfrd`	Fits frequency response with state-space model
`ultidyn`	Creates an uncertain linear time-invariant object
`uss`	Creates an uncertain state-space system

lmiedit

Purpose

Specify or display systems of LMIs as MATLAB® expressions

Syntax

```
lmiedit
```

Description

`lmiedit` is a graphical user interface for the symbolic specification of LMI problems. Typing `lmiedit` calls up a window with two editable text areas and various buttons. To specify an LMI system,

1. Give it a name (top of the window).
2. Declare each matrix variable (name and structure) in the upper half of the window. The structure is characterized by its type (`S` for symmetric block diagonal, `R` for unstructured, and `G` for other structures) and by an additional structure matrix similar to the second input argument of `lmivar`. Please use one line per matrix variable in the text editing areas.
3. Specify the LMIs as MATLAB expressions in the lower half of the window. An LMI can stretch over several lines. However, do not specify more than one LMI per line.

Once the LMI system is fully specified, you can perform the following operations by pressing the corresponding button:

- Visualize the sequence of `lmivar`/`lmiterm` commands needed to describe this LMI system (`view commands` buttons)
- Conversely, display the symbolic expression of the LMI system produced by a particular sequence of `lmivar`/`lmiterm` commands (click the `describe...` buttons)
- Save the symbolic description of the LMI system as a MATLAB string (`save` button). This description can be reloaded later on by pressing the `load` button
- Read a sequence of `lmivar`/`lmiterm` commands from a file (`read` button). The matrix expression of the LMI system specified by these commands is then displayed by clicking on `describe the LMIs...`
- Write in a file the sequence of `lmivar`/`lmiterm` commands needed to specify a particular LMI system (`write` button)
- Generate the internal representation of the LMI system by pressing `create`. The result is written in a MATLAB variable with the same name as the LMI system

Remark

Editable text areas have built-in scrolling capabilities. To activate the scroll mode, click in the text area, maintain the mouse button down, and move the mouse up or down. The scroll mode is only active when all visible lines have been used.

See Also

`lmivar`	Specify the matrix variables in an LMI problem
`lmiterm`	Specify the term content of LMIs
`newlmi`	Attach an identifying tag to LMIs
`lmiinfo`	Interactively retrieve information about the variables and term content of LMIs

lmiinfo

Purpose

Information about variables and term content of LMIs

Syntax

```
lmiinfo(lmisys)
```

Description

`lmiinfo` provides qualitative information about the system of LMIs `lmisys`. This includes the type and structure of the matrix variables, the number of diagonal blocks in the inner factors, and the term content of each block.

`lmiinfo` is an interactive facility where the user seeks specific pieces of information. General LMIs are displayed as

```
N' * L(x) * N < M' * R(x) * M
```

where `N,M` denote the outer factors and `L,R` the left and right inner factors. If the outer factors are missing, the LMI is simply written as

```
L(x) < R(x)
```

If its right-hand side is zero, it is displayed as

```
N' * L(x) * N < 0
```

Information on the block structure and term content of `L(x)` and `R(x)` is also available. The term content of a block is symbolically displayed as

```
C1 + A1*X2*B1 + B1'*X2*A1' + a2*X1 + x3*Q1
```

with the following conventions:

- `X1, X2, x3` denote the problem variables. Upper-case `X` indicates matrix variables while lower-case `x` indicates scalar variables. The labels 1,2,3 refer to the first, second, and third matrix variable in the order of declaration.
- `Cj` refers to constant terms. Special cases are `I` and `I` (`I` = identity matrix).
- `Aj, Bj` denote the left and right coefficients of variable terms. Lower-case letters such as `a2` indicate a scalar coefficient.
- `Qj` is used exclusively with scalar variables as in `x3*Q1`.

The index `j` in `Aj, Bj, Cj, Qj` is a dummy label. Hence `C1` may appear in several blocks or several LMIs without implying any connection between the corresponding constant terms. Exceptions to this rule are the notations `A1*X2*A1'` and `A1*X2*B1 + B1'*X2'*A1'` which indicate symmetric terms and symmetric pairs in diagonal blocks.

Example

Consider the LMI

$$0 \eth \begin{pmatrix} -2X + A^T YB + B^T Y^T A + I & XC \\ C^T X & -zI \end{pmatrix}$$

where the matrix variables are X of Type 1, Y of Type 2, and z scalar. If this LMI is described in `lmis`, information about X and the LMI block structure can be obtained as follows:

```
lmiinfo(lmis)

                  LMI ORACLE
                 ------

This is a system of 1 LMI with 3 variable matrices

Do you want information on
   (v) matrix variables (l) LMIs (q) quit

?> v

Which variable matrix (enter its index k between 1 and 3) ? 1
   X1 is a 2x2 symmetric block diagonal matrix
     its (1,1)-block is a full block of size 2

                      -------

This is a system of 1 LMI with 3 variable matrices
Do you want information on
   (v) matrix variables (l) LMIs (q) quit

?> l

Which LMI (enter its number k between 1 and 1) ? 1

   This LMI is of the form
           0 < R(x)
where the inner factor(s) has 2 diagonal block(s)
```

```
Do you want info on the right inner factor ?

   (w) whole factor (b) only one block
   (o) other LMI (t) back to top level

?> w

Info about the right inner factor

   block (1,1) : I + a1*X1 + A2*X2*B2 + B2'*X2'*A2'

   block (2,1) : A3*X1

   block (2,2) : x3*A4

   (w) whole factor (b) only one block
   (o) other LMI (t) back to top level

                    ------

This is a system of 1 LMI with 3 variable matrices

Do you want information on
   (v) matrix variables (l) LMIs (q) quit

?> q

It has been a pleasure serving you!
```

Note that the prompt symbol is ?> and that answers are either indices or letters. All blocks can be displayed at once with option `(w)`, or you can prompt for specific blocks with option `(b)`.

Remark

`lmiinfo` does not provide access to the numerical value of LMI coefficients.

See Also

`decinfo`	Describe how the entries of a matrix variable X relate to the decision variables
`lminbr`	Return the number of LMIs in an LMI system

`matnbr`	Return the number of matrix variables in a system of LMIs
`decnbr`	Give the total number of decision variables in a system of LMIs

lminbr

Purpose

Return number of LMIs in LMI system

Syntax

```
k = lminbr(lmisys)
```

Description

`lminbr` returns the number k of linear matrix inequalities in the LMI problem described in `lmisys`.

See Also

`lmiinfo`	Interactively retrieve information about the variables and term content of LMIs
`matnbr`	Return the number of matrix variables in a system of LMIs

Purpose

Specify LMI regions for pole placement

Syntax

```
region = lmireg
region = lmireg(reg1,reg2,...)
```

Description

`lmireg` is an interactive facility to specify the LMI regions involved in multi-objective H_∞ synthesis with pole placement constraints (see `msfsyn`). Recall that an LMI region is any convex subset D of the complex plane that can be characterized by an LMI in z and $\bar{z}$, i.e.,

$$D = \{z \in \mathbf{C} : L + Mz + M^T\bar{z} < 0\}$$

for some fixed real matrices M and $L = L^T$. This class of regions encompasses half planes, strips, conic sectors, disks, ellipses, and any intersection of the above.

Calling `lmireg` without argument starts an interactive query/answer session where you can specify the region of your choice. The matrix `region` = [L, M] is returned upon termination. This matrix description of the LMI region can be passed directly to `msfsyn` for synthesis purposes.

The function `lmireg` can also be used to intersect previously defined LMI regions `reg1, reg2,...`. The output region is then the [L, M] description of the intersection of these regions.

See Also

`msfsyn` Multi-model/multi-objective state-feedback synthesis

lmiterm

Purpose

Specify term content of LMIs

Syntax

```
lmiterm(termID,A,B,flag)
```

Description

`lmiterm` specifies the term content of an LMI one term at a time. Recall that *LMI term* refers to the elementary additive terms involved in the block-matrix expression of the LMI. Before using `lmiterm`, the LMI description must be initialized with `setlmis` and the matrix variables must be declared with `lmivar`. Each `lmiterm` command adds one extra term to the LMI system currently described.

LMI terms are one of the following entities:

- outer factors
- constant terms (fixed matrices)
- variable terms AXB or AX^TB where X is a matrix variable and A and B are given matrices called the term coefficients.

When describing an LMI with several blocks, remember to specify **only the terms in the blocks on or below the diagonal** (or equivalently, only the terms in blocks on or above the diagonal). For instance, specify the blocks (1,1), (2,1), and (2,2) in a two-block LMI.

In the calling of `limterm`, `termID` is a four-entry vector of integers specifying the term location and the matrix variable involved.

$$\texttt{termID (1)} = \begin{cases} \texttt{+p} \\ \texttt{-p} \end{cases}$$

where positive p is for terms on the *left-hand side* of the p-th LMI and negative p i s for terms on the *right-hand side* of the p-th LMI.

Recall that, by convention, the left-hand side always refers to the smaller side of the LMI. The index p is relative to the order of declaration and corresponds to the identifier returned by `newlmi`.

$$\texttt{termID (2:3)} = \begin{cases} \texttt{[0, 0]} \text{ for outer factors} \\ \texttt{[i, j]} \text{ for terms in the } (i, j)\text{-th} \\ \qquad\quad \text{block of the left or right inner factor} \end{cases}$$

$$\texttt{termID (4)} = \begin{cases} \texttt{0} & \text{for outer factors} \\ \texttt{x} & \text{for variable terms } AXB \\ -\texttt{x} & \text{for variable terms } AX^TB \end{cases}$$

where `x` is the identifier of the matrix variable `X` as returned by `lmivar`.

The arguments `A` and `B` contain the numerical data and are set according to:

Type of Term	A	B
outer factor N	matrix value of N	omit
constant term C	matrix value of C	omit
variable term AXB or AX^TB	matrix value of A (1 if A is absent)	matrix value of B (1 if B is absent)

Note that identity outer factors and zero constant terms need not be specified.

The extra argument `flag` is optional and concerns only conjugated expressions of the form

$$(AXB) + (AXB)^T = AXB + B^TX^{(T)}A^T$$

in *diagonal blocks*. Setting `flag = 's'` allows you to specify such expressions with a single `lmiterm` command. For instance,

```
lmiterm([1 1 1 X],A,1,'s')
```

adds the symmetrized expression $AX + X^TA^T$ to the (1,1) block of the first LMI and summarizes the two commands

```
lmiterm([1 1 1 X],A,1)
lmiterm([1 1 1  X],1,A')
```

Aside from being convenient, this shortcut also results in a more efficient representation of the LMI.

Example

Consider the LMI

$$\begin{pmatrix} 2AX_2A^T - x_3E + DD^T & B^TX_1 \\ X_1^TB & -I \end{pmatrix} < M^T \begin{pmatrix} CX_1C^T + CX_1^TC^T & 0 \\ 0 & -fX_2 \end{pmatrix} M$$

where X_1, X_2 are matrix variables of Types 2 and 1, respectively, and x_3 is a scalar variable (Type 1).

After initializing the LMI description with `setlmis` and declaring the matrix variables with `lmivar`, the terms on the left-hand side of this LMI are specified by:

```
lmiterm([1 1 1 X2],2*A,A') % 2*A*X2*A'
lmiterm([1 1 1 x3],-1,E) % -x3*E
lmiterm([1 1 1 0],D*D') % D*D'
lmiterm([1 2 1 -X1],1,B) % X1'*B
lmiterm([1 2 2 0],-1) % -I
```

Here `X1, X2, X3` should be the variable identifiers returned by `lmivar`.

Similarly, the term content of the right-hand side is specified by:

```
lmiterm([-1 0 0 0],M) % outer factor M
lmiterm([-1 1 1 X1],C,C','s') % C*X1*C'+C*X1'*C'
lmiterm([-1 2 2 X2],-f,1) % -f*X2
```

Note that $CX_1C^T + CX_1{}^TC^T$ is specified by a single `lmiterm` command with the flag `'s'` to ensure proper symmetrization.

See Also

`setlmis`	Initialize the description of an LMI system
`lmivar`	Specify the matrix variables in an LMI problem
`getlmis`	Get the internal description of an LMI system
`lmiedit`	Specify or display systems of LMIs as MATLAB® expressions
`newlmi`	Attach an identifying tag to LMIs

Purpose

Specify matrix variables in LMI problem

Syntax

```
X = lmivar(type,struct)
[X,n,sX] = lmivar(type,struct)
```

Description

`lmivar` defines a new matrix variable X in the LMI system currently described. The optional output `X` is an identifier that can be used for subsequent reference to this new variable.

The first argument `type` selects among available types of variables and the second argument `struct` gives further information on the structure of X depending on its type. Available variable types include:

type=1: Symmetric matrices with a block-diagonal structure. Each diagonal block is either full (arbitrary symmetric matrix), scalar (a multiple of the identity matrix), or identically zero.

If X has R diagonal blocks, `struct` is an R-by-2 matrix where

- `struct(r,1)` is the size of the `r`-th block
- `struct(r,2)` is the type of the `r`-th block (1 for full, 0 for scalar, –1 for zero block).

type=2: Full m-by-n rectangular matrix. Set `struct = [m,n]` in this case.

type=3: Other structures. With Type 3, each entry of X is specified as zero or $\pm x_n$ where x_n is the n-th decision variable.

Accordingly, `struct` is a matrix of the same dimensions as X such that

- `struct(i,j)=0` if $X(i, j)$ is a hard zero
- `struct(i,j)=n` if $X(i, j) = x_n$
- `struct(i,j)= n` if $X(i, j) = -x_n$

Sophisticated matrix variable structures can be defined with Type 3. To specify a variable X of Type 3, first identify how many *free independent entries* are involved in X. These constitute the set of decision variables associated with X. If the problem already involves n decision variables, label the new free variables as $x_{n+1}, \ldots, x_{n+p}$. The structure of X is then defined in terms of $x_{n+1}, \ldots, x_{n+p}$ as indicated above. To help specify matrix variables of Type 3, `lmivar` optionally returns two extra outputs: (1) the total number n of scalar

decision variables used so far and (2) a matrix `sX` showing the entry-wise dependence of X on the decision variables $x_1, \ldots, x_n$.

Example 1

Consider an LMI system with three matrix variables X_1, X_2, X_3 such that

- X_1 is a 3×3 symmetric matrix (unstructured),
- X_2 is a 2×4 rectangular matrix (unstructured),
- X_3 =

$$\begin{pmatrix} \Delta & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & \delta_2 I_2 \end{pmatrix}$$

  where Δ is an arbitrary 5×5 symmetric matrix, δ_1 and δ_2 are scalars, and I_2 denotes the identity matrix of size 2.

These three variables are defined by

```
setlmis([])
X1 = lmivar(1,[3 1]) % Type 1
X2 = lmivar(2,[2 4]) % Type 2 of dim. 2x4
X3 = lmivar(1,[5 1;1 0;2 0]) % Type 1
```

The last command defines X_3 as a variable of Type 1 with one full block of size 5 and two scalar blocks of sizes 1 and 2, respectively.

Example 2

Combined with the extra outputs `n` and `sX` of `lmivar`, Type 3 allows you to specify fairly complex matrix variable structures. For instance, consider a matrix variable X with structure

$$X = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix}$$

where X_1 and X_2 are 2-by-3 and 3-by-2 rectangular matrices, respectively. You can specify this structure as follows:

1 Define the rectangular variables X_1 and X_2 by

```
setlmis([])
```

```
[X1,n,sX1] = lmivar(2,[2 3])
[X2,n,sX2] = lmivar(2,[3 2])
```

The outputs `sX1` and `sX2` give the decision variable content of X_1 and X_2:

```
sX1

sX1 =
   1   2   3
   4   5   6

sX2

sX2 =
   7   8
   9   10
   11  12
```

For instance, `sX2(1,1)=7` means that the (1,1) entry of X_2 is the seventh decision variable.

2 Use Type 3 to specify the matrix variable X and define its structure in terms of those of X_1 and X_2:

```
[X,n,sX] = lmivar(3,[sX1,zeros(2);zeros(3),sX2])
```

The resulting variable `X` has the prescribed structure as confirmed by

```
sX

sX =
   1   2   3   0   0
   4   5   6   0   0
   0   0   0   7   8
   0   0   0   9   10
   0   0   0   11  12
```

See Also

`setlmis`	Initialize the description of an LMI system
`lmiterm`	Specify the term content of LMIs
`getlmis`	Get the internal description of an LMI system
`lmiedit`	Specify or display systems of LMIs as MATLAB® expressions
`skewdec`	Form a skew-symmetric matrix

`delmvar`	Delete one of the matrix variables of an LMI problem
`setmvar`	Instantiate a matrix variable and evaluate all LMI terms involving this matrix variable

Purpose

Log-log scale plot of `frd` objects

Syntax

```
loglog(sys)
loglog(sys,linetype)
```

Description

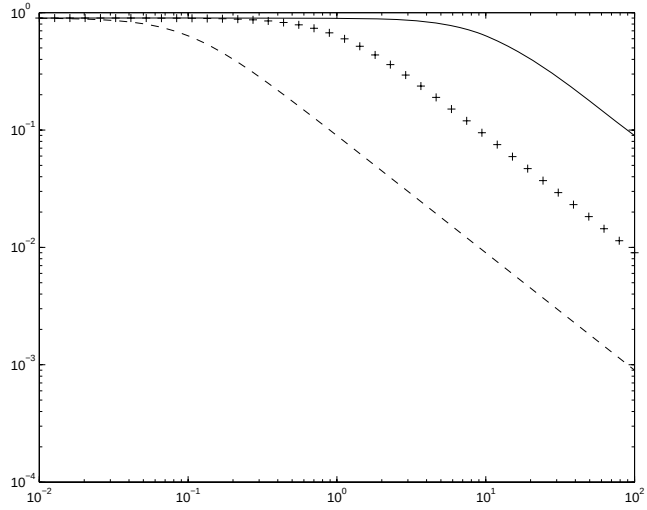
`loglog` is the same as `plot`, except a logarithmic (base 10) scale is used for both the X- and Y-axis.

The argument list consists of (many) *x*-data, *y*-data and line type triples. The *x*-data and *y*-data can be specified individually, as `doubles`, or jointly as an `frd`.

Example

Generate three frequency response objects, `sys1g`, `sys2g` and `sys3g`. Plot the magnitude of these transfer functions versus frequency on a log-log plot.

```
omega = logspace(-2,2,40);
sys1 = tf(0.9,[.1 1]);
sys1g = frd(sys1,omega);
sys2 = tf(0.9,[1 1]);
sys2g = frd(sys2,omega);
sys3 = tf(0.9,[10 1]);
sys3g = frd(sys3,omega);
loglog(abs(sys1g),abs(sys2g),'r+',sys3g.Freq,...

abs(sys3g.Resp(:)),'--');
```



See Also

`plot`	Plots on linear axis
`semilogx`	Plots semi-log scale plot
`semilogy`	Plots semi-log scale plot

Purpose

Comprehensive stability margin analysis of LTI and Simulink® feedback loops

Syntax

```
[cm,dm,mm] = loopmargin(L)
[m1,m2] = loopmargin(L,MFLAG)
[cmi,dmi,mmi,cmo,dmo,mmo,mmio] = loopmargin(P,C)
[m1,m2,m3] = loopmargin(P,C,MFLAG)
```

Description

`[cm,dm,mm] = loopmargin(L)` analyzes the multivariable feedback loop consisting of the loop transfer matrix `L` (size `N-by-N`) in negative feedback with an `N-by-N` identity matrix.

`cm`, or classical gain and phase margins, is an `N-by-1` structure corresponding to loop-at-a-time gain and phase margins for each channel (See `allmargin` for details on the fields of `cm`.).

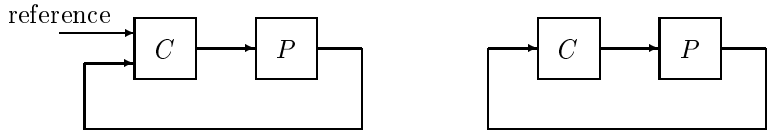
`dm` is an `N-by-1` structure corresponding to loop-at-a-time disk gain and phase margins for each channel. The disk margin for the i-th feedback channel defines a circular region centered on the negative real axis at the average GainMargin (`GM`), e.g. , (GM_{low}+GM_{high})/2, such that `L(i,i)` does not enter that region. Gain and phase disk margin bounds are derived from the radius of the circle, calculated based on the balanced sensitivity function (see Algorithm section).

`mm`, or multiloop disk margin, is a structure corresponding to simultaneous, independent, variations in the individual channels of loop transfer matrix `L`. `mm` calculates the largest region such that for all gain and phase variations, occurring independently in each channel, lie inside the region, that the closed-loop system is stable. Note that `mm` is a single structure, independent of because the number of channels, variations in all channels are handled simultaneously. As in the case for disk margin, the guaranteed bounds are calculated based on a balanced sensitivity function.

If `L` is a `ss/tf/zpk` object, the frequency range and number of frequency points used to calculate `dm` and `mm` margins are chosen automatically.

Output arguments can be limited to only those requested using an optional character string argument. `[m1,m2] = loopmargin(L,'m,c')` returns the multi-loop diskmargin (`'m'`) in `m1`, and the classical margins (`'c'`) in `m2`. Use `'d'` to specify the disk margin. This optional second argument may be any combination, in any order, of the 3 characters `'c'`, `'d'` and `'m'`.

`[cmi,dmi,mmi,cmo,dmo,mmo,mmio] = loopmargin(P,C)` analyzes the multivariable feedback loop consisting of the controller `C` in negative feedback with the plant, `P`. `C` should only be the compensator in the feedback path, without reference channels, if it is a 2-dof architecture. That is, if the closed-loop system has a 2-dof architecture the reference channel of the controller should be eliminated, resulting in a 1-dof architecture, as shown.



2-dof architecture 1-dof architecture

`cmi`,`dmi` and `mmi` structures correspond to the classical loop-at-a-time gain and phase margins, disk margins and multiloop channel margins at the plant input respectively. The structures `cmo`, `dmo` and `mmo` have the same fields as described for `cmi`, `dmi` and `mmi` though they correspond to the plant outputs. `mmio`, or multi-input/multi-output margins, is a structure corresponding to simultaneous, independent, variations in all the individual input and output channels of the feedback loops. `mmio` has the same fields as `mmi` and `mmo`.

If the closed-loop system is an `ss/tf/zpk`, the frequency range and number of points used to calculate `cm`, `dm` and `mm` margins are chosen automatically.

Output arguments can be limited to only those requested using an optional character string argument. `[m1,m2,m3] = loopmargin(L,'mo,ci,mm')` returns the multi-loop diskmargin at the plant output (`'mo'`) in `m1`, the classical margins at the plant input (`` `ci' ``) in `m2`, and the disk margins for simultaneous, independent variations in all input and output channels (`'mm'`) in `m3`. This optional third argument may be any comnination, in any order, of the 7 character pairs `'ci'`, `'di'`, `'mi'`, `'co'`, `'do'`, `'mo'`, and `'mm'`.

Usage with Simulink®

`[cm,dm,mm] = loopmargin(Model,Blocks,Ports)` does a multi-loop stability margin analysis using Simulink® Control Design™ software. `Model` specifies the name of the Simulink diagram for analysis. The margin analysis points are defined at the output ports (`Ports`) of blocks (`Blocks`) within the model. `Blocks` is a cell array of full block path names and `Ports` is a vector of the same

dimension as `Blocks`. If all `Blocks` have a single output port, then `Ports` would be a vector of ones with the same length as `Blocks`.

Three types of stability margins are computed: loop-at-a-time classical gain and phase margins (`cm`), loop-at-a-time disk margins (`dm`) and a multi-loop disk margin (`mm`).

`[cm,dm,mm] = loopmargin(Model,Blocks,Ports,OP)` uses the operating point object `OP` to create linearized systems from the Simulink `Model`.

`[cm,dm,mm,info] = loopmargin(Model,Blocks,Ports,OP)` returns `info` in addition to the margins. `info` is a structure with fields `OperatingPoint`, `LinearizationIO` and `SignalNames` corresponding to the analysis.

Margin output arguments can be limited to only those requested using an optional charcter string argument. INFO is always the last output. For example, `[mm,cm,info] = loopmargin(Model,Blocks,Ports,'m,c')` returns the multi-loop diskmargin (`` `m' ``) in `mm`, the classical margins (`` `c' ``) in `cm`, and the `info` structure.

Basic Syntax

`[cm,dm,mm] = loopmargin(L)` `cm` is calculated using the `allmargin` command and has the same fields as `allmargin`. The `cm` is a structure with the following fields:

Field	Description
`GMFrequency`	All -180 deg crossover frequencies (in rad/s)
`GainMargin`	Corresponding gain margins (`GM = 1/L` where `L` is the gain at crossover)
`PhaseMargin`	Corresponding phase margins (in degrees)
`PMFrequency`	All 0 dB crossover frequencies (in rad/s)

Field	Description
`DelayMargin`	Delay margins (in seconds for continuous-time systems, and multiples of the sample time for discrete-time systems)
`Stable`	1 if nominal closed loop is stable, 0 otherwise. If L is a `frd` or `ufrd` object, the `Stable` flag is set to `NaN`.

`dm`, or Disk Margin, is a structure with the following fields

Field	Description
`GainMargin`	Smallest gain variation (`GM`) such that a disk centered at the point -(`GM(1)` + `GM`(2))/2 would just touch the loop transfer function
`PhaseMargin`	Smallest phase variation, in degrees, corresponding to the disk described in the GainMargin field (degrees)
`Frequency`	Associated with `GainMargin`/`PhaseMargin` fields (rad/s)

`mm` is a structure with the following fields:

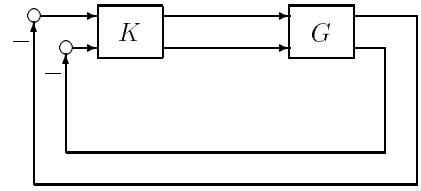
Field	Description
`GainMargin`	Guaranteed bound on simultaneous, independent, gain variations allowed in all plant channels
`PhaseMargin`	Guaranteed bound on simultaneous, independent, phase variations allowed in all plant channels (degrees)
`Frequency`	Associated with `GainMargin`/`PhaseMargin` fields (rad/s)

Example

MIMO Loop-at-a-Time Margins

This example is designed to illustrate that loop-at-a-time margins (gain, phase, and/or distance to –1) can be inaccurate measures of multivariable robustness margins. You will see that margins of the individual loops can be very sensitive to small perturbations within other loops.

The nominal closed-loop system considered here is as follows



G and K are `2-by-2` multiinput/multioutput (MIMO) systems, defined as

$$G: = \frac{1}{s^2 + \alpha^2}\begin{bmatrix} s - \alpha^2 & \alpha(s+1) \\ -\alpha(s+1) & s - \alpha^2 \end{bmatrix}, K = I_2$$

Set $\alpha := 10$, construct G in state-space form, and compute its frequency response.

```
a = [0 10;-10 0];
b = eye(2);
c = [1 8;-10 1];
d = zeros(2,2);
G = ss(a,b,c,d);
K = [1 -2;0 1];
[cmi,dmi,mmi,cmo,dmo,mmo,mmio]=loopmargin(G,K);
```

First consider the margins at the input to the plant. The first input channel has infinite gain margin and 90 degrees of phase margin based on the results from the `allmargin` command, `smi(1)`. The disk margin analysis, `dmi`, of the first channel provides similar results.

```
cmi(1)
ans =
    GMFrequency: [1x0 double]
     GainMargin: [1x0 double]
```

```
    PMFrequency: 21
    PhaseMargin: 90
    DMFrequency: 21
    DelayMargin: 0.0748
         Stable: 1
dmi(1)
ans =
     GainMargin: [0 Inf]
    PhaseMargin: [-90 90]
      Frequency: 1.1168
```

The second input channel has a gain margin of 2.105 and infinite phase margin based on the single-loop analysis, `cmi(2)`. The disk margin analysis, `dmi(2)`, which allows for simultaneous gain and phase variations a loop-at-a-time results in maximum gain margin variations of 0.475 and 2.105 and phase margin variations of +/- 39.18 degs.

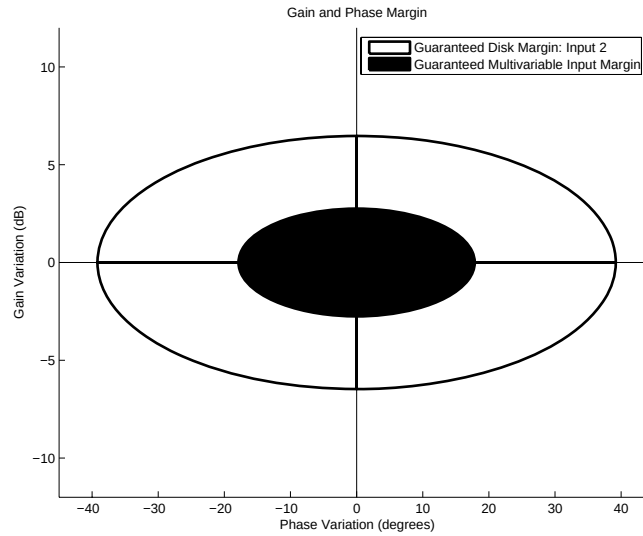
```
cmi(2)
ans =
    GMFrequency: 0
     GainMargin: 2.1053
    PMFrequency: [1x0 double]
    PhaseMargin: [1x0 double]
    DMFrequency: [1x0 double]
    DelayMargin: [1x0 double]
         Stable: 1
dmi(2)
ans =
     GainMargin: [0.4749 2.1056]
    PhaseMargin: [-39.1912 39.1912]
      Frequency: 0.0200
```

The multiple margin analysis of the plant inputs corresponds to allowing simultaneous, independent gain and phase margin variations in each channel. Allowing independent variation of the input channels further reduces the tolerance of the closed-loop system to variations at the input to the plant. The multivariable margin analysis, `mmi`, leads to a maximum allowable gain margin variation of 0.728 and 1.373 and phase margin variations of +/- 17.87 deg. Hence even though the first channel had infinite gain margin and 90

degrees of phase margin, allowing variation in both input channels leads to a factor of two reduction in the gain and phase margin.

```
mmi
mmi =
     GainMargin: [0.7283 1.3730]
    PhaseMargin: [-17.8659 17.8659]
      Frequency: 9.5238e-004
```

The guaranteed region of phase and gain variations for the closed-loop system can be illustrated graphically. The disk margin analysis, `dmi(2)`, indicates the closed-loop system will remain stable for simultaneous gain variations of 0.475 and 2.105 (± 6.465 dB) and phase margin variations of ± 39.18 deg in the second input channel. This is denoted by the region associated with the large ellipse in the following figure. The multivariable margin analysis at the input to the plant, `mmi`, indicates that the closed-loop system will be stable for independent, simultaneous, gain margin variation up to 0.728 and 1.373 (±2.753 dB) and phase margin variations up to ± 17.87 deg (the dark ellipse region) in both input channels.



The output channels have single-loop margins of infinite gain and 90 deg phase variation. The output multivariable margin analysis, `mmo`, leads to a maximum allowable gain margin variation of 0.607 and 1.649 and phase margin variations of +/- 27.53 degs. Hence even though both output channels had infinite gain margin and 90 degrees of phase margin, simultaneous variations in both channels significantly reduce the margins at the plant outputs.

```
mmo
mmo =
     GainMargin: [0.6065 1.6489]
    PhaseMargin: [-27.5293 27.5293]
      Frequency: 0.2287
```

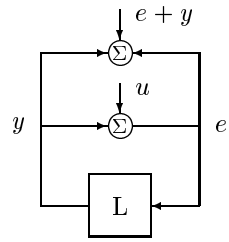
If all the input and output channels are allow to vary independently, `mmio`, the gain margin variation allow are 0.827 and 1.210 and phase margin variations allowed are +/- 10.84 deg.

```
mmio
mmio =
     GainMargin: [0.8267 1.2097]
    PhaseMargin: [-10.8402 10.8402]
      Frequency: 0.2287
```

Algorithm

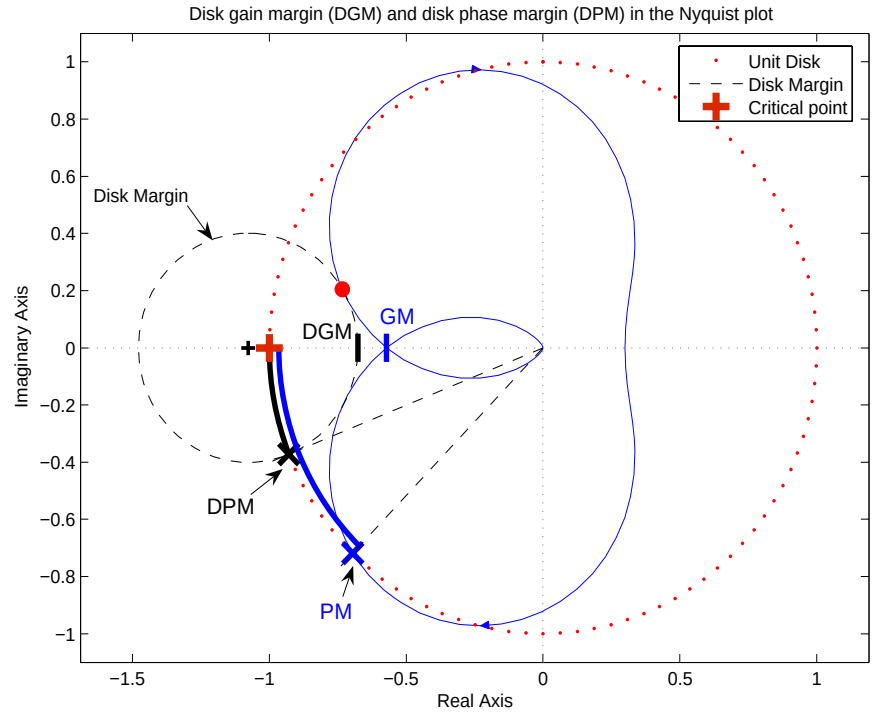
Two well-known loop robustness measures are based on the sensitivity function $S=(I-L)^{-1}$ and the complementary sensitivity function $T=L(I-L)^{-1}$ where L is the loop gain matrix associated with the input or output loops broken simultaneously. In the following figure, S is the transfer matrix from summing junction input u to summing junction output e. T is the transfer matrix from u to y. If signals e and y are summed, the transfer matrix from u to $e+y$ is given by $(I+L)\cdot(I-L)^{-1}$, the balanced sensitivity function. It can be shown (Dailey, 1991, Blight, Daily and Gangass, 1994) that each broken-loop gain can be perturbed by the complex gain $(1+\Delta)(1-\Delta)$ where $|\Delta|<1/\mu(S+T)$ or $|\Delta|<1/\sigma_{max}(S+T)$ at each frequency without causing instability at that frequency. The peak value of $\mu(S+T)$ or $\sigma_{max}(S+T)$ gives a robustness guarantee for all frequencies, and for $\mu(S+T)$ the guarantee is nonconservative (Blight, Daily and Gangass, 1994).

This figure shows a comparison of a disk margin analysis with the



$$\begin{array}{rcccc} e & = & (I-L)^{-1}u & = & Su \\ y & = & L(I-L)^{-1}u & = & Tu \\ e+y & = & (I+L)\cdot(I-L)^{-1}u & = & (S+T)u \end{array}$$

the classical notations of gain and phase margins.



The Nyquist plot is of the loop transfer function L(s)

$$L(s) = \frac{\frac{s}{30} + 1}{(s+1)(s^2 + 1.6s + 16)}$$

- The Nyquist plot of L corresponds to the blue line.
- The unit disk corresponds to the dotted red line.
- GM and PM indicate the location of the classical gain and phase margins for the system L.
- DGM and DPM correspond to the disk gain and phase margins. The disk margins provide a lower bound on classical gain and phase margins.
- The disk margin circle corresponds to the dashed black line. The disk margin corresponds to the largest disk centered at (GMD + 1/GMD)/2 that just touches the loop transfer function L. This location is indicated by the red dot.

The disk margin and multiple channel margins calculation involve the balanced sensitivity function $S+T$. For a given peak value of $\mu(S+T)$, any simultaneous phase and gain variations applied to each loop independently will not destabilize the system if the perturbations remain inside the corresponding circle or disk. This corresponds to the disk margin calculation to find `dmi` and `dmo`.

Similarly, the multiple channel margins calculation involves the balanced sensitivity function $S+T$. Instead of calculating $\mu(S+T)$ a single loop at a time, all the channels are included in the analysis. A μ- analysis problem is formulated with each channel perturbed by an independent, complex perturbation. The peak $\mu(S+T)$ value guarantees that any simultaneous, independent phase and gain variations applied to each loop simultaneously will not destabilize the system if they remain inside the corresponding circle or disk of size $\mu(S+T)$.

Reference

Barrett, M.F., Conservatism with robustness tests for linear feedback control systems, Ph.D. Thesis, Control Science and Dynamical Systems, University of Minnesota, 1980.

Blight, J.D., R.L. Dailey, and D. Gangsass, “Practical control law design for aircraft using multivariable techniques,” *International Journal of Control*, Vol. 59, No. 1, 1994, pp. 93-137.

Bates, D., and I. Postlethwaite, “Robust Multivariable Control of Aerospace Systems,” *Delft University Press,* Delft, The Netherlands, ISBN: 90-407-2317-6, 2002.

See Also

`allmargin`	Finds all stability margins and crossover frequencies
`bode`	`Plots` Bode frequency response of LTI models
`loopsens`	Calculates sensitivity functions of feedback loop
`mussv`	Calculate bounds on the Structured Singular Value (μ)
`robuststab`	Calculate stability margins of uncertain systems
`wcgain`	Calculates worst-case gain of a system
`wcsens`	Calculate worst-case sensitivities for feedback loop
`wcmargin`	Calculate worst-case margins for feedback loop

loopsens

Purpose

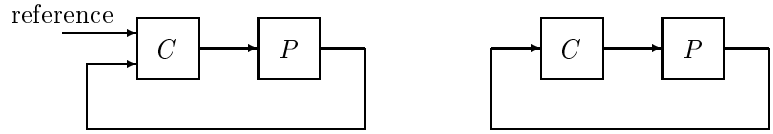
Sensitivity functions of plant-controller feedback loop

Syntax

```
loops = loopsens(P,C)
```

Description

`loops = loopsens(P,C)` creates a `struct`, `loops`, whose fields contain the multivariable sensitivity, complementary and open-loop transfer functions. The closed-loop system consists of the controller `C` in negative feedback with the plant `P`. `C` should only be the compensator in the feedback path, not any reference channels, if it is a 2-dof controller as seen in the figure below. The plant and compensator `P` and `C` can be constant matrices, `double, lti` objects, `frd/ss/tf/zpk`, or uncertain objects `umat/ufrd/uss`.



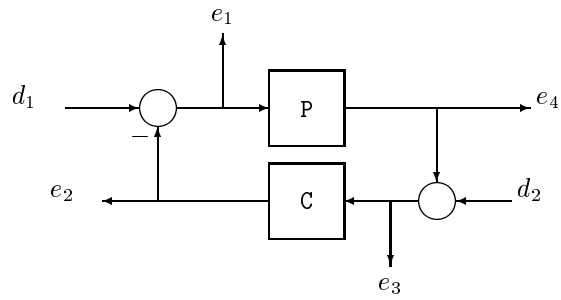
2-dof architecture 1-dof architecture

The `loops` returned variable is a structure with fields:

Field	Description
`Poles`	Closed-loop poles. `NaN` for `frd/ufrd` objects
`Stable`	1 if nominal closed loop is stable, 0 otherwise. `NaN` for `frd/ufrd` objects
`Si`	Input-to-plant sensitivity function
`Ti`	Input-to-plant complementary sensitivity function
`Li`	Input-to-plant loop transfer function
`So`	Output-to-plant sensitivity function
`To`	Output-to-plant complementary sensitivity function
`Lo`	Output-to-plant loop transfer function

Field	Description
PSi	Plant times input-to-plant sensitivity function
CSo	Compensator times output-to-plant sensitivity function

The multivariable closed-loop interconnection structure, shown below, defines the input/output sensitivity, complementary sensitivity, and loop transfer functions.



Description	Equation
Input sensitivity ($TF_{e1 \leftarrow d1}$)	$(I+CP)^{-1}$
Input complementary sensitivity ($TF_{e2 \leftarrow d1}$)	$CP(I+CP)^{-1}$
Output sensitivity ($TF_{e3 \leftarrow d2}$)	$(I+PC)^{-1}$
Output complementary sensitivity ($-T$)$F_{e4 \leftarrow d}$)	$PC(I+PC)^{-1}$
Input loop transfer function	CP
Output loop transfer function	PC

Example

Single Input, Single Output (SISO)

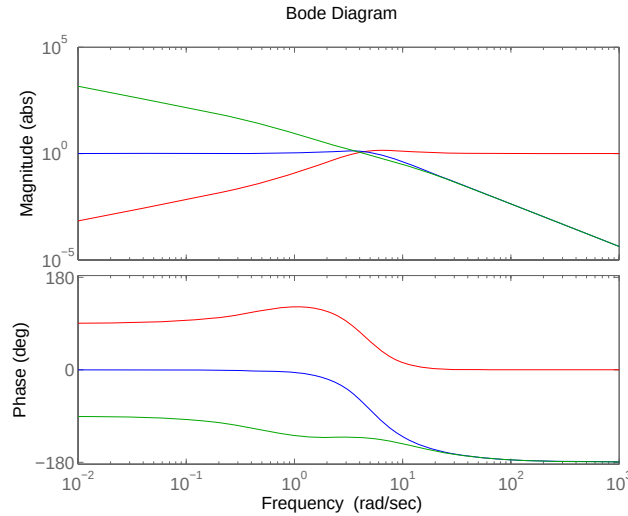
Consider PI controller for a dominantly 1st-order plant, with the closed-loop bandwidth of 2.5 rads/sec. Since the problem is SISO, all gains are the same at input and output.

```
gamma = 2; tau = 1.5; taufast = 0.1;
```

```
P = tf(gamma,[tau 1])*tf(1,[taufast 1]);
tauclp = 0.4;
xiclp = 0.8;
wnclp = 1/(tauclp*xiclp);
KP = (2*xiclp*wnclp*tau - 1)/gamma;
KI = wnclp^2*tau/gamma;
C = tf([KP KI],[1 0]);
```

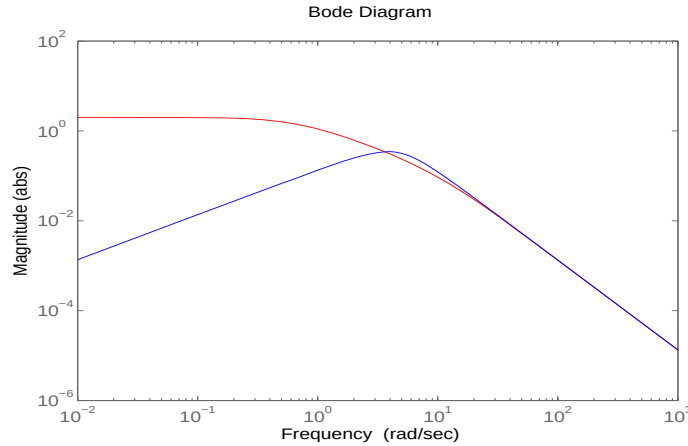
Form the closed-loop (and open-loop) systems with loopsens, and plot Bode plots using the gains at the plant input.

```
loops = loopsens(P,C);
bode(loops.Si,'r',loops.Ti,'b',loops.Li,'g')
```



Finally, compare the open-loop plant gain to the closed-loop value of `PSi`

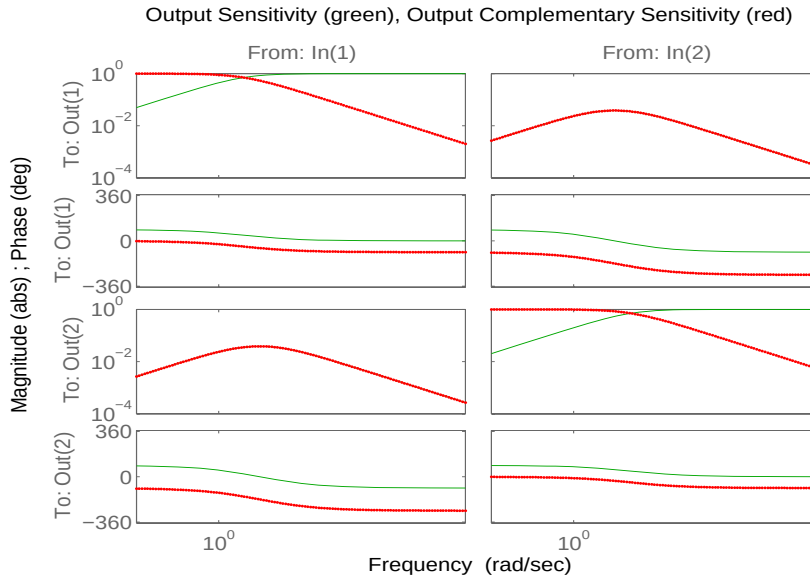
```
bodemag(P,'r',loops.PSi,'b')
```



Multi Input, Multi Output (MIMO)

Consider an integral controller for a constant-gain, 2-input, 2-output plant. For purposes of illustration, the controller is designed via inversion, with different bandwidths in each rotated channel.

```
P = ss([2 3;-1 1]);
BW = diag([2 5]);
[U,S,V] = svd(P.d);  % get SVD of Plant Gain
Csvd = V*inv(S)*BW*tf(1,[1 0])*U'; % inversion based on SVD
loops = loopsens(P,Csvd);
bode(loops.So,'g',loops.To,'r.',logspace(-1,3,120))
```



See Also

`loopmargin`	Performs a comprehensive analysis of feedback loop
`robuststab`	Calculate stability margins of uncertain systems
`wcsens`	Calculate worst-case sensitivities for feedback loop
`wcmargin`	Calculate worst-case margins for feedback loop

Purpose

Compute controller for plant G to optimally match target loop shape Gd

Syntax

```
[K,CL,GAM,INFO]=loopsyn(G,Gd)
[K,CL,GAM,INFO]=loopsyn(G,Gd,RANGE)
```

Description

loopsyn is an H_∞ optimal method for loopshaping control synthesis. It computes a stabilizing H_∞controller K for plant G to shape the sigma plot of the loop transfer function GK to have desired loop shape G_d with accuracy γ=GAM in the sense that if ω_0 is the 0db crossover frequency of the sigma plot of $G_d(j\omega)$, then, roughly,

$$\underline{\sigma}(G(j\omega)K(j\omega)) \geq \frac{1}{\gamma}\ \bar{\sigma}(G_d(j\omega)) \qquad \text{for all } \omega < \omega_0 \tag{5-16}$$

$$\bar{\sigma}(G(j\omega)K(j\omega)) \leq \gamma\ \ \underline{\sigma}(G_d(j\omega)) \qquad \text{for all } \omega > \omega_0 \tag{5-17}$$

The STRUCT array INFO returns additional design information, including a MIMO stable min-phase shaping pre-filter W, the shaped plant $G_s = GW$, the controller for the shaped plant K_s=WK, as well as the frequency range $\{\omega_{min},\omega_{max}\}$ over which the loop shaping is achieved.

Input Arguments:	
G	LTI Plant
Gd	Desired loop-shape (LTI Model)
RANGE	(optional, default {0,Inf}) Desired frequency range for loop-shaping, a 1-by-2 cell array $\{\omega_{min},\omega_{max}\}$; ω_{max} should be at least ten times ω_{min}

Output Arguments:	
K	LTI controller
CL= G*K/(I+GK)	LTI closed-loop system

GAM	loop-shaping accuracy (GAM ≥ 1, with GAM=1 being perfect fit
INFO	additional output information

INFO.W	LTI pre-filter W satisfying $\sigma(G_d)=\sigma(GW)$ for all ω; W is always minimum-phase
INFO.Gs	LTI shaped plant: $G_s = GW$
INFO.Ks	LTI controller for the shaped plant: $K_s=WK$
INFO.range	$\{\omega_{min},\omega_{max}\}$ cell-array containing the approximate frequency range over which loop-shaping could be accurately achieved to with accuracy G. The output INFO.range is either the same as or a subset of the input range

Algorithm

Using the GCD formula of Le and Safonov [1], loopsyn first computes a stable-minimum-phase loop-shaping, squaring-down prefilter W such that the shaped plant $G_s = GW$ is square, and the desired shape G_d is achieved with good accuracy in the frequency range $\{\omega_{min},\omega_{max}\}$ by the shaped plant; i.e.,

$$\sigma(G_d) \approx \sigma(G_s) \quad \text{for all } \omega \in \{\omega_{min},\omega_{max}\}.$$

Then, loopsyn uses the Glover-McFarlane [2] normalized-coprime-factor control synthesis theory to compute an optimal "loop-shaping" controller for the shaped plant via Ks=ncfsyn(Gs), and returns K=W*Ks.

If the plant G is a continuous time LTI and

1. G has a full-rank D-matrix, and
2. no finite zeros on the $j\omega$-axis, and
3. $\{\omega_{min},\omega_{max}\}=[0,\infty]$,

then GW theoretically achieves a perfect accuracy fit $\sigma(G_d) = \sigma(GW)$ for all frequency ω. Otherwise, loopsyn uses a bilinear pole-shifting bilinear transform [3] of the form

```
Gshifted=bilin(G,-1,'S_Tust',[ωmin,ωmax]),
```

which results in a perfect fit for transformed `Gshifted` and an approximate fit over the smaller frequency range $[\omega_{min},\omega_{max}]$ for the original unshifted G provided that $\omega_{max} >> \omega_{min}$. For best results, you should choose ω_{max} to be at least 100 times greater than ω_{min}. In some cases, the computation of the optimal W for `Gshifted` may be singular or ill-conditioned for the range $[\omega_{min},\omega_{max}]$, as when `Gshifted` has undamped zeros or, in the continuous-time case only, `Gshifted` has a D-matrix that is rank-deficient); in such cases, `loopsyn` automatically reduces the frequency range further, and returns the reduced range $[\omega_{min},\omega_{max}]$ as a cell array in the output `INFO.range=`$\{\omega_{min},\omega_{max}\}$

Example

The following code generates the optimal `loopsyn` loopshaping control for the case of a 5-state, 4-output, 5-input plant with a full-rank non-minimum phase zero at s=+10. The result in shown in Figure 5-11.

```
rand('seed',0);randn('seed',0);
s=tf('s'); w0=5; Gd=5/s;           % desired bandwith w0=5
G=((s-10)/(s+100))*rss(3,4,5);     % 4-by-5 non-min-phase plant
[K,CL,GAM,INFO]=loopsyn(G,Gd);
sigma(G*K,'r',Gd*GAM,'k-.',Gd/GAM,'k-.',{.1,100})  % plot result
```

This figure shows that the LOOPSYN controller K optimally fits

```
sigma(G*K) = sigma(Gd)—GAM  %  dB
```

In the above example, `GAM` = 2.0423 = 6.2026 dB.

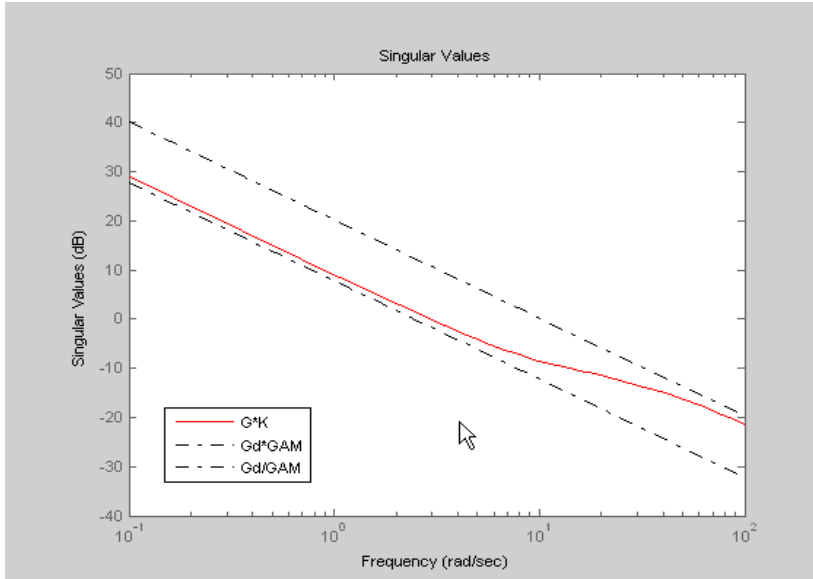


Figure 5-11: LOOPSYN controller

The `loopsyn` controller `K` optimally fits `sigma(G*K)`. As shown in Figure 5-11, it is sandwiched between `sigma(Gd/GAM)` and `sigma(Gd*GAM)` in accordance with the inequalities in Equation 5-16 and Equation 5-17. In the this example, `GAM` = 2.0423 = 6.2026 db.

Limitations

The plant G must be stabilizable and detectable, must have at least as many inputs as outputs, and must be full rank; i.e,

- `size(G,2)` ≥ `size(G,1)`
- `rank(freqresp(G,w))` = `size(G,1)` for some frequency w.

The order of the controller K can be large. Generically, when G_d is given as a SISO LTI, then the order N_K of the controller K satisfies

$$\begin{aligned} N_K &= N_{Gs} + N_W \\ &= N_y N_{Gd} + N_{RHP} + N_W \\ &= N_y N_{Gd} + N_{RHP} + N_G \end{aligned}$$

where

- N_y denotes the number of outputs of the plant G,
- N_{RHP} denotes the total number of non-stable poles and non-minimum-phase zeros of the plant G, including those on the stability boundary and at infinity, and
- N_G, N_{Gs}, N_{Gd} and N_W denote the respective orders of G, G_s, G_d and W.

Model reduction can help reduce the order of K — see `reduce` and `ncfmr`.

References

[1] Le, V.X., and M.G. Safonov. Rational matrix GCD's and the design of squaring-down compensators—a state space theory. *IEEE Trans. Autom.Control*, AC-36(3):384–392, March 1992.

[2] Glover, K., and D. McFarlane. Robust stabilization of normalized coprime factor plant descriptions with H_∞-bounded uncertainty. *IEEE Trans. Autom. Control*, AC-34(8):821–830, August 1992.

[3] Chiang, R.Y., and M.G. Safonov. H_∞ synthesis using a bilinear pole-shifting transform. *AIAA J. Guidance, Control and Dynamics*, 15(5):1111–1115, September–October 1992.

See Also

`loopsyn_demo`	A demo of this function
`mixsyn`	H_∞ mixed-sensitivity controller synthesis
`ncfsyn`	H_∞ - normailized coprime controller synthesis

ltiarray2uss

Purpose

Computes uncertain system bounding given LTI ss array.

Syntax

```
usys = ltiarray2uss(P,Parray,ord)
[usys,wt] = ltiarray2uss(P,Parray,ord)
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord)
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,'InputMult')
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,'OutputMult')
[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,'Additive')
```

Description

The command ltiarray2uss, calculates an uncertain system usys with nominal value P, and whose range of behavior includes the given array of systems, Parray.

usys = ltiarray2uss(P,Parray,ord), usys is formulated as an input multiplicative uncertainty model,
usys = P*(I + wt*ultidyn('IMult',[size(P,2) size(P,2)])), where wt is a stable scalar system, whose magnitude overbounds the relative difference, (P - Parray)/P. The state order of the weighting function used to bound the multiplicative difference between P and Parray is ord. Both P and Parray must be in the classes ss/tf/zpk/frd. If P is an frd then usys will be a ufrd object, otherwise usys will be a uss object. The ultidyn atom is named based on the variable name of Parray in the calling workspace.

[usys,wt] = ltiarray2uss(P,Parray,ord), returns the weight wt used to bound the infinity norm of ((P - Parray)/P).

[usys,wt] = ltiarray2uss(P,Parray,ord,'OutputMult'), uses multiplicative uncertainty at the plant output (as opposed to input multiplicative uncertainty). The formula for usys is
usys = (I + wt*ultidyn('Name',[size(P,1) size(P,1)])*P).

[usys,wt] = ltiarray2uss(P,Parray,ord,'Additive'), uses additive uncertainty.
usys = P + wt*ultidyn('Name',[size(P,1) size(P,2)]). wt is a frequency domain overbound of the infinity norm of (Parray - P).

[usys,wt] = ltiarray2uss(P,Parray,ord,'InputMult'), uses multiplicative uncertainty at the plant input (this is the default). The formula for usys is
usys = P*(I + wt*ultidyn('Name',[size(P,2) size(P,2)])).

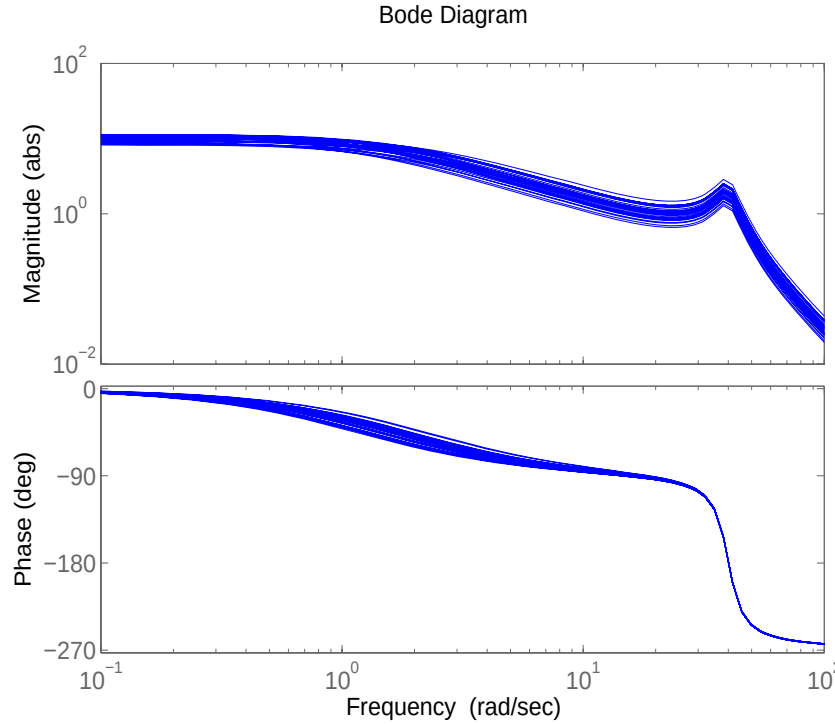
`[usys,wt,diffdata] = ltiarray2uss(P,Parray,ord,type)` returns the norm of the difference (absolute difference for additive, and relative difference for multiplicative uncertainty) between the nominal model `P` and `Parray`. `wt` satisfies `diffdata(w_i) < |wt(w_i)|` at all frequency points.

Example

See the Robust Control Toolbox™ demo entitled "First-cut Robust Design" for a more detailed example of how to use `ltiarray2uss`.

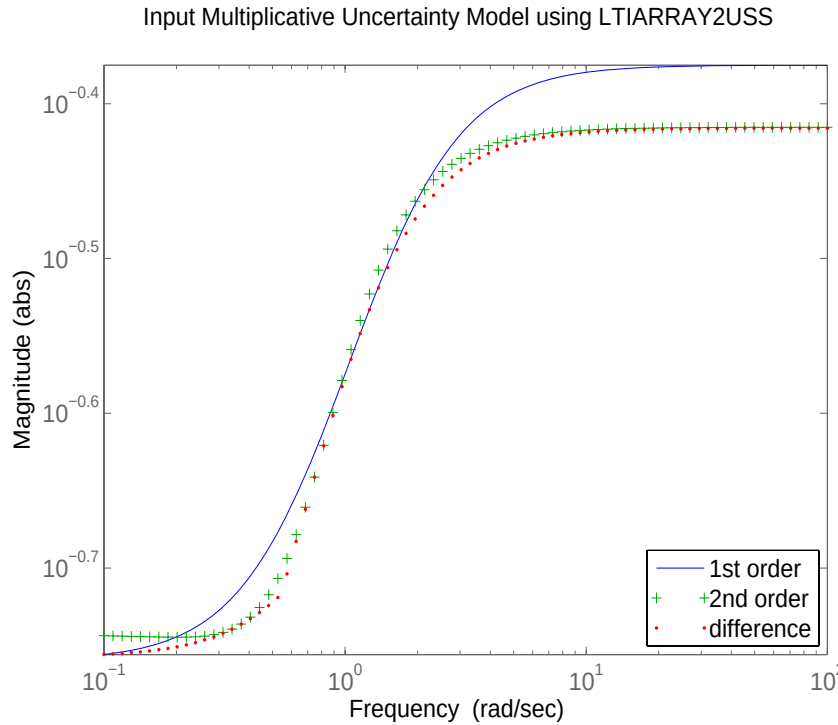
Consider a third order transfer function with an uncertain gain, filter time constant and a lightly damped flexible mode. This model is used to represent a physical system from frequency response data is acquired.

```
gain = ureal('gain',10,'Perc',20);
tau = ureal('tau',.6,'Range',[.42 .9]);
wn = 40;
zeta = 0.1;
usys = tf(gain,[tau 1])*tf(wn^2,[1 2*zeta*wn wn^2]);
sysnom = usys.NominalValue;
parray = usample(usys,30);
om = logspace(-1,2,80);
parrayg = frd(parray,om);
bode(parrayg)
```



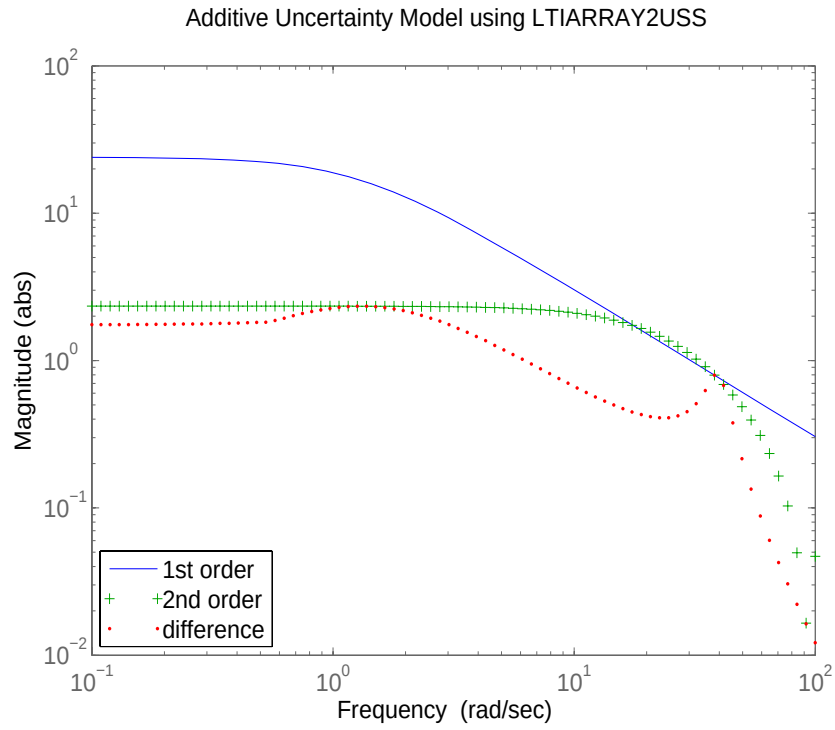
The frequency response data in parray represents 30 experiments performed on the system. The command `ltiarray2uss` is used to generate an uncertain model, umod, based on the frequency response data. Initially an input multiplicative uncertain model is used to characterize the collection of 30 frequency responses. First and second order input multiplicative uncertainty weight are calculated from the data.

```
[umodIn1,wtIn1,diffdataIn] = ltiarray2uss(sysnom,parrayg,1);
[umodIn2,wtIn2,diffdataIn] = ltiarray2uss(sysnom,parrayg,2);
bodemag(wtIn1,'b-',wtIn2,'g+',diffdataIn,'r.',om)
```



Alternatively, an additive uncertain model is used to characterize the collection of 30 frequency responses.

```
[umodAdd1,wtAdd1,diffdataAdd] =
ltiarray2uss(sysnom,parrayg,1,'Additive');
[umodAdd2,wtAdd2,diffdataAdd] =
ltiarray2uss(sysnom,parrayg,2,'Additive');
bodemag(wtAdd1,'b-',wtAdd2,'g+',diffdataAdd,'r.',om)
```



See Also

`fitmagfrd` Fit frequency response magnitude data

Purpose

LQG loop transfer-function recovery (LTR) control synthesis

Syntax

```
[K,SVL,W1] = ltrsyn(G,F,XI,THETA,RHO)
[K,SVL,W1] = ltrsyn(G,F,XI,THETA,RHO,W)
[K,SVL,W1] = ltrsyn(G,F,XI,THETA,RHO,OPT)
[K,SVL,W1] = ltrsyn(G,F,XI,THETA,RHO,W,OPT)
```

Description

`[K,SVL,W1] = ltrsyn(G,F,XI,TH,RHO)` computes a reconstructed-state output-feedback controller K for LTI plant `G` so that `K*G` asymptotically 'recovers' plant-input full-state feedback loop transfer function $L(s) = F(Is-A)^{-1}B+D$; that is, at any frequency `w>0`, `max(sigma(K*G-L, w))` $\rightarrow 0$ as $\rho \rightarrow \infty$, where `L= ss(A,B,F,D)` is the LTI full-state feedback loop transfer function.

`[K,SVL,W1] = ltrsyn(G,F1,Q,R,RHO,'OUTPUT')` computes the solution to the 'dual' problem of filter loop recovery for LTI plant `G` where `F` is a Kalman filter gain matrix. In this case, the recovery is at the plant output, and `max(sigma(G*K-L, w))` $\rightarrow 0$ as $\rho \rightarrow \infty$, where `L1` denotes the LTI filter loop feedback loop transfer function `L1= ss(A,F,C,D)`.

Only the LTI controller `K` for the final value `RHO(end)` is returned.

Inputs	
`G`	LTI plant
`F`	LQ full-state-feedback gain matrix
`XI`	plant noise intensity, or, if `OPT='OUTPUT'` state-cost matrix `XI=Q`,
`THETA`	sensor noise intensity or, if `OPT='OUTPUT'` control-cost matrix `THETA=R`,
`RHO`	vector containing a set of recovery gains
`W`	(optional) vector of frequencies (to be used for plots); if input `W` is not supplied, then a reasonable default is used

Outputs	
K	$K(s)$ — LTI LTR (loop-transfer-recovery) output-feedback, for the last element of RHO (i.e., RHO(end))
SVL	sigma plot data for the 'recovered' loop transfer function if G is MIMO or, for SISO G only, Nyquist loci SVL = [re(1:nr) im(1:nr)]
W1	frequencies for SVL plots, same as W when present

Algorithm

For each value in the vector RHO, [K,SVL,W1] = ltrsyn(G,F,XI,THETA,RHO) computes the full-state-feedback (default OPT='*INPUT*') LTR controller

$$K(s) = [K_c(Is - A + BK_c + K_fC - K_fDK_c)^{-1}K_f]$$

where K_c=F and K_f=lqr(A',C',XI+RHO(i)*B*B',THETA). The 'fictitious noise' term RHO(i)*B*B' results in loop-transfer recovery as RHO(i) $\to \infty$. The Kalman filter gain is $K_f = \Sigma C^T \Theta^{-1}$ where Σ satisfies the Kalman filter Riccati equation $0 = \Sigma A^T + A\Sigma - \Sigma C^T \Theta^{-1} C\Sigma + \Xi + \rho BB^T$. See [1] for further details.

Similarly for the 'dual' problem of filter loop recovery case, [K,SVL,W1] = ltrsyn(G,F,Q,R,RHO,'OUTPUT') computes a filter loop recovery controller of the same form, but with K_f=F is being the input filter gain matrix and the control gain matrix K_c computed as K_c=lqr(A,B,Q+RHO(i)*C'*C,R).

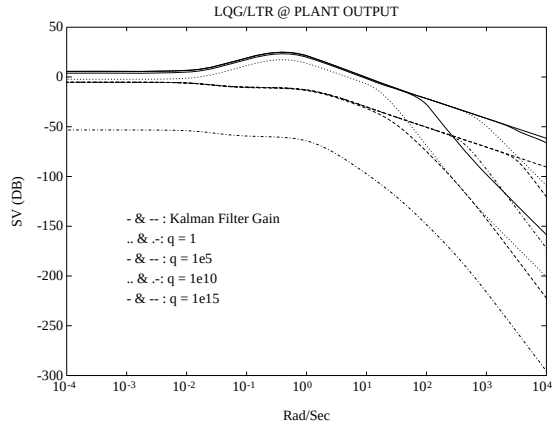


Figure 5-12: Example of LQG/LTR at Plant Output.

Example

```
s=tf('s');G=ss(1e4/((s+1)*(s+10)*(s+100)));[A,B,C,D]=ssdata(G);
F=lqr(A,B,C'*C,eye(size(B,2)));
L=ss(A,B,F,0*F*B);
XI=100*C'*C; THETA=eye(size(C,1));
RHO=[1e3,1e6,1e9,1e12];W=logspace(-2,2);
nyquist(L,'k-.');hold;
[K,SVL,W1]=ltrsyn(G,F,XI,THETA,RHO,W);
```

See also `ltrdemo`

Limitations

The `ltrsyn` procedure may fail for non-minimum phase plants. For full-state LTR (default `OPT='INPUT'`), the plant should not have fewer outputs than inputs. Conversely for filter LTR (when `OPT='OUTPUT'`), the plant should not have fewer inputs than outputs. The plant must be strictly proper, i.e., the D-matrix of the plant should be all zeros. `ltrsyn` is only for continuous time plants (`Ts==0`)

References

[1] Doyle, J., and G. Stein, "Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis," *IEEE Trans. on Automat. Contr.*, AC-26, pp. 4-16, 1981.

See Also

`h2syn`	H_2 controller synthesis
`hinfsyn`	H_∞ controller synthesis

`lqg`	Continuous linear-quadratic-Gaussian control synthesis
`loopsyn`	H_∞ - loop shaping controller synthesis
`ltrdemo`	Demo of LQG/LTR optimal control synthesis
`ncfsyn`	H_∞ - normalized coprime controller synthesis

Purpose

Number of matrix variables in system of LMIs

Syntax

```
K = matnbr(lmisys)
```

Description

`matnbr` returns the number `K` of matrix variables in the LMI problem described by `lmisys`.

See Also

`decnbr`	Give the total number of decision variables in a system of LMIs
`lmiinfo`	Interactively retrieve information about the variables and term content of LMIs
`decinfo`	Describe how the entries of a matrix variable X relate to the decision variables

Purpose

Return the vector of decision variables corresponding to particular values of the matrix variables

Syntax

```
decvec = mat2dec(lmisys,X1,X2,X3,...)
```

Description

Given an LMI system `lmisys` with matrix variables $X_1, \ldots, X_K$ and given values `X1,...,Xk` of $X_1, \ldots, X_K$, `mat2dec` returns the corresponding value `decvec` of the vector of decision variables. Recall that the decision variables are the independent entries of the matrices $X_1, \ldots, X_K$ and constitute the free scalar variables in the LMI problem.

This function is useful, for example, to initialize the LMI solvers `mincx` or `gevp`. Given an initial guess for $X_1, \ldots, X_K$, `mat2dec` forms the corresponding vector of decision variables `xinit`.

An error occurs if the dimensions and structure of `X1,...,Xk` are inconsistent with the description of $X_1, \ldots, X_K$ in `lmisys`.

Example

Consider an LMI system with two matrix variables X and Y such that

- X is A symmetric block diagonal with one 2-by-2 full block and one 2-by-2 scalar block
- Y is a 2-by-3 rectangular matrix

Particular instances of X and Y are

$$X_0 = \begin{pmatrix} 1 & 3 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}, \qquad Y_0 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

and the corresponding vector of decision variables is given by

```
decv = mat2dec(lmisys,X0,Y0)

decv'

ans =
       1    3    -1   5    1    2    3 4 5 6
```

Note that `decv` is of length 10 since Y has 6 free entries while X has 4 independent entries due to its structure. Use `decinfo` to obtain more information about the decision variable distribution in X and Y.

See Also

`dec2mat`	Given values of the decision variables, derive the corresponding values of the matrix variables
`decinfo`	Describe how the entries of a matrix variable X relate to the decision variables
`decnbr`	Give the total number of decision variables in a system of LMIs

mincx

Purpose

Minimize linear objective under LMI constraints

Syntax

```
[copt,xopt] = mincx(lmisys,c,options,xinit,target)
```

Description

The function `mincx` solves the convex program

$$\text{minimize } c^T x \text{ subject to } N^T L(x) N \le M^T R(x) M \qquad \textbf{(5-18)}$$

where x denotes the vector of scalar decision variables.

The system of LMIs (9-18) is described by `lmisys`. The vector `c` must be of the same length as x. This length corresponds to the number of decision variables returned by the function `decnbr`. For linear objectives expressed in terms of the matrix variables, the adequate `c` vector is easily derived with `defcx`.

The function `mincx` returns the global minimum `copt` for the objective $c^T x$, as well as the minimizing value `xopt` of the vector of decision variables. The corresponding values of the matrix variables is derived from `xopt` with `dec2mat`.

The remaining arguments are optional. The vector `xinit` is an initial guess of the minimizer `xopt`. It is ignored when infeasible, but may speed up computations otherwise. Note that `xinit` should be of the same length as `c`. As for `target`, it sets some target for the objective value. The code terminates as soon as this target is achieved, that is, as soon as some feasible x such that $c^T x$ ð `target` is found. Set `options` to `[]` to use `xinit` and `target` with the default options.

Control Parameters

The optional argument `options` gives access to certain control parameters of the optimization code. In `mincx`, this is a five-entry vector organized as follows:

- `options(1)` sets the desired relative accuracy on the optimal value `lopt` (default = 10–2).
- `options(2)` sets the maximum number of iterations allowed to be performed by the optimization procedure (100 by default).
- `options(3)` sets the feasibility radius. Its purpose and usage are as for `feasp`.

- `options(4)` helps speed up termination. If set to an integer value $J > 0$, the code terminates when the objective $c^T x$ has not decreased by more than the desired relative accuracy during the last J iterations.
- `options(5) = 1` turns off the trace of execution of the optimization procedure. Resetting `options(5)` to zero (default value) turns it back on.

Setting `option(i)` to zero is equivalent to setting the corresponding control parameter to its default value. See `feasp` for more detail.

Tip for Speed-Up

In LMI optimization, the computational overhead per iteration mostly comes from solving a least-squares problem of the form

$$\min_x \|Ax - b\|$$

where x is the vector of decision variables. Two methods are used to solve this problem: Cholesky factorization of $A^T A$ (default), and QR factorization of A when the normal equation becomes ill conditioned (when close to the solution typically). The message

```
* switching to QR
```

is displayed when the solver has to switch to the QR mode.

Since QR factorization is incrementally more expensive in most problems, it is sometimes desirable to prevent switching to QR. This is done by setting `options(4) = 1`. While not guaranteed to produce the optimal value, this generally achieves a good trade-off between speed and accuracy.

Memory Problems

QR-based linear algebra (see above) is not only expensive in terms of computational overhead, but also in terms of memory requirement. As a result, the amount of memory required by QR may exceed your swap space for large problems with numerous LMI constraints. In such case, MATLAB® issues the error

```
??? Error using ==> pds
Out of memory. Type HELP MEMORY for your options.
```

You should then ask your system manager to increase your swap space or, if no additional swap space is available, set `options(4) = 1`. This will prevent switching to QR and `mincx` will terminate when Cholesky fails due to numerical instabilities.

Reference

The solver `mincx` implements Nesterov and Nemirovski's Projective Method as described in

Nesterov, Yu, and A. Nemirovski, *Interior Point Polynomial Methods in Convex Programming: Theory and Applications*, SIAM, Philadelphia, 1994.

Nemirovski, A., and P. Gahinet, "The Projective Method for Solving Linear Matrix Inequalities," *Proc. Amer. Contr. Conf.*, 1994, Baltimore, Maryland, pp. 840-844.

The optimization is performed by the C-MEX file `pds.mex`.

See Also

`defcx`	Help specify $c^T x$ objectives for the `mincx` solver
`dec2mat`	Given values of the decision variables, derive the corresponding values of the matrix variables
`decnbr`	Give the total number of decision variables in a system of LMIs
`feasp`	Find a solution to a given system of LMIs
`gevp`	Generalized eigenvalue minimization under LMI constraints

Purpose

H_∞ mixed-sensitivity synthesis method for robust control loopshaping design

Syntax

```
[K,CL,GAM,INFO]=mixsyn(G,W1,W2,W3)
[K,CL,GAM,INFO]=mixsyn(G,W1,W2,W3,KEY1,VALUE1,KEY2,VALUE2,...)
```

Description

`[K,CL,GAM,INFO]=mixsyn(G,W1,W2,W3)` computes a controller K that minimizes the H_∞ norm of the closed-loop transfer function the weighted mixed sensitivity

$$T_{y_1u_1} \triangleq \begin{bmatrix} W_1S \\ W_2R \\ W_3T \end{bmatrix}$$

where S and T are called the *sensitivity* and *complementary sensitivity,* respectively and S, R and T are given by

$$S = (I + GK)^{-1}$$
$$R = K(I + GK)^{-1}$$
$$T = GK(I + GK)^{-1}$$

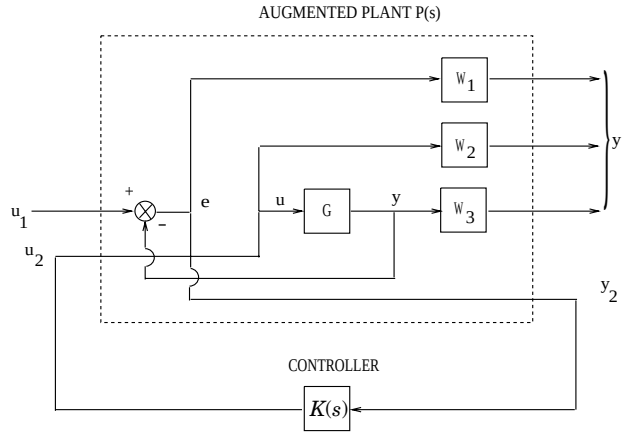


Figure 5-13: Closed-loop transfer function $T_{y_1u_1}$ for mixed sensitivity `mixsyn`.

The returned values of S, R, and T satisfy the following loop shaping inequalities:

$$\bar{\sigma}(S(j\omega)) \le \gamma \ \ \underline{\sigma}(W_1^{-1}(j\omega))$$

$$\bar{\sigma}(R(j\omega)) \le \gamma \ \ \underline{\sigma}(W_2^{-1}(j\omega))$$

$$\bar{\sigma}(T(j\omega)) \le \gamma \ \ \underline{\sigma}(W_3^{-1}(j\omega))$$

where γ=GAM. Thus, W_1, W_3 determine the shapes of sensitivity S and complementary sensitivity T. Typically, you would choose W_1 to be small inside the desired control bandwidth to achieve good disturbance attenuation (i.e., performance), and choose W_3 to be small outside the control bandwidth, which helps to ensure good stability margin (i.e., robustness).

For dimensional compatibility, each of the three weights W_1, W_2 and W_3 must be either empty, scalar (SISO) or have respective input dimensions N_Y, N_U, and N_Y where G is N_Y-by-N_U. If one of the weights is not needed, you may simply assign an empty matrix []; e.g., `P = AUGW(G,W1,[],W3)` is SYS but without the second row (without the row containing `W2`).

Algorithm

`[K,CL,GAM,INFO]=mixsyn(G,W1,W2,W3,KEY1,VALUE1,KEY2,VALUE2,...)`

is equivalent to

```
[K,CL,GAM,INFO]=...
   hinfsyn(augw(G,W1,W2,W3),KEY1,VALUE1,KEY2,VALUE2,...).
```

`mixsyn` accepts all the same key value pairs as `hinfsyn`.

Example

The following code illustrates the use of `mixsyn` for sensitivity and complementary sensitivity 'loop-shaping'.

```
s=zpk('s');
G=(s-1)/(s+1)^2;
W1=0.1*(s+100)/(100*s+1); W2=0.1;
[K,CL,GAM]=mixsyn(G,W1,W2,[]);
L=G*K; S=inv(1+L); T=1-S;
sigma(S,'g',T,'r',GAM/W1,'g-.',GAM*G/ss(W2),'r-.')
```

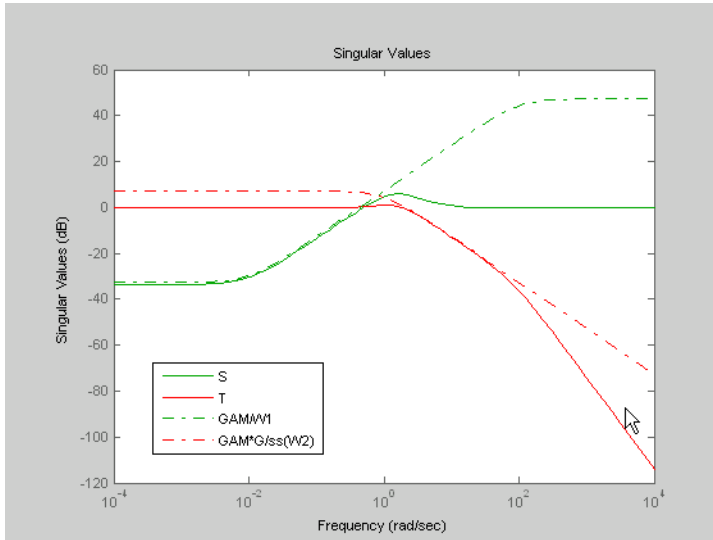


Figure 5-14: `mixsyn(G,W1,W2,[ ])` shapes sigma plots of S and T to conform to γ/W_1 and $\gamma G/W_2$, respectively.

Limitations

The transfer functions G, W_1, W_2 and W_3 must be proper, i.e., bounded as $s \to \infty$ or, in the discrete-time case, as $z \to \infty$. Additionally, W_1, W_2 and W_3 should be stable. The plant G should be stabilizable and detectable; else, `P` will not be stabilizable by any `K`.

See Also

`augw`	Augments plant weights for control design
`hinfsyn`	H_∞ controller synthesis

Purpose

Generate Bessel, Butterworth, Chebyshev, or RC filter

Syntax

```
sys = mkfilter(fc,ord,type)
sys = mkfilter(fc,ord,type,psbndr)
```

Description

`sys = mkfilter(fc,ord,type)` returns a single-input, single-output analog low pass filter `sys` as an `ss` object. The cutoff frequency (Hertz) is `fc` and the filter order is `ord`, a positive integer. The string variable `type` specifies the type of filter and can be one of the following:

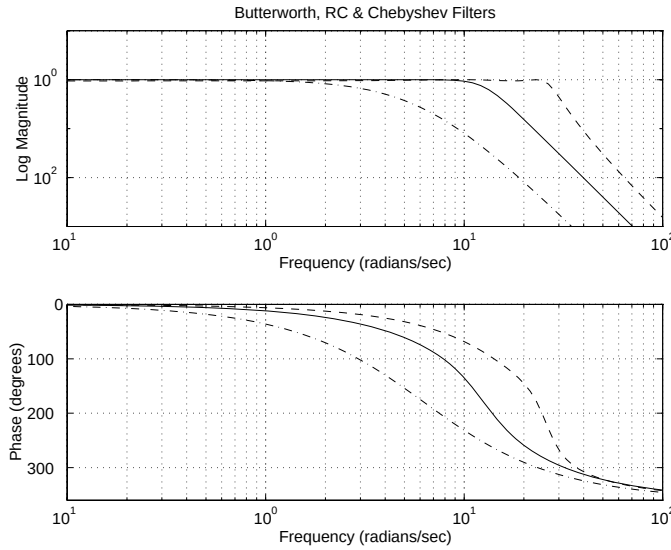
String variable	Description
`'butterw'`	Butterworth filter
`'cheby'`	Chebyshev filter
`'bessel'`	Bessel filter
`'rc'`	Series of resistor/capacitor filters

The dc gain of each filter (except even-order Chebyshev) is set to unity.

`sys = mkfilter(fc,ord,type,psbndr)` contains the input argument `psbndr` that specifies the Chebyshev passband ripple (in dB). At the cutoff frequency, the magnitude is `-psbndr` dB. For even-order Chebyshev filters the DC gain is also `-psbndr` dB.

Example

```
butw = mkfilter(2,4,'butterw');
cheb = mkfilter(4,4,'cheby',0.5);
rc = mkfilter(1,4,'rc');
bode(butw_g,'-',cheb_g,'--',rc_g,'-.')

megend('Butterworth','Chebyshev','RC filter')
```



Limitations

The Bessel filters are calculated using the recursive polynomial formula. This is poorly conditioned for high order filters (order > 8).

See Also

`augw` Augments plant weights for control design

mktito

Purpose

Make two-input-two-output (TITO) system out of MIMO LTI system

Syntax

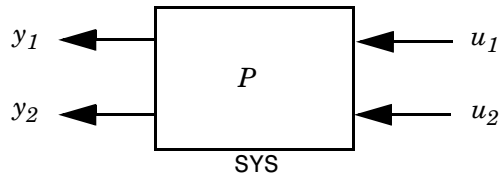
```
SYS=mktito(SYS,NMEAS,NCONT)
```

Description

`SYS=mktito(SYS,NMEAS,NCONT)` tadds TITO (two-input-two-output) partitioning to LTI system `SYS`, assigning OutputGroup and InputGroup properties such that

NMEAS = $\dim(y_2)$

NCONT = $\dim(u_2)$



Any pre-existing OutputGroup or InputGroup properties of `SYS` are overwritten. TITO partitioning simplifies syntax for control synthesis functions like `hinfsyn` and `h2syn`.

Algorithm

```
[r,c]=size(SYS);
set(SYS,'InputGroup', struct('U1',1:c-NCONT,'U2',c-NCONT+1:c));
set(SYS,'OutputGroup',struct('Y1',1:r-NMEAS,'Y2',r-NMEAS+1:r));
```

Example

You can type

```
P=rss(2,4,5); P=mktito(P,2,2);
disp(P.OutputGroup); disp(P.InputGroup);
```

to create a 4-by-5 LTI system `P` with OutputGroup and InputGroup properties

```
U1: [1 2 3]
U2: [4 5]
Y1: [1 2]
Y2: [3 4]
```

See Also

`augw`	Augments plant weights for control design
`hinfsyn`	H_∞ synthesis controller
`h2syn`	H_2 synthesis controller
`ltiprops`	Help on LTI model properties
`sdhinfsyn`	H_∞ discrete-time controller

modreal

Purpose

Modal form realization and projection

Syntax

```
[G1,G2] = modreal(G,cut)
```

Description

`[G1,G2] = modreal(G,cut)` returns a set of state-space LTI objects `G1` and `G2` in modal form given a state-space `G` and the model size of `G1`, `cut`.

The modal form realization has its A matrix in block diagonal form with either 1x1 or 2x2 blocks. The real eigenvalues will be put in 1x1 blocks and complex eigenvalues will be put in 2x2 blocks. These diagonal blocks are ordered in ascending order based on eigenvalue magnitudes.

The complex eigenvalue a+bj is appearing as 2x2 block

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

This table describes input arguments for `modreal`.

Argument	Description
`G`	LTI model to be reduced.
`cut`	(Optional) an integer to split the realization. Without it, a complete modal form realization is returned

This table lists output arguments.

Argument	Description
`G1,G2`	LTI models in modal form

`G` can be stable or unstable. $G_1 = (A_1, B_1, C_1, D_1)$, $G_2 = (A_2, B_2, C_2, D_2)$ and $D_1 = D + C_2(-A_2)^{-1}B_2$ is calculated such that the system DC gain is preserved.

Algorithm

Using a real eigen structure decomposition `reig` and ordering the eigenvectors in ascending order according to their eigenvalue magnitudes, we can form a

similarity transformation out of these ordered real eigenvectors such that he resulting systems G1 and/or G2 are in block diagonal modal form.

Note This routine is extremely useful when model has jω-axis singularities, e.g., rigid body dynamics. It has been incorporated inside Hankel based model reduction routines - hankelmr, balancmr, bstmr, and schurmr to isolate those jω-axis poles from the actual model reduction process.

Example

Given a continuous stable or unstable system, G, the following commands can get a set of modal form realizations depending on the split index -- cut:

```
randn('state',1234); rand('state',5678);
G = rss(50,2,2);
[G1,G2] = modreal(G,2); % cut = 2 for two rigid body modes
G1.d = zeros(2,2); % remove the DC gain of the system from G1
sigma(G,G1,G2)
```

See Also

reduce	Top level model reduction routines
balancmr	Balanced truncation via square-root method
schurmr	Balanced truncation via Schur method
bstmr	Balanced stochastic truncation via Schur method
ncfmr	Balanced truncation for normalized coprime factors
hankelmr	Hankel minimum degree approximation
hankelsv	Hankel singular value

msfsyn

Purpose

Multi-model/multi-objective state-feedback synthesis

Syntax

```
[gopt,h2opt,K,Pcl,X] = msfsyn(P,r,obj,region,tol)
```

Description

Given an LTI plant `P` with state-space equations

$$\begin{cases} \dot{x} &= Ax + B_1 w + B_2 u \\ z_\infty &= C_1 x + D_{11} w + D_{12} u \\ z_2 &= C_2 x + D_{22} u \end{cases}$$

`msfsyn` computes a state-feedback control $u = Kx$ that

- Maintains the RMS gain (H_∞ norm) of the closed-loop transfer function T_∞ from w to z_∞ below some prescribed value $\gamma_0 > 0$
- Maintains the H_2 norm of the closed-loop transfer function T_2 from w to z_2 below some prescribed value $\upsilon_0 > 0$
- Minimizes an H_2/H_∞ trade-off criterion of the form

  $$\alpha \|T_\infty\|_\infty^2 + \beta \|T_2\|_2^2$$

- Places the closed-loop poles inside the LMI region specified by `region` (see `lmireg` for the specification of such regions). The default is the open left-half plane.

Set `r = size(d22)` and `obj = [`γ_0, ν_0, α, β`]` to specify the problem dimensions and the design parameters γ_0, ν_0, α, and β. You can perform pure pole placement by setting `obj = [0 0 0 0]`. Note also that z_∞ or z_2 can be empty.

On output, `gopt` and `h2opt` are the guaranteed H_∞ and H_2 performances, `K` is the optimal state-feedback gain, `Pcl` the closed-loop transfer function from w to $\begin{pmatrix} z_\infty \\ z_2 \end{pmatrix}$, and `X` the corresponding Lyapunov matrix.

The function `msfsyn` is also applicable to multi-model problems where `P` is a polytopic model of the plant:

$$\begin{cases} \dot{x} &= A(t)x + B_1(t)w + B_2(t)u \\ z_\infty &= C_1(t)x + D_{11}(t)w + D_{12}(t)u \\ z_2 &= C_2(t)x + D_{22}(t)u \end{cases}$$

with time-varying state-space matrices ranging in the polytope

$$\begin{pmatrix} A(t) & B_1(t) & B_2(t) \\ C_1(t) & D_{11}(t) & D_{12}(t) \\ C_2(t) & 0 & D_{22}(t) \end{pmatrix} \in \mathrm{Co}\left\{ \begin{pmatrix} A_k & B_k & B_{2k} \\ C_{1k} & D_{11k} & D_{12k} \\ C_{2k} & 0 & D_{22k} \end{pmatrix} : k = 1, \ldots, K \right\}$$

In this context, `msfsyn` seeks a state-feedback gain that robustly enforces the specifications over the entire polytope of plants. Note that polytopic plants should be defined with `psys` and that the closed-loop system `Pcl` is itself polytopic in such problems. Affine parameter-dependent plants are also accepted and automatically converted to polytopic models.

See Also

`lmireg`	Specify LMI regions for pole placement purposes
`psys`	Specification of uncertain state-space models

Purpose

Compute upper and lower bounds on structured singular value (μ) and upper bounds on generalized structured singular value

Syntax

```
bounds = mussv(M,BlockStructure)
[bounds,muinfo] = mussv(M,BlockStructure)
[bounds,muinfo] = mussv(M,BlockStructure,Options)
[ubound,q] = mussv(M,F,BlockStructure)
[ubound,q] = mussv(M,F,BlockStructure,'s')
```

Description

`bounds = mussv(M,BlockStructure)` calculates upper and lower bounds on the structured singular value, or μ, for a given block structure. `M` is a `double`, or `frd` object. If `M` is an N-D array (with $N \geq 3$), then the computation is performed pointwise along the third and higher array dimensions. If `M` is a `frd` object, the computations are performed pointwise in frequency (as well as any array dimensions).

`BlockStructure` is a matrix specifying the perturbation block structure. `BlockStructure` has 2 columns, and as many rows as uncertainty blocks in the perturbation structure. The *i*-th row of `BlockStructure` defines the dimensions of the i'th perturbation block.

- If `BlockStructure(i,:) = [-r 0]`, then the *i*-th block is an `r`-by-`r` repeated, diagonal real scalar perturbation;
- if `BlockStructure(i,:) = [r 0]`, then the *i*-th block is an `r`-by-`r` repeated, diagonal complex scalar perturbation;
- if `BlockStructure(i,:) = [r c]`, then the *i*-th block is an `r`-by-`c` complex full-block perturbation.
- If `BlockStructure` is omitted, its default is `ones(size(M,1),2)`, which implies a perturbation structure of all 1-by-1 complex blocks. In this case, if `size(M,1)` does not equal `size(M,2)`, an error results.

If `M` is a two-dimensional matrix, then `bounds` is a `1-by-2` array containing an upper (first column) and lower (second column) bound of the structured singular value of `M`. For all matrices `Delta` with block-diagonal structure defined by `BlockStructure` and with norm less than `1/bounds(1)` (upper bound), the matrix `I - M*Delta` is not singular. Moreover, there is a matrix `DeltaS` with block-diagonal structure defined by `BlockStructure` and with

norm equal to `1/bounds(2)` (lower bound), for which the matrix `I - M*DeltaS` is singular.

The format used in the 3rd output argument from `lftdata` is also acceptable for describing the block structure.

If `M` is an `frd`, the computations are always performed pointwise in frequency. The output argument `bounds` is a `1-by-2 frd` of upper and lower bounds at each frequency. Note that `bounds.Frequency` equals `M.Frequency`.

If `M` is an N-D array (either `double` or `frd`), the upper and lower bounds are computed pointwise along the 3rd and higher array dimensions (as well as pointwise in frequency, for `frd`). For example, suppose that `size(M)` is $r \times c \times d_1 \times \ldots \times d_F$. Then `size(bounds)` is $1 \times 2 \times d_1 \times \ldots \times d_F$. Using single index notation, `bounds(1,1,i)` is the upper bound for the structured singular value of `M(:,:,i)`, and `bounds(1,2,i)` is the lower bound for the structured singular value of `M(:,:,i)`. Here, any `i` between 1 and $d_1 \cdot d_2 \ldots d_F$ (the product of the d_k) would be valid.

`bounds = mussv(M,BlockStructure,Options)` specifies computation options. `Options` is a character string, containing any combination of the following characters:

Option	Meaning
`'a'`	Upper bound to greatest accuracy, using LMI solver
`'an'`	Same as `'a'`, but without automatic prescaling
`'d'`	Display warnings
`'f'`	Fast upper bound (typically not as tight as the default)
`'g6'`	Use gain-based lower bound (note that the default lower bound employs a power iteration) multiple times (in this case `10+6*10` times). Larger numbers typically give better lower bounds. This is an alternative to the default lower bound which uses a power iteration.
`'i'`	Reinitialize lower bound computation at each new matrix (only relevant if `M` is `ND` array or `frd`)

Option	Meaning
'm7'	Randomly reinitialize lower bound iteration multiple times (in this case 7 times, use 1-9); larger number typically gives better lower bound
'o'	Run "old" algorithms, from version 3.1.1 and before. Included to allow exact replication of earlier calculations.
's'	Suppress progress information (silent)
'U'	Upper-bound "only" (lower bound uses a fast/cheap algorithm).
'x'	Decrease iterations in lower bound computation (faster but not as tight as default). Use 'U' for an even faster lower bound

`[bounds,muinfo] = mussv(M,BlockStructure)` returns `muinfo`, a structure containing more detailed information. The information within `muinfo` must be extracted using `mussvextract`. See `mussvextract` for more details.

Generalized Structured Singular Value

`ubound = mussv(M,F,BlockStructure)` calculates an upper bound on the generalized structured singular value (generalized μ) for a given block structure. `M` is a `double` or `frd` object. `M` and `BlockStructure` are as before. `F` is an additional (`double` or `frd`).

`ubound = mussv(M,F,BlockStructure,'s')` adds an option to run silently. Other options are ignored for generalized μ problems.

Note that in generalized structured singular value computations, only an upper bound is calculated. `ubound` is an upper bound of the generalized structured singular value of the pair `(M,F)`, with respect to the block-diagonal uncertainty described by `BlockStructure`. Consequently `ubound` is 1-by-1 (with additional array dependence, depending on `M` and `F`). For all matrices `Delta` with block-diagonal structure defined by `BlockStructure` and `norm<1/ubound`, the matrix `[I-Delta*M;F]` is guaranteed not to lose column

rank. This is verified by the matrix `Q`, which satisfies `mussv(M+Q*F,BlockStructure,'a')<=ubound`.

Example

See `mussvextract` for a detailed example of the structured singular value.

A simple example for generalized structured singular value can be done with random complex matrices, illustrating the relationship between the upper bound for μ and generalized μ, as well as the fact that the upper bound for generalized μ comes from an optimized μ upper bound.

`M` is a complex 5-by-5 matrix and `F` is a complex 2-by-5 matrix. The block structure `BlockStructure` is an uncertain real parameter δ_1, an uncertain real parameter δ_2, an uncertain complex parameter δ_3 and a twice-repeated uncertain complex parameter δ_4.

```
randn(`state',929)
M = randn(5,5) + sqrt(-1)*randn(5,5);
F = randn(2,5) + sqrt(-1)*randn(2,5);
BlockStructure = [-1 0;-1 0;1 1;2 0];
[ubound,Q] = mussv(M,F,BlockStructure);
bounds = mussv(M,BlockStructure);
optbounds = mussv(M+Q*F,BlockStructure);
```

The quantities `optbounds(1)` and `ubound` should be extremely close, and significantly lower than `bounds(1)` and `bounds(2)`.

```
[optbounds(1) ubound]
ans =
    1.6280 1.6007
[bounds(1)  bounds(2)]
ans =
    3.4827 3.3011
```

Algorithm

The lower bound is computed using a power method, Young and Doyle, 1990, and Packard *et al.* 1988, and the upper bound is computed using the balanced/AMI technique, Young *et al.*, 1992, for computing the upper bound from Fan *et al.*, 1991.

Peter Young and Matt Newlin wrote the original M-files.

The lower-bound power algorithm is from Young and Doyle, 1990, and Packard *et al.* 1988.

The upper-bound is an implementation of the bound from Fan *et al.*, 1991, and is described in detail in Young *et al.*, 1992. In the upper bound computation, the matrix is first balanced using either a variation of Osborne's method (Osborne, 1960) generalized to handle *repeated scalar* and *full* blocks, or a Perron approach. This generates the standard upper bound for the associated complex μ problem. The Perron eigenvector method is based on an idea of Safonov, (Safonov, 1982). It gives the exact computation of μ for positive matrices with scalar blocks, but is comparable to Osborne on general matrices. Both the Perron and Osborne methods have been modified to handle *repeated scalar* and *full* blocks. Perron is faster for small matrices but has a growth rate of n^3, compared with less than n^2 for Osborne. This is partly due to the MATLAB implementation, which greatly favors Perron. The default is to use Perron for simple block structures and Osborne for more complicated block structures. A sequence of improvements to the upper bound is then made based on various equivalent forms of the upper bound. A number of descent techniques are used that exploit the structure of the problem, concluding with general purpose LMI optimization (Boyd *et al.*), 1993, to obtain the final answer.

The optimal choice of `Q` (to minimize the upper bound) in the generalized mu problem is solved by reformulating the optimization into a semidefinite program (Packard *et al.*, 1991).

References

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See Also

`loopmargin`	Comprehensive analysis of feedback loop
`mussvextract`	Extract compressed data returned from `mussv`
`robuststab`	Calculate stability margins of uncertain systems
`robustperf`	Calculate performance margins of uncertain systems
`wcgain`	Calculate worst-case gain of a system
`wcsens`	Calculate worst-case sensitivities for feedback loop
`wcmargin`	Calculate worst-case margins for feedback loop

Purpose

Extract `muinfo` structure returned by `mussv`

Syntax

```
[VDelta,VSigma,VLmi] = mussvextract(muinfo)
```

Description

A structured singular value computation of the form

```
[bounds,muinfo] = mussv(M,BlockStructure)
```

returns detailed information in the structure `muinfo`. `mussvextract` is used to extract the compressed information within `muinfo` into a readable form.

The most general call to `mussvextract` extracts three usable quantities: `VDelta`, `VSigma`, and `VLmi`. `VDelta` is used to verify the lower bound. `VSigma` is used to verify the Newlin/Young upper bound and has fields `DLeft, DRight, GLeft, GMiddle,` and `GRight`. `VLmi` is used to verify the LMI upper bound and has fields `Dr, Dc, Grc`, and `Gcr`. The relation/interpretation of these quantities with the numerical results in `bounds` is described below.

Upper Bound Information

The upper bound is based on a proof that `det(I - M*Delta)` is nonzero for all block-structured matrices `Delta` with norm smaller than `1/bounds(1)`. The Newlin/Young method consists of finding a scalar β and matrices D and G, consistent with `BlockStructure`, such that

$$\bar{\sigma}\left(\left((I+G_l^2)^{-\frac{1}{4}}\left(\frac{D_l M D_r^{-1}}{\beta}-jG_m\right)(I+G_r^2)^{-\frac{1}{4}}\right)\right)\le 1$$

Because some uncertainty blocks and `M` need not be square, the matrices D and G have a few different manifestations. In fact, in the formula above, there are a left and right D and G, as well as a middle G. Any such β is an upper bound of `mussv(M,BlockStructure)`.

It is true that if `BlockStructure` consists only of complex blocks, then all G matrices will be zero, and the expression above simplifies to

$$\bar{\sigma}(D_l M D_r^{-1})\le\beta.$$

The LMI method consists of finding a scalar β and matrices D and G, consistent with `BlockStructure`, such that

$$M'D_{\mathrm{r}}M-\beta^2 D_c+j(G_{\mathrm{cr}}M-M'G_{\mathrm{rc}})\le 0$$

is negative semidefinite. Again, D and G have a few different manifestations to match the row and column dimensions of M. Any such β is an upper bound of `mussv(M,BlockStructure)`. If `BlockStructure` consists only of complex blocks, then all G matrices will be zero, and negative semidefiniteness of $M'D_rM-\beta^2D_c$ is sufficient to derive an upper bound.

Lower Bound Information

The lower bound of `mussv(M,BlockStructure)` is based on finding a "small" (hopefully the smallest) block-structured matrix `VDelta` that causes `det(I - M*VDelta)` to equal 0. Equivalently, the matrix `M*VDelta` has an eigenvalue equal to 1. It will always be true that the lower bound `(bounds(2))` will be the reciprocal of `norm(VDelta)`.

Example

Suppose `M` is a 4-by-4 complex matrix. Take the block structure to be two 1-by-1 complex blocks and one 2-by-2 complex block.

```
randn('state',0)
M = randn(4,4) + sqrt(-1)*randn(4,4);
BlockStructure = [1 1;1 1;2 2];
```

You can calculate bounds on the structured singular value using the `mussv` command and extract the scaling matrices using `mussvextract`.

```
[bounds,muinfo] = mussv(M,BlockStructure);
[VDelta,VSigma,VLmi] = mussvextract(muinfo);
```

You can first verify the Newlin/Young upper bound with the information extracted from `muinfo`. The corresponding scalings are `Dl` and `Dr`.

```
Dl = VSigma.DLeft
Dl =
1.0000e+000            0            0            0
            0  9.9190e-001            0            0
            0            0  1.1255e+000            0
            0            0            0  1.1255e+000
Dr = VSigma.DRight
Dr =
1.0000e+000            0            0            0
            0  9.9190e-001            0            0
            0            0  1.1255e+000            0
            0            0            0  1.1255e+000
```

```
[norm(Dl*M/Dr) bounds(1)]
ans =
4.3420e+000  4.3420e+000
```

You can first verify the LMI upper bound with the information extracted from `muinfo`. The corresponding scalings are `Dr` and `Dc`.

```
Dr = VLmi.Dr;
Dc = VLmi.Dc;
eig(M'*Dr*M - bounds(1)^2*Dc)
ans =
-2.0045e-005 +6.1649e-016i
 -1.4688e+001 -2.4975e-016i
 -2.0436e+001 -4.7583e-016i
 -1.9100e+001 +1.4136e-015i
```

Note that `VDelta` matches the structure defined by `BlockStructure`, and the norm of `VDelta` agrees with the lower bound,

```
VDelta
VDelta =
1.0698e-001 -2.0405e-001i       0                0
0
       0          1.4920e-001 +1.7556e-001i       0
0
           0                          0              -5.4173e-002
-1.0932e-002i -5.7140e-002 +9.6497e-002i
           0                          0               2.8071e-002
-8.0807e-002i -1.3608e-001 -1.0777e-001i
[norm(VDelta) 1/bounds(2)]
ans =
    0.2304 0.2304
```

and that `M*VDelta` has an eigenvalue exactly at 1.

```
eig(M*VDelta)
ans =
1.0000e+000 -8.3267e-017i
 -6.1108e-002 +2.5748e-001i
  4.1427e-018 -5.8578e-018i
 -1.9637e-001 -5.6540e-002i
```

Keep the matrix the same, but change `BlockStructure` to be a 2-by-2 repeated, real scalar block and two complex 1-by-1 blocks. Run `mussv` with the `'C'` option to tighten the upper bound.

```
BlockStructure2 = [-2 0; 1 0; 1 0];
[bounds2,muinfo2] = mussv(M,BlockStructure2,'C');
```

You can compare the computed bounds. Note that `bounds2` should be smaller than `bounds`, because the uncertainty set defined by `BlockStructure2` is a proper subset of that defined by `BlockStructure`.

```
[bounds; bounds2]
ans =
    4.342 4.340
    3.470 3.470
```

You can extract the D, G and `Delta` from `muinfo2` using `mussvextract`.

```
[VDelta2,VSigma2,VLmi2] = mussvextract(muinfo2);
```

As before, you can first verify the Newlin/Young upper bound with the information extracted from `muinfo`. The corresponding scalings are `Dl, Dr, Gl, Gm and Gr`.

```
Dl = VSigma2.DLeft;
Dr = VSigma2.DRight;
Gl = VSigma2.GLeft;
Gm = VSigma2.GMiddle;
Gr = VSigma2.GRight;
dmd = Dl*M/Dr/bounds2(1) - sqrt(-1)*Gm;
SL = (eye(4)+Gl*Gl)^-0.25;
SR = (eye(4)+Gr*Gr)^-0.25;
norm(SL*dmd*SR)
ans =
    1.0000
```

You can first verify the LMI upper bound with the information extracted from `muinfo`. The corresponding scalings are `Dr`, `Dc, Grc` and `Gcr`.

```
Dr = VLmi2.Dr;
Dc = VLmi2.Dc;
Grc = VLmi2.Grc;
Gcr = VLmi2.Gcr;
```

```
eig(M'*Dr*M - bounds(1)^2 *Dc + j*(Gcr*M-M'*Grc))
ans =
-4.4665e-002 -4.4823e-019i
 -5.2486e-004 +1.5623e-018i
 -1.8028e-003 +3.2493e-019i
 -1.2558e-003 +1.2973e-019i
```

`VDelta2` matches the structure defined by `BlockStructure`, and the norm of `VDelta2` agrees with the lower bound,

```
VDelta2
VDelta2 =
   0.2882 0                  0                  0
        0             0.2882 0                  0
        0              0          -0.152 - 0.2448i       0
        0              0              0          -0.0395 -0.2855i
[norm(VDelta2) 1/bounds2(2)]
ans =
    0.2882 0.2882
```

and that `M*VDelta2` has an eigenvalue exactly at 1.

```
eig(M*VDelta2)
ans =
-3.3623e-001 +2.1885e-001i
 -3.6805e-001 -1.5645e-001i
  1.0000e+000 -1.4169e-016i
  4.5066e-001 -3.4481e-001i
```

See Also

`mussv` Calculate bounds on the Structured Singular Value (μ)

Purpose

Calculate normalized coprime stability margin of plant-controller feedback loop

Syntax

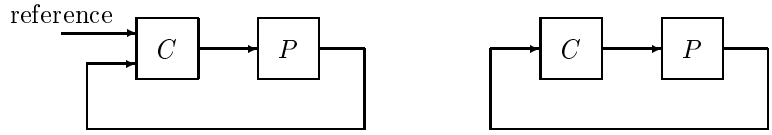
```
[marg,freq] = ncfmargin(P,C)
[marg,freq] = ncfmargin(P,C,tol)
```

Description

`[marg,freq] = ncfmargin(P,C)` calculates the normalized coprime factor/gap metric robust stability margin $b(P, C)$, `marg`, of the multivariable feedback loop consisting of `C` in negative feedback with `P`. The normalized coprime factor $b(P, C)$ is defined as

$$b(P,C) = \left\| \begin{bmatrix} I \\ C \end{bmatrix} (I - PC)^{-1} \begin{bmatrix} P & I \end{bmatrix} \right\|_{\infty}^{-1}.$$

`C` should only be the compensator in the feedback path, such as the *1-dof* architecture shown below (on the right). If the compensator has *2-dof* architecture shown below (on the left), you must eliminate the reference channels before calling `ncfmargin`. `freq` is the frequency associated with the upper bound on `marg`.



2-dof architecture 1-dof architecture

The normalized coprime factor robust stability margin lies between 0 and 1 and is used as an indication of robustness to unstructured perturbations. Values of `marg` greater than 0.3 generally indicate good robustness margins.

`[marg,freq] = ncfmargin(P,C,tol)` calculates the normalized coprime factor/gap metric robust stability of the multivariable feedback loop consisting of `C` in negative feedback with `P`. `tol` specifies a relative accuracy for calculating the normalized coprime factor metric and must be between 10^{-5} and 10^{-2}. `tol=0.001` is the default value.

Example

Consider the plant model `4/(s-0.001)` an unstable first order, and two constant gain controllers, `k1 = 1` and `k2 = 10`. Both controllers stabilize the closed-loop system

```
x = tf(4,[1 0.001]);
clp1 = feedback(x,1)
```

The transfer function `clp1` is shown as is `clp2`.

```
     4
---------
s + 4.001

clp2 = feedback(x,10)

Transfer function:
  4
------
s + 40
```

The closed-loop system with controller `k1`, `clp1`, has a normalized coprime factor robust stability margin of `0.71` that is achieved at infinite frequency. This indicates that the closed-loop system is very robust to unstructured perturbations. The closed-loop system with controller `k2`, `clp2`, has a normalized coprime factor robust stability margin of `0.10`. This indicates that the closed-loop system is not robust to unstructured perturbations.

```
[marg1,freq1] = ncfmargin(x,1)
marg1 =
    0.7071
freq1 =
   Inf
[marg2,freq2] = ncfmargin(x,10)
marg2 =
    0.0995
freq2 =
   Inf
```

Construct an uncertain system, `xu`, by adding an 11% unmodeled dynamics to the nominal system `x`. Calculate the robust stability of the closed-loop system with the feedback gain 1 and 10.

```
xu = x + ultidyn('uncstruc',[1 1],'Bound',0.11);
[stabmarg1, du1, report1] = robuststab(feedback(xu,1));
disp(report1{1})
Uncertain System is robustly stable to modeled uncertainty.
 -- It can tolerate up to 909% of modeled uncertainty.
 -- A destabilizing combination of 909% the modeled uncertainty
exists, causing an instability at 165 rad/s.

[stabmarg10, du10, report10] = robuststab(feedback(xu,10));
disp(report10{1})
Uncertain System is NOT robustly stable to modeled uncertainty.
 -- It can tolerate up to 90.9% of modeled uncertainty.
 -- A destabilizing combination of 90.9% the modeled uncertainty
exists, causing an instability at 1.64e+003 rad/s.
```

The closed-loop system with `K=1` is robustly stable in the presence of the unmodeled dynamics based on the robust stability analysis. In fact, the closed-loop system with `K=1` can tolerate 909% (or 9.09*11%) of the unmodeled LTI dynamics, whereas the closed-loop system is not robustly stable with a constant gain of 10 controller. The closed-loop system with `K=10` implemented can only tolerate 90.9% (or.909*11%) of the unmodeled LTI dynamics.

Algorithm

The computation of the gap amounts to solving 2-block H_∞ problems, Georgiou, Smith, 1988. The particular method used here for solving the H_∞ problems is based on Green *et al.*, 1990. The computation of the `nugap` uses the method of Vinnicombe, 1993.

References

- McFarlane, D.C. and K. Glover, Robust Controller Design using Normalised Coprime Factor Plant Descriptions, Lecture Notes in Control and Information Sciences, Springer Verlag, Vol. 138, 1989.
- McFarlane, D.C., and K. Glover, "A Loop Shaping Design Procedure using Synthesis," IEEE Transactions on Automatic Control, Vol. 37, No. 6, 1992, pp. 759-769.
- Vinnicombe, G., "Measuring Robustness of Feedback Systems," Ph.D. Dissertation, Department of Engineering, University of Cambridge, 1993.

See Also

`loopmargin`	Performs a comprehensive analysis of feedback loop
`gapmetric`	Computes the gap and the Vinnicombe gap metric
`norm`	Computes the norm of a system

`wcmargin`	Calculate worst-case margins for feedback loop

Purpose

Balanced model truncation for normalized coprime factors

Syntax

```
GRED = ncfmr(G)
GRED = ncfmr(G,order)
[GRED,redinfo] = ncfmr(G,key1,value1,...)
[GRED,redinfo] = ncfmr(G,order,key1,value1,...)
```

Description

`ncfmr` returns a reduced order model GRED formed by a set of balanced normalized coprime factors and a struct array redinfo containing the left and right coprime factors of G and their coprime Hankel singular values.

Hankel singular values of coprime factors of such a stable system indicate the respective "state energy" of the system. Hence, reduced order can be directly determined by examining the system Hankel SV's.

With only one input argument `G`, the function will show a Hankel singular value plot of the original model and prompt for model order number to reduce.

The *left and right normalized coprime factors* are defined as [1]

- *Left Coprime Factorization*: $G = M_l^{-1}(s)N_l(s)$
- *Right Coprime Factorization*: $G = N_r(s)M_r^{-1}(s)$

where there exist stable $U_r(s)$, $V_r(s)$, $U_l(s)$, and $V_l(s)$ such that

$$U_r N_r + V_r M_r = I$$

$$N_l U_l + M_l V_l = I$$

The left/right coprime factors are stable, hence implies M_r(s) should contain as RHP-zeros all the RHP-poles of G(s). The comprimeness also implies that there should be no common RHP-zeros in N_r(s) and M_r(s), i.e., when forming $G = N_r(s)M_r^{-1}(s)$, there should be no pole-zero cancellations.

This table describes input arguments for `ncmfr`.

Argument	Description
`G`	LTI model to be reduced (without any other inputs will plot its Hankel singular values and prompt for reduced order)
`ORDER`	(Optional) an integer for the desired order of the reduced model, or optionally a vector packed with desired orders for batch runs

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y`, or a vector of integers. By default, all the anti-stable part of a system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system. `ncfmr` method allows the original model to have jω-axis singularities.

'*MaxError*' can be specified in the same fashion as an alternative for 'ORDER'. In this case, reduced order will be determined when the sum of the tails of the Hankel sv's reaches the '*MaxError*'

Argument	Value	Description
'*MaxError*'	A real number or a vector of different errors	Reduce to achieve H∞ error. When present, '*MaxError*' overides `ORDER` input.
'*Display*'	'*on*' or '*off*'	Display Hankel singular plots (default '`off`').
'*Order*'	integer, vector or cell array	Order of reduced model. Use only if not specified as 2nd argument.

Weights on the original model input and/or output can make the model reduction algorithm focus on some frequency range of interests. But weights have to be stable, minimum phase and invertible.

This table describes output arguments.

Argument	Description
`GRED`	LTI reduced order model, that becomes multi-dimensional array when input is a serial of different model order array.
`REDINFO`	A STRUCT array with 3 fields: • `REDINFO.GL (left coprime factor)` • `REDINFO.GR (right coprime factor)` • `REDINFO.hsv (Hankel singular values)`

`G` can be stable or unstable, continuous or discrete.

Algorithm

Given a state space *(A,B,C,D)* of a system and *k,* the desired reduced order, the following steps will produce a similarity transformation to truncate the original state space system to the k^{th} order reduced model.

1. Find the normalized coprime factors of *G* by solving Hamiltonian described in [1].

$$G_l = \begin{bmatrix} N_l & M_l \end{bmatrix}$$

$$G_r = \begin{bmatrix} N_r \\ M_r \end{bmatrix}$$

2. Perform k^{th} order square root balanced model truncation on G_l (or G_r) [2].
3. The reduced model `GRED` is [2]:

$$\left[\begin{array}{c|c} \hat{A} & \hat{B} \\ \hline \hat{C} & \hat{D} \end{array}\right] = \left[\begin{array}{c|c} A_c - B_m C_l & B_n - B_m D_l \\ \hline C_l & D_l \end{array}\right]$$

where

$$N_l := (A_c, B_n, C_c, D_n)$$

$$M_l := (A_c, B_m, C_c, D_m)$$

$$C_l = (D_m)^{-1} C_c$$

$$D_l = (D_m)^{-1} D_n$$

Example

Given a continuous or discrete, stable or unstable system, `G`, the following commands can get a set of reduced order models based on your selections:

```
rand('state',1234); randn('state',5678);
G = rss(30,5,4); G.d = zeros(5,4);
[g1, redinfo1] = ncfmr(G); % display Hankel SV plot
                           % and prompt for order (try 15:20)
[g2, redinfo2] = ncfmr(G,20);
[g3, redinfo3] = ncfmr(G,[10:2:18]);
[g4, redinfo4] = ncfmr(G,'MaxError',[0.01, 0.05]);
for i = 1:4
    figure(i); eval(['sigma(G,g' num2str(i) ');']);
end
```

Reference

[1] M. Vidyasagar. *Control System Synthesis - A Factorization Approach.* London: The MIT Press, 1985.

[2] M. G. Safonov and R. Y. Chiang, "A Schur Method for Balanced Model Reduction," *IEEE Trans. on Automat. Contr.*, vol. AC-2, no. 7, July 1989, pp. 729-733.

See Also

`reduce`	Top level model reduction routines
`balancmr`	Balanced truncation via square-root method
`schurmr`	Balanced truncation via Schur method
`bstmr`	Balanced stochastic truncation via Schur method
`hankelmr`	Hankel minimum degree approximation
`hankelsv`	Hankel singular value

Purpose

Loop shaping design using Glover-McFarlane method

Syntax

```
[K,CL,GAM,INFO]=ncfsyn(G)
[K,CL,GAM,INFO]=ncfsyn(G,W1)
[K,CL,GAM,INFO]=ncfsyn(G,W1,W2)
[K,CL,GAM,INFO]=ncfsyn(G,W1,W2,'ref')
```

Description

`ncfsyn` is a method for designing controllers that uses a combination of loop shaping and robust stabilization as proposed in McFarlane and Glover [1]-[2]. The first step is for you to select a pre- and post-compensator W_1 and W_2, so that the gain of the 'shaped plant' $G_s := W_2 G W_1$ is sufficiently high at frequencies where good disturbance attenuation is required and is sufficiently low at frequencies where good robust stability is required. The second step is to use `ncfsyn` to compute an optimal *positive* feedback controllers `K`.

The optimal `Ks` has the property that the `sigma` plot of the shaped loop

```
Ls=W2*G*W1*Ks
```

matches the target loopshape G_s optimally, roughly to within plus or minus `20*log10(GAM)` decibels. The number margin `GAM=1/ncfmargin(Gs,K)` and is always greater than 1. `GAM` gives a good indication of robustness of stability to a wide class of unstructured plant variations, with values in the range 1<`GAM`<3 corresponding to satisfactory stability margins for most typical control system designs.

`[K,CL,GAM,INFO]=ncfsyn(G,W1,W2,'ref')` computes the Glover-McFarlane H_∞ normalized co-prime factor loop-shaping controller `K`, with a reference command, for `lti` plant `G`, weights `W1` and `W2` if the `'ref'` option is included. The closed-loop system returned, `CL`, represents the transfer matrix from the reference and disturbance to the feedback error and output of `W1`.

Algorithm

`K=W2*Ks*W1`, where `Ks` =K_∞ is an optimal H_∞ controller that simultaneously minimizes the two H_∞ cost functions

$$\gamma := \min_{K} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I - G_s K)^{-1} [G_s, I] \right\|_\infty$$

$$\gamma := \min_{K} \left\| \begin{bmatrix} I \\ G_s \end{bmatrix} (I - KG_s)^{-1} [K, I] \right\|_\infty$$

Roughly speaking, this means for most plants that

$$\sigma(W_2 G W_1 K_\infty), \text{ db} = \sigma(W_2 G W_1), \text{ db} \pm \gamma, \text{ db}$$

$$\sigma(K_\infty W_2 G W_1), \text{ db} = \sigma(W_2 G W_1), \text{ db} \pm \gamma, \text{ db},$$

so you can use the weights W_1 and W_2 for loopshaping. For a more precise bounds on loopshaping accuracy, see Theorem 16.12 of Zhou and Glover [1].

Theory ensures that if $G_s=NM^{-1}$is a normalized coprime factorization (NCF) of the weighted plant model G_s satisfying

$$G_s=N(jw)^*N(jw) + M(jw)^*M(jw) = I,$$

then the control system will remain robustly stable for any perturbation $\tilde{G}_s$ to the weighted plant model G_s that can be written

$$\tilde{G}_s = (N + \Delta_1)(M + \Delta_2)^{-1}$$

for some stable pair Δ_1, Δ_2 satisfying

$$\left\| \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \right\|_\infty < \text{MARG:=1/GAM} .$$

The closed-loop H_∞-norm objective has the standard signal gain interpretation. Finally it can be shown that the controller, K_∞, does not substantially affect the loop shape in frequencies where the gain of W_2GW_1 is either high or low, and will guarantee satisfactory stability margins in the frequency region of gain cross-over. In the regulator set-up, the final controller to be implemented is $K=W_1K_\infty W_2$.

Input arguments

`G` LTI plant to be controlled

`W1,W2` Stable minimum-phase LTI weights, either SISO or MIMO. Default is $W_1=I$, $W_2=I$

`` `ref' `` Reference input to controller. Default is no reference input is included.

Output arguments

`K` LTI controller `K= W1*Ks*W2`

`CL` $\begin{bmatrix} I \\ K_\infty \end{bmatrix}(I - W_2 G W_1 K_\infty)^{-1}[W_2 G W_1, I]$, LTI H_∞ optimal closed loop

`GAM` H_∞ optimal cost $\gamma = \dfrac{1}{b(W_2 G W_1, K_\infty)}$ = `hinfnorm(CL)` ≥ 1

`INFO` structure array containing additional information

Additional output `INFO` fields:

`INFO.emax`	`nugap` robustness `emax=1/GAM=ncfmargin(Gs,-Ks)`$=b(W_2 G W_1, K_\infty)$
`INFO.Gs`	`'shaped plant' Gs=W2*G*W1`
`INFO.Ks`	`Ks = K = NCFSYN(Gs) = NCFSYN(W2*G*W1)`

`[MARG,FREQ] = ncfmargin(G,K,TOL)` calculates the normalized coprime factor/gap metric robust stability margin assuming *negative* feedback.

$$\texttt{MARG}=b(G,-K) = 1/\left\| \begin{bmatrix} I \\ -K \end{bmatrix}(I+GK)^{-1}[G\ , I] \right\|_\infty .$$

where `G` and `K` are LTI plant and controller, and `TOL` (default=.001) is the tolerance used to compute the H_∞ norm. `FREQ` is the peak frequency — i.e., the frequency at which the infinity norm is reached to within `TOL`.

Algorithm

See the McFarlane and Glover [1]-[2] for details.

Example

The following code shows how `ncfsyn` can be used for loop-shaping. The achieved loop `G*K` has a `sigma plot` is equal to that of the target loop `G*W1` to within plus or minus `20*log10(GAM)` decibels.

```
s=zpk('s');
G=(s-1)/(s+1)^2;
W1=0.5/s;
[K,CL,GAM]=ncfsyn(G,W1);
sigma(G*K,'r',G*W1,'r-.',G*W1*GAM,'k-.',G*W1/GAM,'k-.')
```

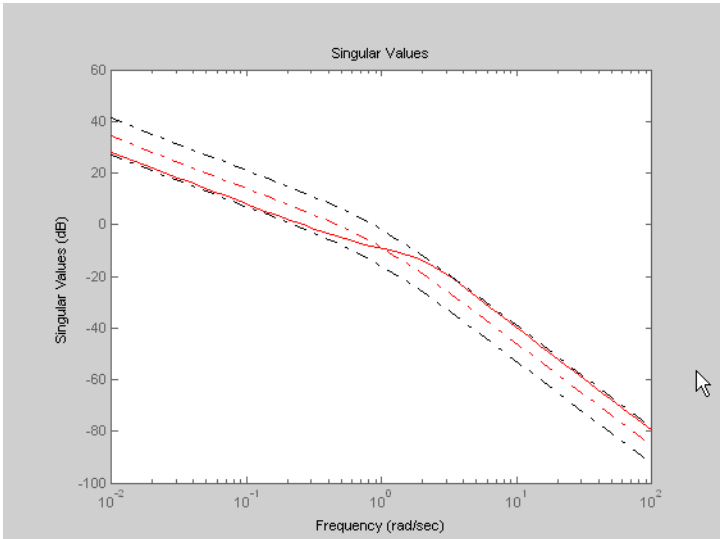


Figure 5-15: Achieved loop G*K and shaped loop Gs, ±20log(GAM) db

Reference

[1] McFarlane, D.C., and K. Glover, Robust Controller Design using Normalised Coprime Factor Plant Descriptions, Springer Verlag, *Lecture Notes in Control and Information Sciences,* vol. 138, 1989.

[2] McFarlane, D.C., and K. Glover, "A Loop Shaping Design Procedure using Synthesis," *IEEE Transactions on Automatic Control,* vol. 37, no. 6, pp. 759–769, June 1992.

[3] Vinnicombe, G., "Measuring Robustness of Feedback Systems," PhD dissertation, Department of Engineering, University of Cambridge, 1993.

[4] Zhou, K., and J.C. Doyle, Essentials of Robust Control. NY: Prentice-Hall, 1998.

See Also

`gapmetric`	Computes the gap and the Vinnicombe gap metric
`hinfsyn`	H_∞ controller synthesis
`loopsyn`	H_∞ - loop shaping controller synthesis
`ncfmargin`	Normalized coprime stability margin of the plant-controller feedback loop

newlmi

Purpose

Attach identifying tag to LMIs

Syntax

```
tag = newlmi
```

Description

`newlmi` adds a new LMI to the LMI system currently described and returns an identifier tag for this LMI. This identifier can be used in `lmiterm`, `showlmi`, or `dellmi` commands to refer to the newly declared LMI. Tagging LMIs is *optional* and only meant to facilitate code development and readability.

Identifiers can be given mnemonic names to help keep track of the various LMIs. Their value is simply the ranking of each LMI in the system (in the order of declaration). They prove useful when some LMIs are deleted from the LMI system. In such cases, the identifiers are the safest means of referring to the remaining LMIs.

See Also

`setlmis`	Initialize the description of an LMI system
`lmivar`	Specify the matrix variables in an LMI problem
`lmiterm`	Specify the term content of LMIs
`getlmis`	Get the internal description of an LMI system
`lmiedit`	Specify or display systems of LMIs as MATLAB® expressions
`dellmi`	Remove an LMI from a given system of LMIs

Purpose

Converts a normalized value of a `ureal` atom into its corresponding actual value

Syntax

```
avalue = normalizedactual2(A,NV)
```

Description

`Converts a normalized value NV of a ureal atom to its corresponding actual (unnormalized) value.`

If `NV` is an array of values, then `avalue` will be an array of the same dimension.

Example

Create uncertain real parameters with a range that is symmetric about the nominal value, where each endpoint is 1 unit from the nominal. Points that lie inside the range are less than 1 unit from the nominal, while points that lie outside the range are greater than 1 unit from the nominal.

```
a = ureal('a',3,'range',[1 5]);
actual2normalized(a,[1 3 5])
ans =
   -1.0000   -0.0000    1.0000
normalized2actual(a,[-1 1])
ans =
   1.0000    5.0000
normalized2actual(a,[-1.5 1.5])
ans =
   0.0000    6.0000
```

See Also

`actual2normalized`	Converts uncertain atom nominal value to actual value
`robuststab`	Calculates robust stability margin
`robustperf`	Calculates robust performance margin

Purpose

Assess robust stability of polytopic or parameter-dependent system

Syntax

```
[tau,Q0,Q1,...] = pdlstab(pds,options)
```

Description

`pdlstab` uses parameter-dependent Lyapunov functions to establish the stability of uncertain state-space models over some parameter range or polytope of systems. Only sufficient conditions for the existence of such Lyapunov functions are available in general. Nevertheless, the resulting robust stability tests are always less conservative than quadratic stability tests when the parameters are either time-invariant or slowly varying.

For an affine parameter-dependent system

$$E(p)\dot{x} = A(p)x + B(p)u$$

$$y = C(p)x + D(p)u$$

with $p = (p_1, \ldots, p_n) \in \mathbf{R}^n$, `pdlstab` seeks a Lyapunov function of the form

$$V(x, p) = x^T Q(p)^{-1} x,\ Q(p) = Q_0 + p_1 Q_1 + \ldots p_n Q_n$$

such that $dV(x, p)/dt < 0$ along all admissible parameter trajectories. The system description `pds` is specified with `psys` and contains information about the range of values and rate of variation of each parameter p_i.

For a *time-invariant* polytopic system

$$E\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

with

$$\begin{pmatrix} A + jE & B \\ C & D \end{pmatrix} = \sum_{i=1}^{n} \alpha_i \begin{pmatrix} A + jE_i & B_i \\ C_i & D_i \end{pmatrix}, \qquad \alpha_i \geq 0, \qquad \sum_{i=1}^{n} \alpha_i = 1, \tag{5-19}$$

`pdlstab` seeks a Lyapunov function of the form

$$V(x, \alpha) = x^T Q(\alpha)^{-1} x,\ Q(\alpha) = \alpha_1 Q_1 + \ldots + \alpha_n Q_n$$

such that $dV(x, \alpha)/dt < 0$ for all polytopic decompositions (9-19).

Several options and control parameters are accessible through the optional argument `options`:

- Setting `options(1)=0` tests robust stability (default)
- When `options(2)=0`, `pdlstab` uses simplified sufficient conditions for faster running times. Set `options(2)=1` to use the least conservative conditions

Remark

For affine parameter-dependent systems with *time-invariant* parameters, there is equivalence between the robust stability of

$$E(p)\dot{x} = A(p)x \qquad \textbf{(5-20)}$$

and that of the dual system

$$E(p)^T\dot{z} = A(p)^T z \qquad \textbf{(5-21)}$$

However, the second system may admit an affine parameter-dependent Lyapunov function while the first does not.

In such case, `pdlstab` automatically restarts and tests stability on the dual system (9-21) when it fails on (9-20).

See Also

`quadstab`	Quadratic stability of polytopic or affine parameter-dependent systems

pdsimul

Purpose

Time response of parameter-dependent system along given parameter trajectory

Syntax

```
pdsimul(pds,'traj',tf,'ut',xi,options)
[t,x,y] = pdsimul(pds,pv,'traj',tf,'ut',xi,options)
```

Description

`pdsimul` simulates the time response of an affine parameter-dependent system

$$E(p)\dot{x} = A(p)x + B(p)u$$

$$y = C(p)x + D(p)u$$

along a parameter trajectory $p(t)$ and for an input signal $u(t)$. The parameter trajectory and input signals are specified by two time functions `p=traj(t)` and `u=ut(t)`. If `'ut'` is omitted, the response to a step input is computed by default.

The affine system `pds` is specified with `psys`. The function `pdsimul` also accepts the polytopic representation of such systems as returned by `aff2pol(pds)` or `hinfgs`. The final time and initial state vector can be reset through `tf` and `xi` (their respective default values are 5 seconds and 0). Finally, `options` gives access to the parameters controlling the ODE integration (type `help gear` for details).

When invoked without output arguments, `pdsimul` plots the output trajectories $y(t)$. Otherwise, it returns the vector of integration time points `t` as well as the state and output trajectories `x,y`.

See Also

`psys`	Specification of uncertain state-space models
`pvec`	Quantification of uncertainty on physical parameters

Purpose

Compute polytopic coordinates with respect to box corners

Syntax

```
vertx = polydec(PV)
[C,vertx] = polydec(PV,P)
```

Description

`vertx = polydec(PV)` takes an uncertain parameter vector `PV` taking values ranging in a box, and returns the corners or vertices of the box as columns of the matrix `vertx`.

`[C,vertx] = polydec(PV,P)` takes an uncertain parameter vector `PV` and a value `P` of the parameter vector `PV`, and returns the convex decomposition `C` of `P` over the set `VERTX` of box corners:

```
P = c1*VERTX(:,1) + ... + cn*VERTX(:,n)
cj >=0 ,              c1 + ... + cn = 1
```

The list `vertx` of corners can be obtained directly by typing

```
vertx = polydec(PV)
```

See Also

`pvec`	Quantification of uncertainty on physical parameters
`pvinfo`	Describe a parameter vector specified with `pvec`
`aff2pol`	Convert affine parameter-dependent models to polytopic ones
`hinfgs`	Synthesis of gain-scheduled H_∞ controllers

popov

Purpose

Perform Popov robust stability test

Syntax

```
[t,P,S,N] = popov(sys,delta,flag)
```

Description

`popov` uses the Popov criterion to test the robust stability of dynamical systems with possibly nonlinear and/or time-varying uncertainty. The uncertain system must be described as the interconnection of a nominal LTI system `sys` and some uncertainty `delta`.

The command

```
[t,P,S,N] = popov(sys,delta)
```

tests the robust stability of this interconnection. Robust stability is guaranteed if `t` < 0. Then `P` determines the quadratic part x^TPx of the Lyapunov function and `D` and `S` are the Popov multipliers.

If the uncertainty `delta` contains real parameter blocks, the conservatism of the Popov criterion can be reduced by first performing a simple loop transformation. To use this refined test, call `popov` with the syntax

```
[t,P,S,N] = popov(sys,delta,1)
```

See Also

`quadstab`	Quadratic stability of polytopic or affine parameter-dependent systems
`pdlstab`	Robust stability of polytopic or affine parameter-dependent systems (P-system)

Purpose

Inquire about polytopic or parameter-dependent systems created with psys

Syntax

```
psinfo(ps)
[type,k,ns,ni,no] = psinfo(ps)
pv = psinfo(ps,'par')
sk = psinfo(ps,'sys',k)
sys = psinfo(ps,'eval',p)
```

Description

psinfo is a multi-usage function for queries about a polytopic or parameter-dependent system ps created with psys. It performs the following operations depending on the calling sequence:

- psinfo(ps) displays the type of system (affine or polytopic); the number k of SYSTEM matrices involved in its definition; and the numbers of ns, ni, no of states, inputs, and outputs of the system. This information can be optionally stored in MATLAB® variables by providing output arguments.
- pv = psinfo(ps,'par') returns the parameter vector description (for parameter-dependent systems only).
- sk = psinfo(ps,'sys',k) returns the k-th SYSTEM matrix involved in the definition of ps. The ranking k is relative to the list of systems syslist used in psys.
- sys = psinfo(ps,'eval',p) instantiates the system for a given vector p of parameter values or polytopic coordinates.

  For *affine parameter-dependent* systems defined by the SYSTEM matrices S_0, $S_1, \ldots, S_n$, the entries of p should be real parameter values $p_1, \ldots, p_n$ and the result is the LTI system of SYSTEM matrix

  $$S(p) = S_0 + p_1 S_1 + \ldots + p_n S_n$$

  For *polytopic* systems with SYSTEM matrix ranging in

  $$\mathrm{Co}\{S_1, \ldots, S_n\},$$

  the entries of p should be polytopic coordinates $p_1, \ldots, p_n$ satisfying $p_j \geq 0$ and the result is the interpolated LTI system of SYSTEM matrix

  $$S = \frac{p_1 S_1 + \ldots + p_n S_n}{p_1 + \ldots + p_n}$$

See Also

psys — Specification of uncertain state-space models

Purpose

Specify polytopic or parameter-dependent linear systems

Syntax

```
pols = psys(syslist)
affs = psys(pv,syslist)
```

Description

`psys` specifies state-space models where the state-space matrices can be uncertain, time-varying, or parameter-dependent.

Two types of uncertain state-space models can be manipulated in the LMI Control Toolbox:

- *Polytopic* systems

  $$E(t)\dot{x} = A(t)x + B(t)u$$

  $$y = C(t)x + D(t)u$$

  whose `SYSTEM` matrix takes values in a fixed polytope:

  $$\underbrace{\begin{bmatrix} A(t)+jE(t) & B(t) \\ C(t) & D(t) \end{bmatrix}}_{S(t)} \in \mathrm{Co}\left\{ \underbrace{\begin{bmatrix} A_1+jE_1 & B_1 \\ C_1 & D_1 \end{bmatrix}}_{S_1}, \ldots, \underbrace{\begin{bmatrix} A_k+jE_k & B_k \\ C_k & D_k \end{bmatrix}}_{S_k} \right\}$$

  where $S_1, \ldots, S_k$ are given "vertex" systems and

  $$\mathrm{Co}\{S_1, \ldots, S_k\} = \left\{ \sum_{i=1}^{k} \alpha_i S_i \ : \ \alpha_i \geq 0, \ \sum_{i=1}^{k} \alpha_i = 1 \right\}$$

  denotes the convex hull of $S_1, \ldots, S_k$ (polytope of matrices with vertices $S_1, \ldots, S_k$)

- *Affine parameter-dependent* systems

  $$E(p)\dot{x} = A(p)x + B(p)u$$

  $$y = C(p)x + D(p)u$$

  where $A(\cdot)$; $B(\cdot), \ldots, E(\cdot)$ are fixed affine functions of some vector $p = (p_1, \ldots, p_n)$ of real parameters, i.e.,

$$\underbrace{\begin{bmatrix} A(p)+jE(p) & B(p) \\ C(p) & D(p) \end{bmatrix}}_{S(p)} =$$

$$\underbrace{\begin{bmatrix} A_0+jE_0 & B_0 \\ C_0 & D_0 \end{bmatrix}}_{S_0} + p_1 \underbrace{\begin{bmatrix} A_1+jE_1 & B_1 \\ C_1 & D_1 \end{bmatrix}}_{S_1} + \ldots + p_n \underbrace{\begin{bmatrix} A_n+jE_n & B_n \\ C_n & D_n \end{bmatrix}}_{S_n}$$

where S_0, S_1, . . ., S_n are given `SYSTEM` matrices. The parameters p_i can be time-varying or constant but uncertain.

Both types of models are specified with the function `psys`. The argument `syslist` lists the `SYSTEM` matrices S_i characterizing the polytopic value set or parameter dependence. In addition, the description `pv` of the parameter vector (range of values and rate of variation) is required for affine parameter-dependent models (see `pvec` for details). Thus, a polytopic model with vertex systems S_1, . . ., S_4 is created by

```
pols = psys([s1,s2,s3,s4])
```

while an affine parameter-dependent model with 4 real parameters is defined by

```
affs = psys(pv,[s0,s1,s2,s3,s4])
```

The output is a structured matrix storing all the relevant information.

See Also

`psinfo`	Inquire about polytopic or parameter-dependent systems created with `psys`
`pvec`	Quantification of uncertainty on physical parameters
`aff2pol`	Convert affine parameter-dependent models to polytopic ones

pvec

Purpose

Specify range and rate of variation of uncertain or time-varying parameters

Syntax

```
pv = pvec('box',range,rates)
pv = pvec('pol',vertices)
```

Description

`pvec` is used in conjunction with psys to specify parameter-dependent systems. Such systems are parametrized by a vector $p = (p_1, \ldots, p_n)$ of uncertain or time-varying real parameters p_i. The function `pvec` defines the range of values and the rates of variation of these parameters.

The type `'box'` corresponds to independent parameters ranging in intervals

$$\underline{p}_j \le p_j \le \bar{p}_j$$

The parameter vector p then takes values in a hyperrectangle of $\mathbf{R}^n$ called the parameter box. The second argument range is an n-by-2 matrix that stacks up the extremal values $\underline{p}_j$ and $\bar{p}_j$ of each p_j. If the third argument `rates` is omitted, all parameters are assumed time-invariant. Otherwise, `rates` is also an n-by-2 matrix and its j-th row specifies lower and upper bounds $\underline{v}_j$ and $\bar{v}_j$ on $\frac{dp_j}{dt}$:

$$\underline{v}_j \le \frac{dp_j}{dt} \le \bar{v}_j$$

Set $\underline{v}_j =$ `Inf` and $\bar{v}_j =$ `Inf` if $p_j(t)$ can vary arbitrarily fast or discontinuously.

The type `'pol'` corresponds to parameter vectors p ranging in a polytope of the parameter space $\mathbf{R}^n$. This polytope is defined by a set of vertices $V_1, \ldots, V_n$ corresponding to "extremal" values of the vector p. Such parameter vectors are declared by the command

```
pv = pvec('pol',[v1,v2, . . ., vn])
```

where the second argument is the concatenation of the vectors `v1,...,vn`.

The output argument `pv` is a structured matrix storing the parameter vector description. Use `pvinfo` to read the contents of `pv`.

Example

Consider a problem with two time-invariant parameters

$p_1 \in [-1, 2], p_2 \in [20, 50]$

The corresponding parameter vector $p = (p_1, p_2)$ is specified by

```
pv = pvec('box',[-1 2;20 50])
```

Alternatively, this vector can be regarded as taking values in the rectangle drawn in Figure 9.2. The four corners of this rectangle are the four vectors

$$v_1 = \begin{pmatrix} -1 \\ 20 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -1 \\ 50 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 2 \\ 20 \end{pmatrix}, \quad v_4 = \begin{pmatrix} 2 \\ 50 \end{pmatrix}$$

Hence, you could also specify p by

```
pv = pvec('pol',[v1,v2,v3,v4])
```

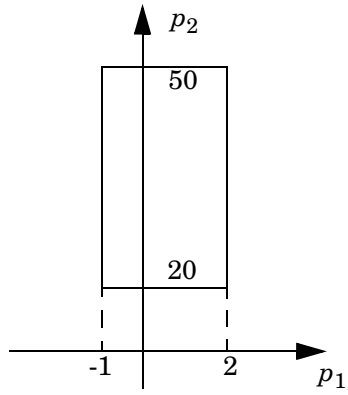


Figure 5-16: Parameter box

See Also

`pvinfo`	Describe a parameter vector specified with `pvec`
`psys`	Specification of uncertain state-space models

pvinfo

Purpose

Describe parameter vector specified with `pvec`

Syntax

```
[typ,k,nv] = pvinfo(pv)
[pmin,pmax,dpmin,dpmax] = pvinfo(pv,'par',j)
vj = pvinfo(pv,'par',j)
p = pvinfo(pv,'eval',c)
```

Description

`pvec` retrieves information about a vector $p = (p_1, \ldots, p_n)$ of real parameters declared with `pvec` and stored in `pv`. The command `pvinfo(pv)` displays the type of parameter vector (`'box'` or `'pol'`), the number n of scalar parameters, and for the type `'pol'`, the number of vertices used to specify the parameter range.

For the type `'box'`:

```
[pmin,pmax,dpmin,dpmax] = pvinfo(pv,'par',j)
```

returns the bounds on the value and rate of variations of the j-th real parameter p_j. Specifically,

$$\texttt{pmin} \le p_j(t) \le \texttt{pmax} \ , \ \texttt{dpmin} \le \frac{dp_j}{dt} \le \texttt{dpmax}$$

For the type `'pol'`:

```
pvinfo(pv,'par',j)
```

returns the j-th vertex of the polytope of $\mathbf{R}^n$ in which p ranges, while

```
pvinfo(pv,'eval',c)
```

returns the value of the parameter vector p given its barycentric coordinates `c` with respect to the polytope vertices $(V_1, \ldots, V_k)$. The vector `c` must be of length k and have nonnegative entries. The corresponding value of p is then given by

$$p = \frac{\sum_{i=1}^{k} c_i V_i}{\sum_{i=1}^{k} c_i}$$

See Also

`pvec`	Quantification of uncertainty on physical parameters
`psys`	Specification of uncertain state-space models

Purpose

Compute quadratic H_∞ performance of polytopic or parameter-dependent system

Syntax

```
[perf,P] = quadperf(ps,g,options)
```

Description

The RMS gain of the time-varying system

$$E(t)\dot{x} = A(t)x + B(t)u, \qquad y = C(t)x + D(t)u \tag{5-22}$$

is the smallest $\gamma > 0$ such that

$$\|y\|_{L_2} \le \gamma \|u\|_{L_2} \tag{5-23}$$

for all input $u(t)$ with bounded energy. A sufficient condition for (9-23) is the existence of a quadratic Lyapunov function

$$V(x) = x^T P x, \qquad P > 0$$

such that

$$\forall u \in L_2, \quad \frac{dV}{dt} + y^T y - \gamma^2 u^T u < 0$$

Minimizing γ over such quadratic Lyapunov functions yields the quadratic H_∞ performance, an upper bound on the true RMS gain.

The command

```
[perf,P] = quadperf(ps)
```

computes the quadratic H_∞ performance `perf` when (9-22) is a polytopic or affine parameter-dependent system `ps` (see `psys`). The Lyapunov matrix P yielding the performance `perf` is returned in `P`.

The optional input `options` gives access to the following task and control parameters:

- If `options(1)=1`, `perf` is the largest portion of the parameter box where the quadratic RMS gain remains smaller than the positive value `g` (for affine parameter-dependent systems only). The default value is 0
- If `options(2)=1`, `quadperf` uses the least conservative quadratic performance test. The default is `options(2)=0` (fast mode)

- `options(3)` is a user-specified upper bound on the condition number of P (the default is 109).

See Also

`quadstab`	Quadratic stability of polytopic or affine parameter-dependent systems
`psys`	Specification of uncertain state-space models

Purpose

Quadratic stability of polytopic or affine parameter-dependent systems

Syntax

```
[tau,P] = quadstab(ps,options)
```

Description

For affine parameter-dependent systems

$$E(p)\dot{x} = A(p)x, \qquad p(t) = (p_1(t), \ldots, p_n(t))$$

or polytopic systems

$$E(t)\dot{x} = A(t)x, \qquad (A, E) \in \mathrm{Co}\{(A_1, E_1), \ldots, (A_n, E_n)\},$$

`quadstab` seeks a fixed Lyapunov function $V(x) = x^T P x$ with $P > 0$ that establishes quadratic stability. The affine or polytopic model is described by `ps` (see `psys`).

The task performed by `quadstab` is selected by `options(1)`:

- if `options(1)=0` (default), `quadstab` assesses quadratic stability by solving the LMI problem

  Minimize τ over $Q = Q^T$ such that

  $A^T Q E + E Q A^T < \tau I$ for all admissible values of (A, E)

  $$Q > I$$

  The global minimum of this problem is returned in `tau` and the system is quadratically stable if `tau` < 0
- if `options(1)=1`, `quadstab` computes the largest portion of the specified parameter range where quadratic stability holds (only available for affine models). Specifically, if each parameter p_i varies in the interval

  $p_i \in [p_{i0} - \delta_i, p_{i0} + \delta_i]$,

  `quadstab` computes the largest $\theta > 0$ such that quadratic stability holds over the parameter box

  $p_i \in [p_{i0} - \theta\delta_i, p_{i0} + \theta\delta_i]$

  This "quadratic stability margin" is returned in `tau` and `ps` is quadratically stable if `tau` Š 1.

Given the solution Q_{opt} of the LMI optimization, the Lyapunov matrix P is given by $P = Q_{\mathrm{opt}}^{-1}$. This matrix is returned in `P`.

Other control parameters can be accessed through `options(2)` and `options(3)`:

- if `options(2)=0` (default), `quadstab` runs in fast mode, using the least expensive sufficient conditions. Set `options(2)=1` to use the least conservative conditions
- `options(3)` is a bound on the condition number of the Lyapunov matrix P. The default is 10^9.

See Also

`pdlstab`	Robust stability of polytopic or affine parameter-dependent systems (P-system)
`decay`	Quadratic decay rate of polytopic or affine P-systems
`quadperf`	Compute the quadratic H_∞ performance of a polytopic or parameter-dependent system
`psys`	Specification of uncertain state-space models

Purpose

Generate random uncertain `atom` objects

Syntax

```
A = randatom(Type)
A = randatom(Type,sz)
A = randatom
```

Description

`A = randatom(Type)` generates a 1-by-1 `type` uncertain object. Valid values for `Type` include `'ureal'`, `'ultidyn'`, `'ucomplex'`, and `'ucomplexm'`.

`A = randatom(Type,sz)` generates an `sz(1)`-by-`sz(2)` uncertain object. Valid values for `Type` include `'ultidyn'` or `'ucomplexm'`. If `Type` is set to `'ureal'` or `'ucomplex'`, the size variable is ignored and A is a 1-by-1 uncertain object.

`A = randatom`, where `randatom` has no input arguments, results in a 1-by-1 uncertain object. The class is of this object is randomly selected between `'ureal'`,`'ultidyn'` and `'ucomplex'`.

In general, both `rand` and `randn` are used internally. You can control the result of `randatom` by setting seeds for both random number generators before calling the function.

Example

The following statement creates the `ureal` uncertain object `xr`. Note that your display can differ because a random seed is used.

```
xr = randatom('ureal')
Uncertain Real Parameter: Name BMSJA, NominalValue -6.75,
Range [-7.70893  -1.89278]
```

The following statement creates the variable `ultidyn` uncertain object `xlti` with three inputs and four outputs. You will get the results shown below if both the random variable seeds are set to 29.

```
rand('seed',29);
randn('seed',29);
xlti = randatom('ultidyn',[4 3])
Uncertain GainBounded LTI Dynamics: Name OOJGS, 4x3,
Gain Bound = 0.646
```

See Also

`rand`	Generates uniformly distributed random numbers
`randn`	Generates normally distributed random numbers
`randumat`	Creates a random uncertain matrix

`randuss`	Creates a random uncertain system
`ucomplex`	Creates an uncertain complex parameter
`ucomplexm`	Creates an uncertain complex matrix
`ultidyn`	Creates an uncertain linear time-invariant object

Purpose

Generate random uncertain umat objects

Syntax

```
um = randumat(ny,nu)
um = randumat
```

Description

`um = randumat(ny,nu)` generates an uncertain matrix of size `ny-by-nu`. `randumat` randomly selects from uncertain atoms of type `'ureal'`, `'ultidyn'`, and `'ucomplex'`.

`um = randumat` results in a 1-by-1 `umat` uncertain object, including up to four uncertain objects.

Example

The following statement creates the `umat` uncertain object `x1` of size 2-by-3. Note that your result can differ because a random seed is used.

```
x1 = randumat(2,3)
UMAT: 2 Rows, 3 Columns
  ROQAW: complex, nominal = 9.92+4.84i, radius = 0.568, 1
occurrence
  UEPDY: real, nominal = -5.81, variability = [-1.98681
0.133993], 3 occurrences
  VVNHL: complex, nominal = 5.64-6.13i, radius = 1.99, 2
occurrences
```

The following statement creates the `umat` uncertain object `x2` of size 4-by-2 with the seed 91.

```
rand('seed',91); randn('seed',91);
x2 = randumat(4,2)
UMAT: 4 Rows, 2 Columns
  SSAFF: complex, nominal = -0.366+2.81i, radius = 1.76, 3
occurrences
 VDTIH: complex, nominal = -3.03-3i, +/- 27.5%, 2 occurrences
  XOLLJ: real, nominal = 0.0628, range = [-3.73202  4.28174], 1
occurrence
```

See Also

`rand`	Generate uniformly distributed random numbers
`randn`	Generate normally distributed random numbers
`randatom`	Create a random uncertain atom
`randuss`	Create a random uncertain system
`ucomplex`	Creates an uncertain complex parameter

`ultidyn`	Creates an uncertain linear time-invariant object

Purpose

Generate stable, random uss objects

Syntax

```
usys = randuss(n)
usys = randuss(n,p)
usys = randuss(n,p,m)
usys = randuss(n,p,m,Ts)
usys = randuss
```

Description

`usys = randuss(n)` generates an `n`th order single-input/single-output uncertain continuous-time system. `randuss` randomly selects from uncertain atoms of type `'ureal'`, `'ultidyn'`, and `'ucomplex'`.

`usys = randuss(n,p)` generates an `n`th order single-input uncertain continuous-time system with `p` outputs.

`usys = randuss(n,p,m)` generates an `n`th order uncertain continuous-time system with `p` outputs and `m` inputs.

`usys = randuss(n,p,m,Ts)` generates an `n`th order uncertain discrete-time system with `p` outputs and `m` inputs. The sample time is `Ts`.

`usys = randuss` (without arguments) results in a 1-by-1 uncertain continuous-time `uss` object with up to four uncertain objects.

In general, both `rand` and `randn` are used internally. You can control the result of `randuss` by setting seeds for both random number generators before calling the function.

Example

The statement creates a fifth order, continuous-time uncertain system `s1` of size `2-by-3`. Note your display can differ because a random seed is used.

```
s1 = randuss(5,2,3)
USS: 5 States, 2 Outputs, 3 Inputs, Continuous System
 CTPQV: 1x1 LTI, max. gain = 2.2, 1 occurrence
  IGDHN: real, nominal = -4.03, variability = [-3.74667
22.7816]%, 1 occurrence
 MLGCD: complex, nominal = 8.36+3.09i, +/- 7.07%, 1 occurrence
  OEDJK: complex, nominal = -0.346-0.296i, radius = 0.895, 1
occurrence
```

See Also

`rand`	Generates uniformly distributed random numbers
`randn`	Generates normally distributed random numbers
`randatom`	Creates a random uncertain atom
`randumat`	Creates a random uncertain matrix
`ucomplex`	Creates an uncertain complex parameter
`ultidyn`	Creates an uncertain linear time-invariant object

Purpose

LAPACK reciprocal condition estimator of `frd` object

Syntax

```
r = rcond(x)
```

Description

`rcond(x)` is an estimate for the reciprocal of the condition of the `frd` object `x` in the 1-norm obtained by the LAPACK condition estimator. `rcond` operates on `x.ReponseData` of the `x frd` at each frequency to construct `r`. If `x` is well conditioned, `rcond(x)` is near 1.0. If `x` is badly conditioned, `rcond(x)` is near EPS.

`r=rcond(x)` returns `r` as an `frd` object.

See Also

`cond`	Calculates condition number with respect to inversion
`norm`	Calculates matrix or vector norm
`condest`	Calculates a 1-norm condition number estimate
`normest`	Calculates a matrix 2-norm estimate

reduce

Purpose

Simplified access to Hankel singular value based model reduction functions

Syntax

```
GRED = reduce(G)
GRED = reduce(G,order)
[GRED,redinfo] = reduce(G,'key1','value1',...)
[GRED,redinfo] = reduce(G,order,'key1','value1',...)
```

Description

`reduce` returns a reduced order model `GRED` of `G` and a struct array `redinfo` containing the error bound of the reduced model, Hankel singular values of the original system and some other relevant model reduction information.

An error bound is a measure of how close `GRED` is to `G` and is computed based on either *additive error,* $\| G-GRED \|_\infty$, *multiplicative error,* $\| G^{-1}(G-GRED) \|_\infty$, or *nugap error* (ref.: `ncfmr`) [1],[4],[5].

Hankel singular values of a stable system indicate the respective state energy of the system. Hence, reduced order can be directly determined by examining the system Hankel SV's. Model reduction routines, which based on Hankel SV's are grouped by their error bound types. In many cases, the additive error method `GRED=reduce(G,ORDER)` is adequate to provide a good reduced order model. But for systems with lightly damped poles and/or zeros, a multiplicative error method (namely, `GRED=reduce(G,ORDER,'ErrorType','mult')`) that minimizes the relative error between `G` and `GRED` tends to produce a better fit.

This table describes input arguments for `reduce`.

Argument	Description
`G`	LTI model to be reduced (without any other inputs will plot its Hankel singular values and prompt for reduced order).
`ORDER`	(Optional) an integer for the desired order of the reduced model, or optionally a vector packed with desired orders for batch runs.

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y`, or a vector of integers. By default, all the anti-stable part of a physical system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system.

'*MaxError*' can be specified in the same fashion as an alternative for 'ORDER' after an '*ErrorType*' is selected. In this case, reduced order will be determined when the sum of the tails of the Hankel SV's reaches the '*MaxError*'.

Argument	Value	Description
'*Algorithm*'	'*balance*' '*schur*' '*hankel*' '*bst*' '*ncf*'	Default for 'add' (balancmr) Option for 'add' (schurmr) Option for 'add' (hankelmr) Default for 'mult' (bstmr) Default for 'ncf' (ncfmr)
'*ErrorType*'	'*add*' '*mult*' '*ncf*'	Additive error (default) Multiplicative error at model output NCF nugap error
'*MaxError*'	A real number or a vector of different errors	Reduce to achieve H∞ error. When present, '*MaxError*' overrides ORDER input.
'*Weights*'	{Wout,Win} cell array	Optimal 1x2 cell array of LTI weights Wout (output) and Win (input); default is both identity; used only with '*ErrorType*', '*add*'. Weights must be invertible.
'*Display*'	'*on*' or '*off*'	Display Hankel singular plots (default '*off*').
'*Order*'	Integer, vector or cell array	Order of reduced model. Use only if not specified as 2nd argument.

Weights on the original model input and/or output can make the model reduction algorithm focus on some frequency range of interests. But weights have to be stable, minimum phase and invertible.

This table describes output arguments.

Argument	Description
GRED	LTI reduced order model. Becomes multi-dimensional array when input is a serial of different model order array.
REDINFO	A STRUCT array with 3 fields: • REDINFO.ErrorBound • REDINFO.StabSV • REDINFO.UnstabSV For 'hankel' algorithm, STRUCT array becomes: • REDINFO.ErrorBound • REDINFO.StabSV • REDINFO.UnstabSV • REDINFO.Ganticausal For 'ncf' option, STRUCT array becomes: • REDINFO.GL • REDINFO.GR • REDINFO.hsv

G can be stable or unstable. G and GRED can be either continuous or discrete.

A successful model reduction with a well-conditioned original model G will ensure that the reduced model GRED satisfies the infinity norm error bound.

Example

Given a continuous or discrete, stable or unstable system, G, the following commands can get a set of reduced order models based on your selections:

```
rand('state',1234); randn('state',5678);G = rss(30,5,4);
[g1, redinfo1] = reduce(G); % display Hankel SV plot
                            % and prompt for order
[g2, redinfo2] = reduce(G,20); % default to balancmr
[g3, redinfo3] = reduce(G,[10:2:18],'algorithm','schur');
                            % select schurmr
```

```
[g4, redinfo] = reduce(G,'ErrorType','mult','MaxError',[0.01,
0.05]);
rand('state',12345); randn('state',6789);
wt1 = rss(6,5,5); wt1.d = eye(5)*2;
wt2 = rss(6,4,4); wt2.d = 2*eye(4);
[g5, redinfo5] = reduce(G, [10:2:18], 'weight',{wt1,wt2});
[g6, redinfo6] =
reduce(G,'ErrorType','add','algorithm','hankel', ...
      'maxerror',[0.01]);
for i = 1:6
    figure(i); eval(['sigma(G,g' num2str(i) ');']);
end
```

Reference

[1] K. Glover, "All Optimal Hankel Norm Approximation of Linear Multivariable Systems, and Their $\mathbf{L}_\infty$ – error Bounds," Int. J. Control, vol. 39, no. 6, pp. 1145-1193, 1984.

[2] M. G. Safonov and R. Y. Chiang, "A Schur Method for Balanced Model Reduction," *IEEE Trans. on Automat. Contr.*, vol. AC-2, no. 7, July 1989, pp. 729-733.

[3] M. G. Safonov, R. Y. Chiang and D. J. N. Limebeer, "Optimal Hankel Model Reduction for Nonminimal Systems," *IEEE Trans. on Automat. Contr.*, vol. 35, No. 4, April, 1990, pp. 496-502.

[4] M. G. Safonov and R. Y. Chiang, "Model Reduction for Robust Control: A Schur Relative-Error Method," *International Journal of Adaptive Control and Signal Processing*, vol. 2, pp. 259-272, 1988.

[5] K. Zhou, "Frequency weighted L• error bounds," Syst. Contr. Lett., Vol. 21, 115–125, 1993.

See Also

`balancmr`	Balanced truncation via square-root method
`schurmr`	Balanced truncation via Schur method
`bstmr`	Balanced stochastic truncation via Schur method
`ncfmr`	Balanced truncation for normalized coprime factors
`hankelmr`	Hankel minimum degree approximation
`hankelsv`	Hankel singular value

repmat

Purpose

Replicate and tile array

Syntax

```
B = repmat(A,M,N)
```

Description

`B = repmat(A,M,N)` creates a large matrix `B` consisting of an `M`-by-`N` tiling of copies of `A`.

`B = repmat(A,[M N])` accomplishes the same result as `repmat(A,M,N)`.

`B = repmat(A,[M N P ...])` tiles the array `A` to produce an `M`-by-`N`-by-`P`-by-... block array. `A` can be N-D.

`repmat(A,M,N)` for scalar `A` is commonly used to produce an `M`-by-`N` matrix filled with values of `A`.

Example

Simple examples of using `repmat` are

```
repmat(randumat(2,2),2,3)
repmat(ureal('A',6),[4 2])
```

Purpose

Create options object for use with `robuststab` and `robustperf`

Syntax

```
opts = robopt
opts = robopt('name1',value1,'name2',value2,...)
```

Description

`opts = robopt` (with no input arguments) creates an options object with all the properties set to their default values.

`opts = robopt('name1',value1,'name2',value2,...)` creates a `robopt` object in which specified properties have the given values. Any unspecified property is set to its default value. It is sufficient to type only enough leading characters to define the property name uniquely. Case is ignored for property names.

`robopt` with no input or output arguments displays a complete list of option properties and their default values.

Fields

The following are the `robopt` object properties:

Object Property	Description
`Display`	Displays progress of computations `{`on';'off'}`. Default is `'off'`
`Sensitivity`	Computes margin sensitivity to individual uncertainties `{`on';'off'}`. Default is `'on'`.
`VaryUncertainty`	Percentage variation of uncertainty used as a stepsize in finite-difference calculations to estimate sensitivity. Default is 25.
`Mussv`	Option used in internal structured singular value calculations (when calling `mussv`). Default is `'sm9'`.

Object Property	Description
`Default`	Structure, field names are `robopt` properties, and values are the default values.
`Meaning`	Structure, field names are `robopt` properties, and values are the text description of the property.

Example

You can create a `robopt` options object called `opt` with all default values.

```
opt = robopt
Property Object Values:
            Display: 'off'
        Sensitivity: 'on'
    VaryUncertainty: 25
              Mussv: 'sm9'
            Default: [1x1 struct]
            Meaning: [1x1 struct]
```

An elementary finite-difference scheme is used in estimating local sensitivities. The property `VaryUncertainty` denotes the step size used in estimating the derivatives necessary in computing sensitivities.

In the following statements, you are requesting that the sensitivity of the robust stability margin calculation to a 50% variation in individual uncertainties be calculated. The `robopt` options properties `'Sensitivity'` and `'VaryUncertainty'` are set individually.

```
opt = robopt;
opt.VaryUncertainty = 50;
opt
Property Object Values:
            Display: 'off'
        Sensitivity: 'on'
    VaryUncertainty: 50
              Mussv: 'sm9'
            Default: [1x1 struct]
            Meaning: [1x1 struct]
```

See Also

`dkitopt` Creates an options object for `dksyn`

`robuststab`	Calculates stability margins of uncertain systems
`robustperf`	Calculates performance margins of uncertain systems
`wcgopt`	Creates a `wcgain` option object
`wcsens`	Calculates worst-case sensitivities for a feedback loop
`wcmargin`	Calculates worst-case margins for a feedback loop

robustperf

Purpose

Calculates robust performance margin of uncertain multivariable system

Syntax

```
perfmarg = robustperf(sys)
[perfmarg,wcu,report,info] = robustperf(sys)
[perfmarg,wcu,report,info] = robustperf(sys,opt)
```

Description

The performance of a nominally stable uncertain system model will generally degrade for specific values of its uncertain elements. `robustperf`, largely included for historical purposes, computes the robust performance margin, which is one measure of the level of degradation brought on by the modeled uncertainty. The relationship between `robustperf` and other measures, such as `robuststab` and `wcgain`, is described in Chapter 2, "Generalized Robustness Analysis."

As with other *uncertain-system* analysis tools, only bounds on the performance margin are computed. The exact robust performance margin is guaranteed to lie between these upper and lower bounds.

The computation used in `robustperf` is a frequency domain calculation. If the input system `sys` is a `ufrd`, then the analysis is performed on the frequency grid within the `ufrd`. If the input system `sys` is a `uss`, then an appropriate frequency grid is generated (automatically), and the analysis performed on that frequency grid. In all discussion that follows, *N* denotes the number of points in the frequency grid.

The computation used in `robustperf` is a frequency-domain calculation. Coupled with stability of the nominal system, this frequency domain calculation determines robust performance of `sys`. If the input system `sys` is a `ufrd`, then the analysis is performed on the frequency grid within the `ufrd`. Note that the stability of the nominal system is not verified by the computation. If the input system sys is a `uss`, then the stability of the nominal system is first checked, an appropriate frequency grid is generated (automatically), and the analysis performed on that frequency grid. In all discussion that follows, N denotes the number of points in the frequency grid.

Basic Syntax

Suppose `sys` is a `ufrd` or `uss` with *M* uncertain elements. The results of

```
[perfmarg,perfmargunc,Report] = robustperf(sys)
```

are such that `perfmarg` is a structure with the following fields:

Field	Description
`LowerBound`	Lower bound on robust performance margin, positive scalar.
`UpperBound`	Upper bound on robust performance margin, positive scalar.
`CriticalFrequency`	The value of frequency at which the performance degradation curve crosses the y=1/x curve. See "Generalized Robustness Analysis" in the online documentation.

`perfmargunc` is a `struct` of values of uncertain elements associated with the intersection of the performance degradation curve and the $y=1/x$ curve. See Chapter 2, "Generalized Robustness Analysis." There are M field names, which are the names of uncertain elements of `sys`.

`Report` is a text description of the robust performance analysis results.

Example

Create a plant with a nominal model of an integrator, and include additive unmodeled dynamics uncertainty of a level of 0.4 (this corresponds to 100% model uncertainty at 2.5 rads/s).

```
P = tf(1,[1 0]) + ultidyn('delta',[1 1],'bound',0.4);
```

Design a "proportional" controller K that puts the nominal closed-loop bandwidth at 0.8 rad/s. Roll off K at a frequency 25 times the nominal closed-loop bandwidth. Form the closed-loop sensitivity function.

```
BW = 0.8;
K = tf(BW,[1/(25*BW) 1]);
S = feedback(1,P*K);
```

Assess the performance margin of the closed-loop sensitivity function. Because the nominal gain of the sensitivity function is 1, and the performance

degradation curve is monotonically increasing (see Chapter 2, "Generalized Robustness Analysis"), the performance margin should be less than 1.

```
[perfmargin,punc] = robustperf(S);
perfmargin
perfmargin =
           UpperBound: 7.4305e-001
           LowerBound: 7.4305e-001
    CriticalFrequency: 5.3096e+000
```

You can verify that the upper bound of the performance margin corresponds to a point on or above the $y=1/x$ curve. First, compute the normalized size of the value of the uncertain element, and check that this agrees with the upper bound.

```
nsize = actual2normalized(S.Uncertainty.delta, punc.delta)
nsize =
perfmargin.UpperBound
ans =
  7.4305e-001
```

Compute the system gain with that value substituted, and verify that the product of the normalized size and the system gain is greater than or equal to 1.

```
gain = norm(usubs(S,punc),inf,.00001);
nsize*gain
ans =
  1.0000e+000
```

Finally, as a sanity check, verify that the robust performance margin is less than the robust stability margin (it should always be, as described in Chapter 4, "Robustness Analysis").

```
[stabmargin] = robuststab(S);
stabmargin
stabmargin =
                UpperBound: 3.1251e+000
                LowerBound: 3.1251e+000
    DestabilizingFrequency: 4.0862e+000
```

While the robust stability margin is easy to describe (poles migrating from stable region into unstable region), describing the robust performance margin

is less elementary. See the diagrams and figures in Chapter 2, "Generalized Robustness Analysis." Rather than finding values for uncertain elements that lead to instability, the analysis finds values of uncertain elements "corresponding to the intersection point of the performance degradation curve with a $y=1/x$ hyperbola." This characterization, mentioned above in the description of `perfmarg.CriticalFrequency` and `perfmargunc`, is used often in the descriptions below.

Basic Syntax with Fourth Output Argument

A fourth output argument yields more specialized information, including sensitivities and frequency-by-frequency information.

```
[perfmarg,perfmargunc,Report,Info] = robustperf(sys)
```

In addition to the first 3 output arguments, described previously, `Info` is a structure with the following fields:

Field	Description
`Sensitivity`	A `struct` with *M* fields, field names are names of uncertain elements of `sys`. Values of fields are positive and contain the local sensitivity of the overall Stability Margin to that element's uncertainty range. For instance, a value of 25 indicates that if the uncertainty range is enlarged by 8%, then the stability margin should drop by about 2% (25% of 8). If the `Sensitivity` property of the `robopt` object is `'off'`, the values are set to `NaN`.
`Frequency`	*N*-by-1 frequency vector associated with analysis.
`BadUncertainValues`	*N*-by-1 cell array, with one entry for each frequency point. The `k`th entry `Info.BadUncertainValues{k}` is a `struct` of values of uncertain elements resulting from a robust performance analysis at frequency `Info.Frequency(k)`.

Field	Description
MussvBnds	A 1-by-2 frd, with upper and lower bounds from mussv. The (1,1) entry is the μ-upper bound (corresponds to perfmarg.LowerBound) and the (1,2) entry is the μ-lower bound (for perfmarg.UpperBound).
MussvInfo	Structure of compressed data from mussv.

Options (e.g., controlling what is displayed during the computation, turning on/off the sensitivity computation, setting the step size in the sensitivity computation, and controlling the option argument used in the underlying call to mussv) is specified using the robustness analysis options robopt object. For instance, you can turn the display on and turn off the sensitivity by executing

```
opt = robopt('Sensitivity','off','Display','on');
[PerfMarg,Destabunc,Report,Info] = robustperf(sys,opt)
```

Handling Array Dimensions

If sys has array dimensions (for example, suppose that the size of sys is $r \times c \times d_1 \times d_2 \times \ldots \times d_F$, refer to the $d_1 \times d_2 \times \ldots \times d_F$ as the *array dimensions*) then the margin calculation is performed "pointwise" (individually, at each and every array value) and the computed answers all have array dimensions as well. Details are described below. Again, assume that there are N frequency points and M uncertain elements.

The results of

```
[perfmarg,perfmargunc,Report,Info] = robustperf(sys,opt)
```

are `perfmarg` a structure with the following fields

Field	Description
`LowerBound`	$d_1 \times \ldots \times d_F$, lower bound on stability margin across the array dimensions.
`UpperBound`	$d_1 \times \ldots \times d_F$, upper bound on performance margin across the array dimensions. Using single indexing, for each `i`, the upper bound on the performance margin of `sys(:,:,i)` is `perfmarg.UpperBound(i)`.
`CriticalFrequency`	$d_1 \times \ldots \times d_F$, the value of frequency at which the performance degradation curve crosses the $y=1/x$ curve. Using single indexing, for each `i`, the frequency at which the performance degradation curve crosses the $y=1/x$ curve in robust performance analysis of `sys(:,:,i)` is `perfmarg.CriticalFrequency(i)`. See Chapter 2, "Generalized Robustness Analysis."

`perfmargunc` is a $d_1 \times \ldots \times d_F$ structure array of values of uncertain elements, associated with the intersection of the performance degradation curve and the $y=1/x$ curve. See "Generalized Robustness Analysis" in the online documentation. Using single indexing, for each `i`, the `struct` of values of uncertain elements for uncertain system `sys(:,:,i)` is `perfmargunc(i)`.

`Report` is a character array, dimensions 3, 4, ..., `F+2` are $d_1 \times \ldots \times d_F$, containing text description of the robustness analysis results at each grid in the array dimensions.

In addition to the first 3 output arguments, described previously, `Info` is a structure with the following fields

Field	Description
`Sensitivity`	$A\ d_1 \times \ldots \times d_F$ `struct`, field names are names of uncertain elements of `sys`. Using single indexing notation, `Sensitivity(i)` contains the sensitivities of `perfmarg.UpperBound(i)` for the uncertain system `sys(:,:,i)`.
`Frequency`	N-by-1 frequency vector associated with analysis.
`BadUncertainValues`	N-by-1 cell array, with one entry for each frequency point. The `k`th entry `Info.BadUncertainValues{k}` is a $d_1 \times \ldots \times d_F$ `struct` of values of uncertain elements resulting from a $d_1 \times \ldots \times d_F$ family of robust performance computations at frequency `Info.Frequency(k)`.
`MussvBnds`	$1 \times 2 \times d_1 \times \ldots \times d_F$ `frd`, with upper and lower bounds from `mussv`. Using single indexing for the dimensions associated with the array dimensions, it follows that the `(1,1,i)` entry is the μ-upper bound (reciprocal of `perfmarg.UpperBound(i)`) while the `(1,2,i)` entry is the μ-lower bound (reciprocal of `perfmarg.UpperBound(i)`).
`MussvInfo`	Structure of compressed data from `mussv`.

The smallest performance margin over all array dimensions is computed `min(perfmarg.UpperBound(:))`. Computing

```
i = find(UpperBound==min(UpperBound(:)))
```

and then selecting `perfmargunc(i)` yields values for an uncertainty corresponding to the smallest performance margin across all array dimensions.

Algorithm

A rigorous robust performance analysis consists of two steps:

1 Verify that the nominal system is stable;

2 Robust performance analysis on an augmented system.

The algorithm in `robustperf` follows this in spirit, but might require user attention.

If `sys` is a `uss` object, then the first requirement of stability of nominal value is explicitly checked within `robustperf`. However, if `sys` is an `ufrd`, then the verification of nominal stability from the nominal frequency response data is not performed, and is instead *assumed*.

The exact performance margin is guaranteed to be no larger than `UpperBound` (some uncertain elements associated with this magnitude cause instability – one instance is returned in the structure `perfmargunc`). The instability created by `perfmargunc` occurs at the frequency value in `CriticalFrequency`.

Similarly, the exact performance margin is guaranteed to be no smaller than `LowerBound`.

Limitations

Because the calculation is carried out with a frequency gridding, it is possible (likely) that the true critical frequency is missing from the frequency vector used in the analysis. This is similar to the problem in `robuststab`. However, in comparing to `robuststab`, the problem in `robustperf` is less acute. The robust performance margin, considered a function of problem data and frequency, is typically a continuous function (unlike the robust stability margin, described in the Robust Control Toolbox™ demo called *Getting Reliable Estimates of Robustness Margins* in the online documentation). Hence, in robust performance margin calculations, increasing the density of the frequency grid will always increase the accuracy of the answers, and in the limit, answers arbitrarily close to the actual answers are obtainable with finite frequency grids.

See Also

`loopmargin`	Comprehensive analysis of feedback loop.
`mussv`	Calculate bounds on the Structured Singular Value (μ)
`norm`	Calculate LTI system norms
`robopt`	Create a `robuststab`/`robustperf` options object
`robuststab`	Calculates stability margins of uncertain systems
`actual2normalized`	Normalizes range of uncertain atoms
`wcgain`	Calculate worst-case gain of uncertain systems
`wcsens`	Calculate worst-case sensitivities for feedback loop
`wcmargin`	Calculate worst-case margins for feedback loop

robuststab

Purpose

Calculate robust stability margins of uncertain multivariable system

Syntax

```
[stabmarg,destabunc,report,info] = robuststab(sys)
[stabmarg,destabunc,report,info] = robuststab(sys,opt)
```

Description

A nominally stable uncertain system is generally unstable for specific values of its uncertain elements. Determining the values of the uncertain elements closest to their nominal values for which instability occurs is a *robust stability* calculation.

If the uncertain system is stable for all values of uncertain elements within their allowable ranges (ranges for `ureal`, norm bound or positive-real constraint for `ultidyn`, radius for `ucomplex`, weighted ball for `ucomplexm`), the uncertain system is *robustly stable*. Conversely, if there is a combination of element values that cause instability, and all lie within their allowable ranges, then the uncertain system is not robustly stable.

`robuststab` computes the margin of stability robustness for an uncertain system. A stability robustness margin greater than 1 means that the uncertain system is stable for all values of its modeled uncertainty. A stability robustness margin less than 1 implies that certain allowable values of the uncertain elements, within their specified ranges, lead to instability.

Numerically, a margin of 0.5 (for example) implies two things: the uncertain system remains stable for all values of uncertain elements that are less than 0.5 normalized units away from their nominal values and, there is a collection of uncertain elements that are less than or equal to 0.5 normalized units away from their nominal values that results in instability. Similarly, a margin of 1.3 implies that the uncertain system remains stable for all values of uncertain elements up to 30% outside their modeled uncertain ranges. See `actual2normalized` for converting between actual and normalized deviations from the nominal value of an uncertain element.

As with other *uncertain-system* analysis tools, only bounds on the exact stability margin are computed. The exact robust stability margin is guaranteed to lie in between these upper and lower bounds.

The computation used in `robuststab` is a frequency-domain calculation, which determines whether poles can migrate (due to variability of the uncertain atoms) across the stability boundary (imaginary axis for continuous-time,

unit circle for discrete-time). Coupled with stability of the nominal system, determining that no migration occurs constitutes robust stability. If the input system `sys` is a `ufrd`, then the analysis is performed on the frequency grid within the `ufrd`. Note that the stability of the nominal system is not verified by the computation. If the input system sys is a `uss`, then the stability of the nominal system is first checked, an appropriate frequency grid is generated (automatically), and the analysis performed on that frequency grid. In all discussion that follows, N denotes the number of points in the frequency grid.

Basic Syntax

Suppose `sys` is a `ufrd` or `uss` with M uncertain elements. The results of

```
[stabmarg,destabunc,Report] = robuststab(sys)
```

are that `stabmarg` is a structure with the following fields

Field	Description
`LowerBound`	Lower bound on stability margin, positive scalar. If greater than 1, then the uncertain system is guaranteed stable for all values of the modeled uncertainty. If the nominal value of the uncertain system is unstable, then `stabmarg.UpperBound` and `stabmarg.LowerBound` will equal $-\infty$.

Field	Description
UpperBound	Upper bound on stability margin, positive scalar. If less than 1, the uncertain system is not stable for all values of the modeled uncertainty.
DestabilizingFrequency	The critical value of frequency at which instability occurs, with uncertain elements closest to their nominal values. At a particular value of uncertain elements (see destabunc below), the poles migrate across the stability boundary (imaginary-axis in continuous-time systems, unit-disk in discrete-time systems) at the frequency given by DestabilizingFrequency.

destabunc is a structure of values of uncertain elements, closest to nominal, that cause instability. There are *M* field names, which are the names of uncertain elements of sys. The value of each field is the corresponding value of the uncertain element, such that when jointly combined, lead to instability. The command pole(usubs(sys,destabunc)) shows the instability. If A is an uncertain atom of sys, then

```
actual2normalized(destabunc.A,sys.Uncertainty.A)
```

will be less than or equal to UpperBound, and for at least one uncertain element of sys, this normalized distance will be equal to UpperBound, proving that UpperBound is indeed an upper bound on the robust stability margin.

Report is a text description of the robustness analysis results.

Example

Construct a feedback loop with a second-order plant and a PID controller with approximate differentiation. The second-order plant has frequency-dependent uncertainty, in the form of additive unmodeled dynamics, introduced with an ultidyn object and a shaping filter.

robuststab is used to compute the stability margins of the closed-loop system with respect to the plant model uncertainty.

```
P = tf(4,[1 .8 4]);
```

```
delta = ultidyn('delta',[1 1],'SampleStateDim',5);
Pu = P + 0.25*tf([1],[.15 1])*delta;
C = tf([1 1],[.1 1]) + tf(2,[1 0]);
S = feedback(1,Pu*C);
[stabmarg,destabunc,report,info] = robuststab(S);
```

You can view the `stabmarg` variable.

```
stabmarg
stabmarg =
                UpperBound: 0.8181
                LowerBound: 0.8181
    DestabilizingFrequency: 9.1321
```

As the margin is less than 1, the closed-loop system is not stable for plant models covered by the uncertain model `Pu`. There is a specific plant within the uncertain behavior modeled by `Pu` (actually about 82% of the modeled uncertainty) that leads to closed-loop instability, with the poles migrating across the stability boundary at 9.1 rads/s.

The `report` variable is specific, giving a plain-language version of the conclusion.

```
report
report =
Uncertain System is NOT robustly stable to modeled uncertainty.
 -- It can tolerate up to 81.8% of modeled uncertainty.
 -- A destabilizing combination of 81.8% the modeled uncertainty
exists, causing an instability at 9.13 rad/s.
 -- Sensitivity with respect to uncertain element ...
    'delta' is 100%.  Increasing 'delta' by 25% leads to a 25%
decrease in the margin.
```

Because the problem has only one uncertain element, the stability margin is completely determined by this element, and hence the margin exhibits 100% sensitivity to this uncertain element.

You can verify that the destabilizing value of `delta` is indeed about 0.82 normalized units from its nominal value.

```
actual2normalized(S.Uncertainty.delta,destabunc.delta)
ans =
    0.8181
```

Use `usubs` to substitute the specific value into the closed-loop system. Verify that there is a closed-loop pole near `j9.1`, and plot the unit-step response of the nominal closed-loop system, as well as the unstable closed-loop system.

```
Sbad = usubs(S,destabunc);
pole(Sbad)
ans =
  1.0e+002 *
  -3.2318
  -0.2539
  -0.0000 + 0.0913i
  -0.0000 - 0.0913i
  -0.0203 + 0.0211i
  -0.0203 - 0.0211i
  -0.0106 + 0.0116i
  -0.0106 - 0.0116i
step(S.NominalValue,'r--',Sbad,'g',4);
```

Finally, as an ad-hoc test, set the gain bound on the uncertain `delta` to 0.81 (slightly less than the stability margin). Sample the closed-loop system at 100 values, and compute the poles of all these systems.

```
S.Uncertainty.delta.Bound = 0.81;
S100 = usample(S,100);
p100 = pole(S100);
max(real(p100(:)))
ans =
 -6.4647e-007
```

As expected, all poles have negative real parts.

Basic Syntax with Fourth Output Argument

A fourth output argument yields more specialized information, including sensitivities and frequency-by-frequency information.

```
[StabMarg,Destabunc,Report,Info] = robuststab(sys)
```

In addition to the first 3 output arguments, described previously, `Info` is a structure with the following fields

Field	Description
`Sensitivity`	`A struct` with *M* fields, Field names are names of uncertain elements of `sys`. Values of fields are positive, each the local sensitivity of the overall stability margin to that element's uncertainty range. For instance, a value of 25 indicates that if the uncertainty range is enlarged by 8%, then the stability margin should drop by about 2% (25% of 8). If the `Sensitivity` property of the `robopt` object is `'off'`, the values are set to `NaN`.
`Frequency`	*N*-by-1 frequency vector associated with analysis.
`BadUncertainValues`	*N*-by-1 cell array, with one entry for each frequency point. The `k`th entry`Info.BadUncertainValues{k}` is a `struct` of values of uncertain elements, closest to their nominal values, which cause the system poles to migrate across the stability boundary at frequency `Info.Frequency(k)`. The command `pole(usubs(sys,Info.BadUncertainValues{k}))` shows the migration. The command `usubs(sys,cat(1,Info.BadUncertainValues{:}))` generates an *N*-by-1 `ss` array. Each instance is unstable, with poles on the stability boundary at frequencies given by the vector `Info.Frequency`. This migration to instability has been achieved with the smallest normalized deviations in the uncertain elements from their nominal values.

Field	Description
`MussvBnds`	A 1-by-2 `frd`, with upper and lower bounds from `mussv`. The (1,1) entry is the μ-upper bound (corresponds to `stabmarg.LowerBound`) and the (1,2) entry is the μ-lower bound (for `stabmarg.UpperBound`).
`MussvInfo`	Structure of compressed data from `mussv`.

Options (e.g., controlling what is displayed during the computation, turning on/off the sensitivity computation, setting the step-size in the sensitivity computation, and controlling the option argument used in the underlying call to `mussv`) can be specified using the robustness analysis options `robopt` object. For instance, you can turn the display on, and the sensitivity calculation off by executing

```
opt = robopt('Sensitivity','off','Display','on');
[StabMarg,Destabunc,Report,Info] = robuststab(sys,opt)
```

Handling Array Dimensions

If `sys` has array dimensions (for example, suppose that the size of `sys` is $r \times c \times d_1 \times d_2 \times \ldots \times d_F$, refer to the $d_1 \times d_2 \times \ldots \times d_F$ as the *array dimensions*) then the margin calculation is performed pointwise (individually, at each and every array value) and the computed answers all have array dimensions as well. Details are described below. Again, assume that there are N frequency points and M uncertain elements.

The results of

```
[stabmarg,destabunc,Report,Info] = robuststab(sys,opt)
```

are `stabmarg` is a structure with the following fields:

Field	Description
`LowerBound`	$d_1 \times \ldots \times d_F$, lower bound on stability margin across the array dimensions.
`UpperBound`	$d_1 \times \ldots \times d_F$, upper bound on stability margin across the array dimensions. Using single-indexing, for each `i`, the upper bound on the stability margin of `sys(:,:,i)` is `stabmarg.UpperBound(i)`.
`DestabilizingFrequency`	$d_1 \times \ldots \times d_F$, frequency at which instability occurs, associated with `stabmarg.UpperBound`. Using single-indexing, for each `i`, the frequency at which instability occurs in robust stability analysis of `sys(:,:,i)` is `stabmarg.DestabilizingFrequency(i)`.

`destabunc` is a $d_1 \times \ldots \times d_F$ structure array of values of uncertain elements, that cause instability. Using single-indexing, for each `i`, the destabilizing values of uncertain elements for uncertain system `sys(:,:,i)` is `destabunc(i)`.

`Report` is a character array, dimensions 3, 4, ... , `F+2` are $d_1 \times \ldots \times d_F$, containing text description of the robustness analysis results at each grid in the array dimensions.

In addition to the first 3 output arguments, described previously, `Info` is a structure with the following fields

Field	Description
`Sensitivity`	A $d_1 \times \ldots \times d_F$ `struct`, field names are names of uncertain elements of `sys`. Using single indexing notation, `Sensitivity(i)` contains the sensitivities of `stabmarg.UpperBound(i)` for the uncertain system `sys(:,:,i)`.
`Frequency`	$N \times 1$ frequency vector associated with analysis.
`BadUncertainValues`	N-by-1 cell array, with one entry for each frequency point. The `k`'th entry `Info.BadUncertainValues{k}` is a $d_1 \times \ldots \times d_F$ `struct` of values of uncertain elements, closest to their nominal values, that cause the system poles to migrate across the stability boundary at frequency `Info.Frequency(k)`. The command `usubs(sys,Info.BadUncertainValues{k})` produces an `ss` array of size $d_1 \times \ldots \times d_F$ with the substitutions made. Alternatively, `usubs(sys,cat(F+1,Info.BadUncertainValues{:}))` produces an `ss` array of size $d_1 \times \ldots \times d_F \times N$ with the substitutions made.
`MussvBnds`	A $1 \times 2 \times d_1 \times \ldots \times d_f$ `frd`, with upper and lower bounds from `mussv`. Using single-indexing for the dimensions associated with the array dimensions, it follows that the `(1,1,i)` entry is the μ-upper bound (corresponding to `stabmarg.LowerBound(i)`) while the `(1,2,i)` entry is the μ-lower bound (which corresponds to `stabmarg.UpperBound(i)`).
`MussvInfo`	Structure of compressed data from `mussv`.

You can compute the smallest stability margin over all array dimensions via

```
min(stabmarg.UpperBound(:)).
```

Computing `i = find(UpperBound==min(UpperBound(:)))` and then `destabunc(i)` yields values for an uncertainty corresponding to the smallest stability margin across all array dimensions.

Algorithm

A rigorous robust stability analysis consists of two steps:

1. Verify that the nominal system is stable;
2. Verify that no poles cross the stability boundary as the uncertain elements vary within their ranges.

Because the stability boundary is also associated with the frequency response, the second step can be interpreted (and carried out) as a frequency domain calculation. This amounts to a classical μ-analysis problem.

The algorithm in `robuststab` follows this in spirit, but might require user attention.

If `sys` is a `uss` object, then the first requirement of stability of nominal value is explicitly checked within `robuststab`. However, if `sys` is an `ufrd`, then the verification of nominal stability from the nominal frequency response data is not performed, and is instead assumed.

In the second step (monitoring the stability boundary for the migration of poles), rather than check all points on stability boundary, the algorithm only detects migration of poles across the stability boundary at the frequencies in `info.Frequency`.

See the "Limitations" section below about issues related to migration detection.

The exact stability margin is guaranteed to be no larger than `UpperBound` (some uncertain elements associated with this magnitude cause instability – one instance is returned in the structure `destabunc`). The instability created by `destabunc` occurs at the frequency value in `DestabilizingFrequency`.

Similarly, the exact stability margin is guaranteed to be no smaller than `LowerBound`. In other words, for all modeled uncertainty with magnitude up to `LowerBound`, the system is guaranteed stable. These bounds are derived using the upper bound for the structured singular value, which is essentially optimally-scaled, small-gain theorem analysis.

Limitations

Under most conditions, the robust stability margin that occurs at each frequency is a continuous function of the problem data at that frequency.

Because the problem data, in turn, is a continuous function of frequency, it follows that finite frequency grids are usually adequate in correctly assessing robust stability bounds, assuming the frequency grid is dense enough.

Nevertheless, there are simple examples that violate this. In some problems, the migration of poles from stable to unstable *only* occurs at a finite collection of specific frequencies (generally unknown to you). Any frequency grid that excludes these critical frequencies (and almost every grid will exclude them) will result in undetected migration and misleading results, namely stability margins of ∞.

See the Robust Control Toolbox™ demo titled "Getting Reliable Estimates of Robustness Margins" in the online documentation about circumventing the problem in an engineering-relevant fashion.

See Also

`loopmargin`	Comprehensive analysis of a feedback loop
`mussv`	Calculates bounds on the Structured Singular Value (μ)
`robopt`	Creates a `robuststab`/`robustperf` options object
`robustperf`	Calculates performance margins of uncertain systems
`wcgain`	Calculates worst-case gain of uncertain systems
`wcsens`	Calculates worst-case sensitivities for a feedback loop
`wcmargin`	Calculates worst-case margins for a feedback loop

Purpose

Schur decomposition of `frd` object

Syntax

```
[u,t] = schur(x)
    t = schur(x)
[u,t] = schur(x,0)
    t = schur(x,0)
[u,t] = schur(x,'econ')
    t = schur(x,'econ')
```

Description

`frd/schur` applies the `schur` command to `frd` objects. `[u,t] = schur(x)` operates on the `x.ReponseData` of the `frd` object at each frequency point to construct `u` and `t`. `u` and `t` are `frd` objects. `x` must be square. See the built-in `schur` command for details.

See Also

`qz`	Creates a QZ factorization for generalized eigenvalues
`schur`	Calculates a Schur decomposition

schurmr

Purpose

Balanced model truncation via Schur method

Syntax

```
GRED = schurmr(G)
GRED = schurmr(G,order)
[GRED,redinfo] = schurmr(G,key1,value1,...)
[GRED,redinfo] = schurmr(G,order,key1,value1,...)
```

Description

`schurmr` returns a reduced order model `GRED` of G and a struct array redinfo containing the error bound of the reduced model and Hankel singular values of the original system.

The error bound is computed based on Hankel singular values of `G`. For a stable system Hankel singular values indicate the respective state energy of the system. Hence, reduced order can be directly determined by examining the system Hankel SV's, σι.

With only one input argument `G`, the function will show a Hankel singular value plot of the original model and prompt for model order number to reduce.

This method guarantees an error bound on the infinity norm of the *additive error* || `G-GRED` ||∞ for well-conditioned model reduced problems [1]:

$$\|G - Gred\|_\infty \le 2 \sum_{k+1}^{n} \sigma_i$$

This table describes input arguments for `schurmr`.

Argument	Description
`G`	LTI model to be reduced (without any other inputs will plot its Hankel singular values and prompt for reduced order).
`ORDER`	(Optional) an integer for the desired order of the reduced model, or optionally a vector packed with desired orders for batch runs

A batch run of a serial of different reduced order models can be generated by specifying `order = x:y`, or a vector of integers. By default, all the anti-stable

part of a system is kept, because from control stability point of view, getting rid of unstable state(s) is dangerous to model a system.

'*MaxError*' can be specified in the same fashion as an alternative for 'ORDER'. In this case, reduced order will be determined when the sum of the tails of the Hankel sv's reaches the '*MaxError*'.

Argument	Value	Description
'MaxError'	A real number or a vector of different errors	Reduce to achieve H∞ error. When present, '*MaxError*' overides ORDER input.
'Weights'	{Wout,Win} cell array	Optimal 1x2 cell array of LTI weights Wout (output) and Win (input); default is both identity; Weights must be invertible.
'Display'	'*on*' or '*off*'	Display Hankel singular plots (default 'off').
'Order'	Integer, vector or cell array	Order of reduced model. Use only if not specified as 2nd argument.

Weights on the original model input and/or output can make the model reduction algorithm focus on some frequency range of interests. But weights have to be stable, minimum phase and invertible.

This table describes output arguments.

Argument	Description
`GRED`	LTI reduced order model. Becomes multi-dimensional array when input is a serial of different model order array.
`REDINFO`	A STRUCT array with 3 fields: • `REDINFO.ErrorBound` • `REDINFO.StabSV` • `REDINFO.UnstabSV`

`G` can be stable or unstable. `G` and `GRED` can be either continuous or discrete.

Algorithm

Given a state space *(A,B,C,D)* of a system and *k,* the desired reduced order, the following steps will produce a similarity transformation to truncate the original state space system to the k^{th} order reduced model [16].

1. Find the controllability and observability grammians P and Q.
2. Find the Schur decomposition for PQ in both ascending and descending order, respectively,

$$V_A^T PQ V_A = \begin{bmatrix} \lambda_1 & \cdots & \cdots \\ 0 & \cdots & \cdots \\ 0 & 0 & \lambda_n \end{bmatrix}$$

$$V_D^T PQ V_D = \begin{bmatrix} \lambda_n & \cdots & \cdots \\ 0 & \cdots & \cdots \\ 0 & 0 & \lambda_1 \end{bmatrix}$$

3. Find the left/right orthonormal eigen-bases of PQ associated with the k^{th} big Hankel singular values.

$$V_A = [V_{R,SMALL}, \overbrace{V_{L,BIG}}]$$

4. Find the SVD of $(V^T_{L,BIG} V_{R,BIG}) = U \Sigma V^T$

$$V_D = [\overbrace{V_{R,BIG}}, V_{L,SMALL}]$$

5 Form the left/right transformation for the final k^{th} order reduced model

$$S_{L,BIG} = V_{L,BIG}\, U\Sigma(1{:}k,1{:}k)^{-1/2}$$

$$S_{R,BIG} = V_{R,BIG} V\Sigma(1{:}k,1{:}k)^{-1/2}$$

6 Finally,

$$\left[\begin{array}{c|c} \hat{A} & \hat{B} \\ \hline \hat{C} & \hat{D} \end{array}\right] = \left[\begin{array}{c|c} S^T_{L,BIG} A S_{R,BIG} & S^T_{L,BIG} B \\ \hline C S_{R,BIG} & D \end{array}\right]$$

The proof of the Schur balance truncation algorithm can be found in [2].

Example

Given a continuous or discrete, stable or unstable system, `G`, the following commands can get a set of reduced order models based on your selections:

```
rand('state',1234); randn('state',5678);G = rss(30,5,4);
[g1, redinfo1] = schurmr(G); % display Hankel SV plot
                             % and prompt for order (try 15:20)
[g2, redinfo2] = schurmr(G,20);
[g3, redinfo3] = schurmr(G,[10:2:18]);
[g4, redinfo4] = schurmr(G,'MaxError',[0.01, 0.05]);
rand('state',12345); randn('state',6789);
wt1 = rss(6,5,5); wt1.d = eye(5)*2;
wt2 = rss(6,4,4); wt2.d = 2*eye(4);
[g5, redinfo5] = schurmr(G, [10:2:18], 'weight',{wt1,wt2});
for i = 1:5
     figure(i); eval(['sigma(G,g' num2str(i) ');']);
end
```

Reference

[1] K. Glover, “All Optimal Hankel Norm Approximation of Linear Multivariable Systems, and Their $\mathbf{L}_\infty$– error Bounds,” Int. J. Control, vol. 39, no. 6, pp. 1145-1193, 1984.

[2] M. G. Safonov and R. Y. Chiang, “A Schur Method for Balanced Model Reduction,” *IEEE Trans. on Automat. Contr.*, vol. 34, no. 7, July 1989, pp. 729-733.

See Also

`reduce` Top level model reduction routines

`balancmr`	Balanced truncation via square-root method
`bstmr`	Balanced stochastic truncation via Schur method
`ncfmr`	Balanced truncation for normalized coprime factors
`hankelmr`	Hankel minimum degree approximation
`hankelsv`	Hankel singular value

Purpose

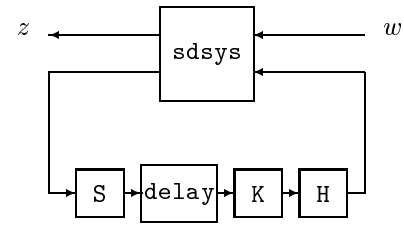
Compute L_2 norm of continuous-time system in feedback with discrete-time system

Syntax

```
[gaml,gamu] = sdhinfnorm(sdsys,k)
[gaml,gamu] = sdhinfnorm(sdsys,k,delay)
[gaml,gamu] = sdhinfnorm(sdsys,k,delay,tol)
```

Description

`[gaml,gamu] = sdhinfnorm(sdsys,k)` computes the L_2 induced norm of a continuous-time LTI plant, `sdsys`, in feedback with a discrete-time controller, `k`, connected through an ideal sampler and a zero-order hold (see figure below). `sdsys` must be strictly proper, such that the constant feedback gain must be zero. The outputs, `gamu` and `gaml`, are upper and lower bounds on the induced L_2 norm of the sampled-data closed-loop system.



`[gaml,gamu] = sdhinfnorm(sdsys,k,h,delay)` includes the input argument `delay`. `delay` is a nonnegative integer associated with the number of computational delays of the controller. The default value of the delay is 0.

`[gaml,gamu] = sdhinfnorm(sdsys,k,h,delay,tol)` includes the input argument, `tol`, which defines the difference between upper and lower bounds when search terminates. The default value of `tol` is 0.001.

Example

Consider an open-loop, continuous-time transfer function `p = 30/s(s+30)` and a continuous-time controller `k = 4/(s+4)`. The closed-loop continuous-time system has a peak magnitude across frequency of 1.

```
p = ss(tf(30,[1 30])*tf([1],[1 0]));
k = ss(tf(4,[1 4]));
cl = feedback(p,k);
norm(cl,'inf')
ans =
```

```
       1
```

Initially the controller is to be implemented at a sample rate of 1.5 Hz. The sample-data norm of the closed-loop system with the discrete-time controller is 1.0.

```
kd = c2d(k,0.75,'zoh');
[gu,gl] = sdhinfnorm([1; 1]*p*[1 1],-kd);
[gu gl]
ans =
    3.7908    3.7929
```

Because of the large difference in norm between the continuous-time and sampled-data closed-loop system, the sample rate of the controller is increased from 1.5 Hz to 5 Hz. The sample-data norm of the new closed-loop system is 3.79.

```
kd = c2d(k,0.2,'zoh');
[gu,gl] = sdhinfnorm([1; 1]*p*[1 1],-kd);
[gu gl]
ans =
    1.0044    1.0049
```

Algorithm

`sdhinfnorm` uses variations of the formulas described in the Bamieh and Pearson paper to obtain an equivalent discrete-time system. (These variations are done to improve the numerical conditioning of the algorithms.) A preliminary step is to determine whether the norm of the continuous-time system over one sampling period without control is less than the given value. This requires a search and is, computationally, a relatively expensive step.

Reference

Bamieh, B.A., and J.B. Pearson, "A General Framework for Linear Periodic Systems with Applications to Sampled-Data Control," *IEEE Transactions on Automatic Control,* Vol. AC–37, 1992, pp. 418-435.

See Also

`gapmetric`	Computes the gap and the Vinnicombe gap metric
`hinfsyn`	Synthesizes a H_∞ optimal controller
`norm`	Calculates the system norm of an LTI object
`sdhinfsyn`	Synthesizes a sample-data H_∞ optimal controller
`sdlsim`	Simulates response of a sampled-data feedback system

Purpose

Compute H_∞ controller for sampled-data system

Syntax

```
[K,GAM]=sdhinfsyn(P,NMEAS,NCON)
[K,GAM]=sdhinfsyn(P,NMEAS,NCON, KEY1,VALUE1,KEY2,VALUE2,...)
```

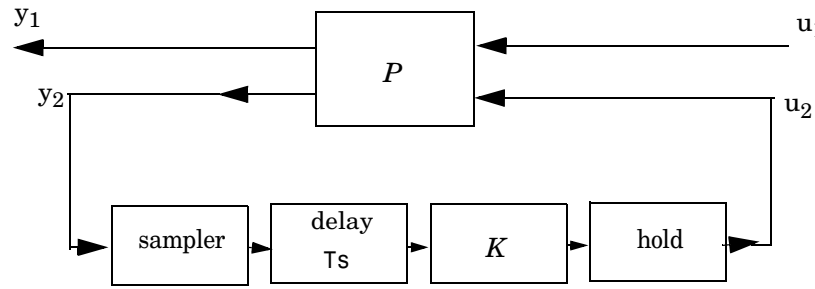
Description

`sdhinfsyn` controls a continuous-time LTI system P with a discrete-time controller `K`. The continuous-time LTI plant `P` has a state-space realization partitioned as follows:

$$P = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & 0 \\ C_2 & 0 & 0 \end{array}\right]$$

where the continuous-time disturbance inputs enter through B_1, the outputs from the controller are held constant between sampling instants and enter through B_2, the continuous-time errors (to be kept small) correspond to the C_1 partition, and the output measurements that are sampled by the controller correspond to the C_2 partition. B_2 has column size `ncon` and C_2 has row size `nmeas`. Note that the D matrix must be zero.

`sdhinfsyn` synthesizes a discrete-time LTI controller `K` to achieve a given norm (if possible) or find the minimum possible norm to within tolerance `TOLGAM`.



Similar to `hinfsyn`, the function `sdhinfsyn` employs a γ iteration. Given a high and low value of γ, `GMAX` and `GMIN`, the bisection method is used to iterate on the value of γ in an effort to approach the optimal H_∞ control design. If `GMAX = GMIN`, only one γ value is tested. The stopping criterion for the bisection algorithm

requires that the relative difference between the last γ value that failed and the last γ value that passed be less than `TOLGAM`.

Input arguments:

`P`	LTI plant
`NMEAS`	Number of measurements output to controller
`NCON`	Number of control inputs

Optional input arguments (`KEY, VALUE`) pairs are similar to `hinfsyn`, but with additional `KEY` values `'Ts'` and `'DELAY'`.

KEY	VALUE	Meaning
`'GMAX'`	real	Initial upper bound on GAM (default=`Inf`)
`'GMIN'`	real	Initial lower bound on GAM (default=0)
`'TOLGAM'`	real	Relative error tolerance for GAM (default=.01)
`'Ts'`	real	(Default=1) sampling period of the controller to be designed
`'DELAY'`	integer	(Default=0) a nonnegative integer giving the number of sample periods delay for the control computation
`'DISPLAY'`	`'off'` `'on'`	(Default) no command window display, or the command window displays synthesis progress information

Output arguments:

`K`	H_∞ controller
`GAM`	Final γ value of H_∞ cost achieved

Algorithm

`sdhinfsyn` uses a variation of the formulas described in the Bamieh and Pearson paper [1] to obtain an equivalent discrete-time system. (This is done to improve the numerical conditioning of the algorithms.) A preliminary step is

to determine whether the norm of the continuous-time system over one sampling period without control is less than the given γ-value. This requires a search and is computationally a relatively expensive step.

Reference

[1] Bamieh, B.A., and J.B. Pearson, "A General Framework for Linear Periodic Systems with Applications to Sampled-Data Control," *IEEE Transactions on Automatic Control,* Vol. AC–37, 1992, pp. 418-435.

See Also

`norm`	System norm of an LTI object
`hinfsyn`	Synthesize an H_∞ optimal controller
`sdhinfnorm`	Calculate norm of sampled-data feedback system

sdlsim

Purpose

Time response of sampled-data feedback system

Syntax

```
sdlsim(p,k,w,t,tf)
sdlsim(p,k,w,t,tf,x0,z0)
sdlsim(p,k,w,t,tf,x0,z0,int)
[vt,yt,ut,t] = sdlsim(p,k,w,t,tf)
[vt,yt,ut,t] = sdlsim(p,k,w,t,tf,x0,z0,int)
```

Description

`sdlsim(p,k,w,t,tf)` plots the time response of the hybrid feedback system. `lft(p,k)`, is forced by the continuous input signal described by `w` and `t` (values and times, as in `lsim`). `p` must be a continuous-time LTI system, and `k` must be discrete-time LTI system with a specified sampling time (the unspecified sampling time -1 is not allowed). The final time is specified with `tf`.

`sdlsim(p,k,w,t,tf,x0,z0)` specifies the initial state vector `x0` of `p`, and `z0` of `k`, at time `t(1)`.

`sdlsim(p,k,w,t,tf,x0,z0,int)` specifies the continuous-time integration step size `int`. `sdlsim` forces `int = (k.Ts)/N int` where `N`>4 is an integer. If any of these optional arguments is omitted, or passed as empty matrices, then default values are used. The default value for `x0` and `z0` is zero. Nonzero initial conditions are allowed for `p` (and/or `k`) only if `p` (and/or `k`) is an `ss` object.

If `p` and/or `k` is an LTI array with consistent array dimensions, then the time simulation is performed pointwise across the array dimensions.

`[vt,yt,ut,t] = sdlsim(p,k,w,t,tf)` computes the continuous-time response of the hybrid feedback system `lft(p,k)` forced by the continuous input signal defined by `w` and `t` (values and times, as in `lsim`). `p` must be a continuous-time system, and `k` must be discrete-time, with a specified sampling time (the unspecified sampling time -1 is not allowed). The final time is specified with `tf`. The outputs `vt`, `yt` and `ut` are 2-by-1 cell arrays: in each the first entry is a time vector, and the second entry is the signal values. Stored in this manner, the signal `vt` is plotted by using one of the following commands:

```
plot(vt{1},vt{2})
plot(vt{:})
```

Signals `yt` and `ut` are respectively the input to `k` and output of `k`.

If p and/or k are LTI arrays with consistent array dimensions, then the time simulation is performed pointwise across the array dimensions. The outputs are 2-by-1-by-array dimension cell arrays. All responses can be plotted simultaneously, for example, plot(vt).

[vt,yt,ut,t] = sdlsim(p,k,w,t,tf,x0,z0,int) The optional arguments are int (integration step size), x0 (initial condition for p), and z0 (initial condition for k). sdlsim forces int = (k.Ts)/N, where N>4 is an integer. If any of these arguments is omitted, or passed as empty matrices, then default values are used. The default value for x0 and z0 is zero. Nonzero initial conditions are allowed for p (and/or k) only if p (and/or k) is an ss object.

Example

To illustrate the use of sdlsim, consider the application of a discrete controller to an integrator with near integrator. A continuous plant and a discrete controller are created. A sample and hold equivalent of the plant is formed and the discrete closed-loop system is calculated. Simulating this with lsim gives the system response at the sample points. sdlsim is then used to calculate the intersample behavior.

```
P = tf(1,[1, 1e-5,0]);
T = 1.0/20;
C = ss([-1.5 T/4; -2/T -.5],[ .5 2;1/T 1/T],...
   [-1/T^2  -1.5/T], [1/T^2  0],T);
Pd = c2d(P,T,'zoh');
```

The closed-loop digital system is now set up. You can use sysic to construct the interconnected feedback system.

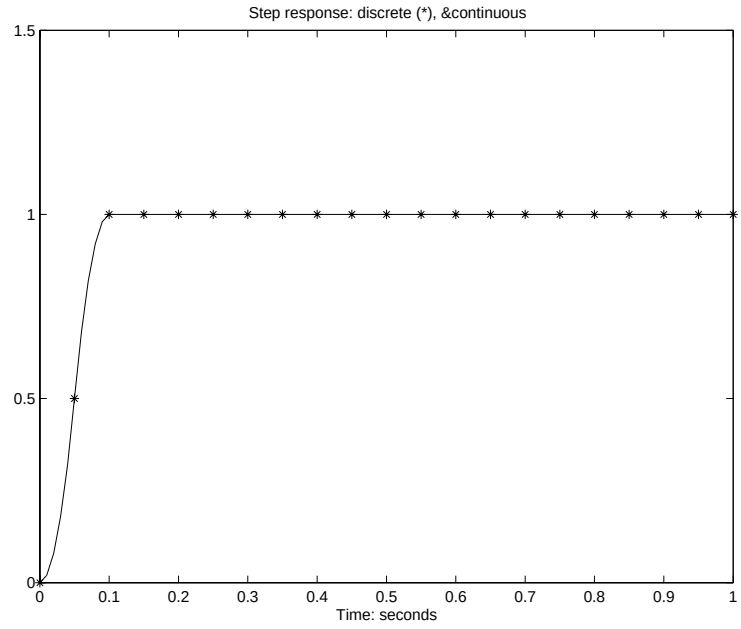
```
systemnames = 'Pd C';
inputvar = '[ref]';
outputvar = '[Pd]';
input_to_Pd = '[C]';
input_to_C = '[ref ; Pd]';
sysoutname = 'dclp';
cleanupsysic = 'yes';
sysic;
```

lsim is used to simulate the digital step response.

```
[yd,td] = step(dclp,20*T);
```

The continuous interconnection is set up and the sampled data response is calculated with `sdlsim`.

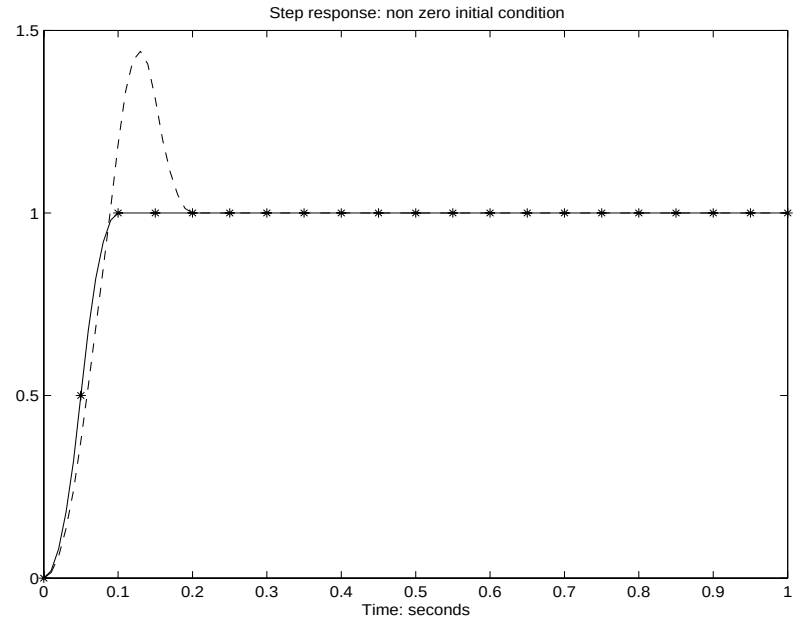
```
M = [0,1;1,0;0,1]*blkdiag(1,P);
t = [0:.01:1]';
u = ones(size(t));
y1 = sdlsim(M,C,u,t);
plot(td,yd,'r*',y1{:},'b-')
axis([0,1,0,1.5])
xlabel('Time: seconds')
title('Step response: discrete (*), &continuous')
```



You can see the effect of a nonzero initial condition in the continuous-time system. Note how examining the system at only the sample points will underestimate the amplitude of the overshoot.

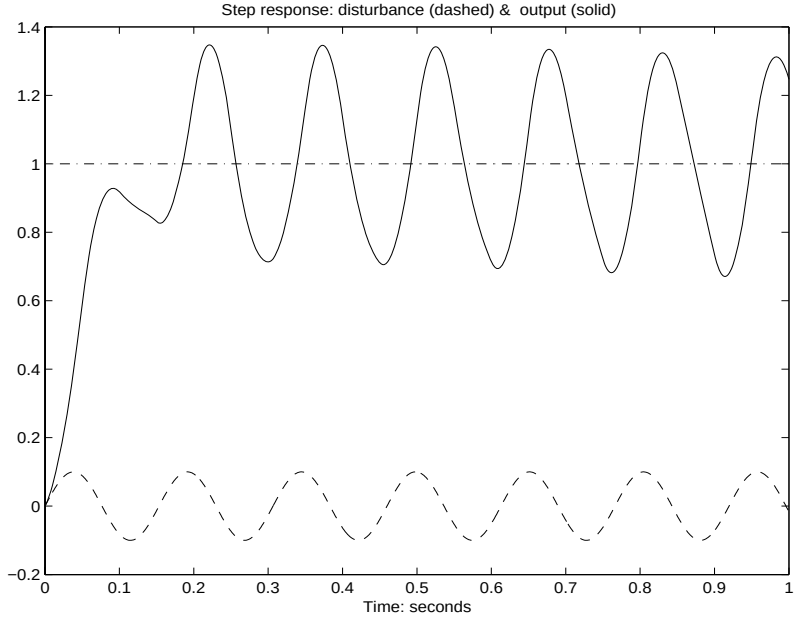
```
y2 = sdlsim(M,C,u,t,1,0,[0.25;0]);
plot(td,yd,'r*',y1{:},'b-',y2{:},'g--')
```

```
axis([0,1,0,1.5])
xlabel('Time: seconds')
title('Step response: non zero initial condition')
```



Finally, you can examine the effect of a sinusoidal disturbance at the continuous-time plant output. This controller is not designed to reject such a disturbance and the system does not contain antialiasing filters. Simulating the effect of antialiasing filters is easily accomplished by including them in the continuous interconnection structure.

```
M2 = [0,1,1;1,0,0;0,1,1]*blkdiag(1,1,P);
t = [0:.001:1]';
dist = 0.1*sin(41*t);
u = ones(size(t));
[y3,meas,act] = sdlsim(M2,C,[u dist],t,1);
plot(y3{:},'-',t,dist,'b--',t,u,'g-.')
xlabel('Time: seconds')
title('Step response: disturbance (dashed) &  output (solid)')
```



Algorithm

`sdlsim` oversamples the continuous-time, `N` times the sample rate of the controller `k`.

See Also

`gapmetric`	Computes the gap and the Vinnicombe gap metric
`hinfsyn`	Synthesizes a H_∞ optimal controller
`norm`	Computes the system norm of an LTI object
`sdhinfnorm`	Calculates the norm of a sampled-data feedback system
`sdhinfsyn`	Synthesizes a sample-data H_∞ optimal controller

Purpose

State-space sector bilinear transformation

Syntax

```
[G,T] = sectf(F,SECF,SECG)
```

Description

`[G,T] = sectf(F,SECF,SECG)` computes a linear fractional transform `T` such that the system `lft(F,K)` is in sector `SECF` if and only if the system `lft(G,K)` is in sector `SECG` where

```
G=lft(T,F,NU,NY)
```

where `NU` and `NY` are the dimensions of u_{T2} and y_{T2}, respectively—see Figure 5-17.

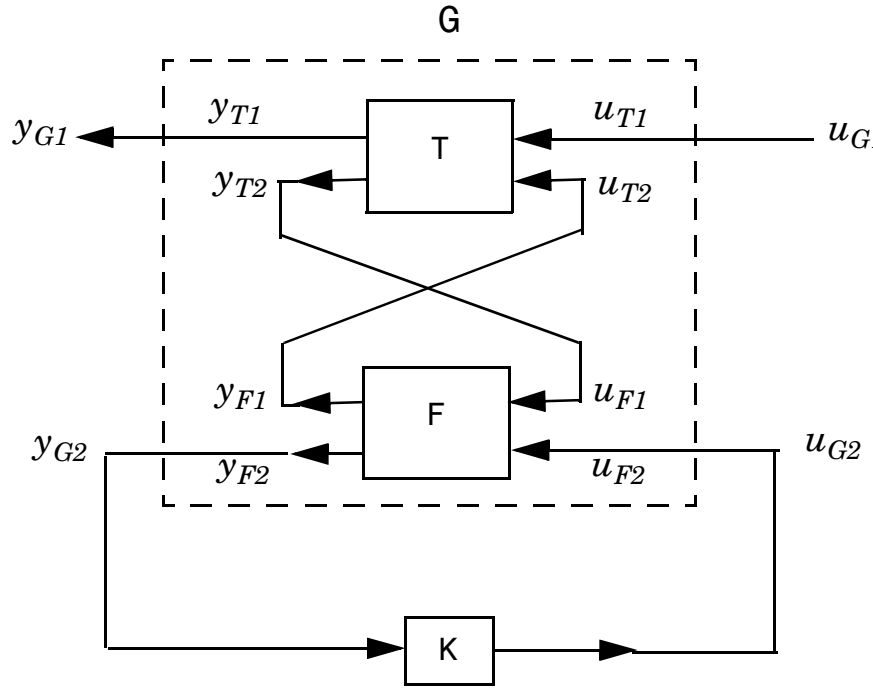


Figure 5-17: Sector transform `G=lft(T,F,NU,NY)`.

`sectf` are used to transform general conic-sector control system performance specifications into equivalent H_∞-norm performance specifications.

Input Arguments:		
`F`		LTI state-space plant
`SECG, SECF:`		Conic Sector:
	`[-1,1]` or `[-1;1]`	$\|y\|^2 \le \|u\|^2$
	`[0,Inf]` or `[0;Inf]`	$0 \le Re[y^*u]$
	`[A,B]` or `[A;B]`	$0 \ge \mathrm{Re}[(y - Au)^*(y - Bu)]$
	`[a,b]` or `[a;b]`	$0 \ge \mathrm{Re}[(y - diag(a)u)^*(y - diag(b)u)]$
	`S`	$0 \ge \mathrm{Re}[(S_{11}u + S_{12}y)^*(S_{21}u + S_{22}y)]$
	`S`	$0 \ge \mathrm{Re}[(S_{11}u + S_{12}y)^*(S_{21}u + S_{22}y)]$

where `A,B` are scalars in [–∞, ∞] or square matrices; `a,b` are vectors; `S=[S11 S12;S21,S22]` is a square matrix whose blocks `S11,S12,S21,S22` are either scalars or square matrices; `S` is a two-port system `S=mksys(a,b1,b2, ,'tss')` with transfer function

$$S(s) = \begin{bmatrix} S_{11}(s) & S_{12}(s) \\ S_{21}(s) & S_{22}(s) \end{bmatrix}$$

Output Arguments:	
`G`	Transformed plant $G(s)$=`lftf(T,F)`
`T`	LFT sector transform, maps conic sector `SECF` into conic sector `SECG`

Output variables `are`:

`G`	The transformed plant $G(s)$=`lftf(T,F)`:
`T`	The linear fractional transformation $T(s)$=`T`

Examples

The statement G(jω) inside *sector*[–1, 1] is equivalent to the H_∞ inequality

$$\sup_{\omega} \bar{\sigma}(G(j\omega)) = \|G\|_\infty \leq 1$$

Given a two-port open-loop plant $P(s)$:= `P`, the command `P1 = sectf(P,[0,Inf],[-1,1])` computes a transformed $P_1(s)$:= `P1` such that if `lft(G,K)` is inside *sector*[–1, 1] if and only if `lft(F,K)` is inside *sector*[0, ∞]. In other words, `norm(lft(G,K),inf)<1` if and only if `lft(F,K) is` strictly positive real. See Figure 5-19

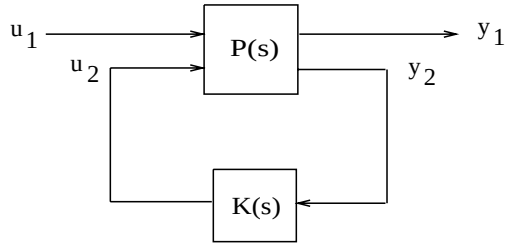


Figure 5-18: Sector Transform Block Diagram.

Here is a simple example of the sector transform.

$$P(s) = \frac{1}{s+1} \in \text{sec}tor[-1, 1] \rightarrow P_1(s) = \frac{s+2}{s} \in \text{sec}tor[0,\infty].$$

You can compute this by simply executing the following commands:

```
P = ss(tf(1,[1 1]));
P1 = sectf(P,[-1,1],[0,Inf]);
```

The Nyquist plots for this transformation are depicted in Figure 5-19, Example of Sector Transform.. The condition $P_1(s)$ inside [0, ∞] implies that $P_1(s)$ is stable and $P_1(j\omega)$ is *positive real*, i.e.,

$$P_1^*(j\omega) + P_1(j\omega) \geq 0 \quad \forall \omega$$

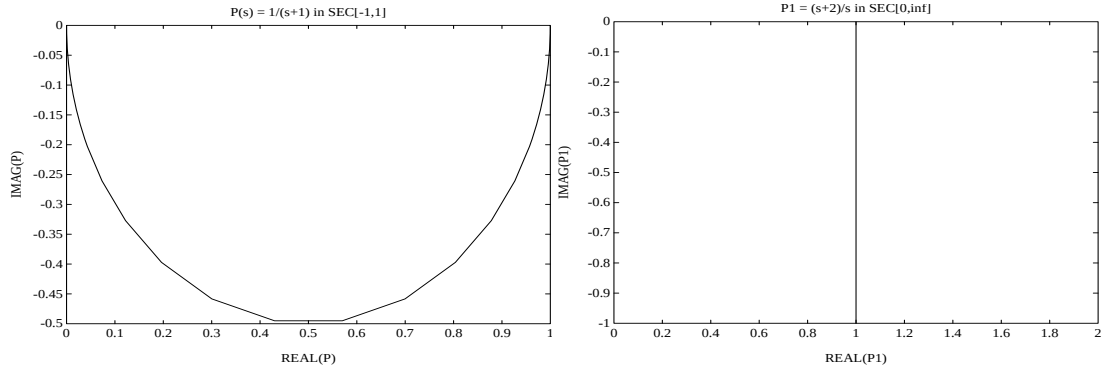


Figure 5-19: Example of Sector Transform.

Algorithm

`sectf` uses the generalization of the sector concept of [3] described by [1]. First the sector input data `Sf= SECF` and `Sg=SECG` is converted to two-port state-space form; non-dynamical sectors are handled with empty *a*, *b1*, *b2*, *c1*, *c2* matrices. Next the equation

$$S_g(s)\begin{bmatrix} u_{g_1} \\ y_{g_1} \end{bmatrix} = S_f(s)\begin{bmatrix} u_{f_1} \\ y_{f_1} \end{bmatrix}$$

is solved for the two-port transfer function $T(s)$ from $u_{g_1} y_{f_1}$ to $u_{f_1} y_{g_1}$. Finally, the function `lftf` is used to compute $G(s)$ as G=lftf(T,F).

Limitations

A well-posed conic sector must have $\det(B - A) \neq 0$ or $\det\left(\begin{bmatrix} s_{11} & s_{12}; \\ s_{21} & s_{22} \end{bmatrix}\right) \neq 0$.

Also, you must have $\dim(u_{F1}) = \dim(y_{F1})$ since sectors are only defined for square systems.

References

[1] Safonov, M.G., *Stability and Robustness of Multivariable Feedback Systems*. Cambridge, MA: MIT Press, 1980.

[2] Safonov, M.G., E.A. Jonckheere, M. Verma and D.J.N. Limebeer, "Synthesis of Positive Real Multivariable Feedback Systems," *Int. J. Control*, vol. 45, no. 3, pp. 817-842, 1987.

[3] Zames, G., "On the Input-Output Stability of Time-Varying Nonlinear Feedback Systems ≥— Part I: Conditions Using Concepts of Loop Gain, Conicity, and Positivity," *IEEE Trans. on Automat. Contr.*, AC-11, pp. 228-238, 1966.

See Also

`lft`	Forms Redheffer star product of systems
`hinfsyn`	H_∞ controller synthesis

frd/semilogx

Purpose

Semilog scale plot of `frd` object

Syntax

```
semilogx(...) same as plot
semilogy(...) same as plot
```

Description

`semilogx(...)` is the same as `plot(...)`, except a logarithmic (base 10) scale is used for the x-axis.

`semilogy(...)` is the same as `plot(...)`, except a logarithmic (base 10) scale is used for the y-axis.

See Also

`loglog`	Plot `frd` object on a log-log scale
`plot`	Plot `frd` object on a linear scale
`semilogy`	Plot `frd` object on a semi-log scale

Purpose

Initialize description of LMI system

Syntax

```
setlmis(lmi0)
```

Description

Before starting the description of a new LMI system with `lmivar` and `lmiterm`, type

```
setlmis([])
```

to initialize its internal representation.

To add on to an existing LMI system, use the syntax

```
setlmis(lmi0)
```

where `lmi0` is the internal representation of this LMI system. Subsequent `lmivar` and `lmiterm` commands will then add new variables and terms to the initial LMI system `lmi0`.

See Also

`getlmis`	Get the internal description of an LMI system
`lmivar`	Specify the matrix variables in an LMI problem
`lmiterm`	Specify the term content of LMIs
`newlmi`	Attach an identifying tag to LMIs

Purpose

Instantiate matrix variable and evaluate all LMI terms involving this matrix variable

Syntax

```
newsys = setmvar(lmisys,X,Xval)
```

Description

`setmvar` sets the matrix variable X with identifier `X` to the value `Xval`. All terms involving X are evaluated, the constant terms are updated accordingly, and X is removed from the list of matrix variables. A description of the resulting LMI system is returned in `newsys`.

The integer `X` is the identifier returned by `lmivar` when X is declared. Instantiating X with `setmvar` does not alter the identifiers of the remaining matrix variables.

The function `setmvar` is useful to freeze certain matrix variables and optimize with respect to the remaining ones. It saves time by avoiding partial or complete redefinition of the set of LMI constraints.

Example

Consider the system

$$\dot{x} = Ax + Bu$$

and the problem of finding a stabilizing state-feedback law $u = Kx$ where K is an unknown matrix.

By the Lyapunov Theorem, this is equivalent to finding $P > 0$ and K such that

$$(A + BK)P + P(A + BK)^T + I < 0.$$

With the change of variable $Y := KP$, this condition reduces to the LMI

$$AP + PA^T + BY + Y^TB^T + I < 0.$$

This LMI is entered by the commands

```
n = size(A,1) % number of states
ncon = size(B,2) % number of inputs

setlmis([])
P = lmivar(1,[n 1]) % P full symmetric
Y = lmivar(2,[ncon n]) % Y rectangular

lmiterm([1 1 1 P],A,1,'s') % AP+PA'
```

```
lmiterm([1 1 1 Y],B,1,'s') % BY+Y'B'
lmiterm([1 1 1 0],1) % I
lmis = getlmis
```

To find out whether this problem has a solution K for the particular Lyapunov matrix $P = I$, set P to I by typing

```
news = setmvar(lmis,P,1)
```

The resulting LMI system `news` has only one variable $Y = K$. Its feasibility is assessed by calling `feasp`:

```
[tmin,xfeas] = feasp(news)
Y = dec2mat(news,xfeas,Y)
```

The computed Y is feasible whenever `tmin` < 0.

See Also

`evallmi`	Given a particular instance of the decision variables, evaluate all variable terms in the system of LMIs
`delmvar`	Delete one of the matrix variables of an LMI problem

showlmi

Purpose

Return left- and right-hand sides of LMI after evaluation of all variable terms

Syntax

```
[lhs,rhs] = showlmi(evalsys,n)
```

Description

For given values of the decision variables, the function `evallmi` evaluates all variable terms in a system of LMIs. The left- and right-hand sides of the `n`-th LMI are then constant matrices that can be displayed with `showlmi`. If `evalsys` is the output of `evallmi`, the values `lhs` and `rhs` of these left- and right-hand sides are given by

```
[lhs,rhs] = showlmi(evalsys,n)
```

An error is issued if `evalsys` still contains variable terms.

Example

See the description of `evallmi`.

See Also

`evallmi`	Given a particular instance of decision variables, evaluate all variable terms in an LMI system
`setmvar`	Instantiate a matrix variable and evaluate all LMI terms involving this matrix variable

Purpose

Simplify representation of uncertain object

Syntax

```
B = simplify(A)
B = simplify(A,'full')
B = simplify(A,'basic')
B = simplify(A,'class')
```

Description

`B = simplify(A)` performs model-reduction-like techniques to detect and eliminate redundant copies of uncertain elements. Depending on the result, the class of `B` may be lower than `A`. The `AutoSimplify` property of each uncertain element in `A` governs what reduction methods are used. After reduction, any uncertain element which does not actually affect the result is deleted from the representation.

`B = simplify(A,'full')` overrides all uncertain element's `AutoSimplify` property, and uses `'full'` reduction techniques.

`B = simplify(A,'basic')` overrides all uncertain element's `AutoSimplify` property, and uses `'basic'` reduction techniques.

`B = simplify(A,'class')` does not perform reduction. However, any uncertain elements in `A` with zero occurences are eliminated, and the class of `B` may be lower than the class of `A`.

Example

Create a simple `umat` with a single uncertain real parameter. Select specific elements, note that result remains in class `umat`. Simplify those same elements, and note that class changes.

```
p1 = ureal('p1',3,'Range',[2 5]);
L = [2 p1];
L(1)
UMAT: 1 Rows, 1 Columns
L(2)
UMAT: 1 Rows, 1 Columns
  p1: real, nominal = 3, range = [2  5], 1 occurrence
simplify(L(1))
ans =
     2
simplify(L(2))
```

```
  Uncertain Real Parameter: Name p1, NominalValue 3, Range [2  5]
```

Create four uncertain real parameters, with a default value of `AutoSimplify ('basic')`, and define a high order polynomial [1].

```
m = ureal('m',125000,'Range',[100000 150000]);
xcg = ureal('xcg',.23,'Range',[.15 .31]);
zcg = ureal('zcg',.105,'Range',[0 .21]);
va = ureal('va',80,'Range',[70 90]);
cw = simplify(m/(va*va)*va,'full')
UMAT: 1 Rows, 1 Columns
   m: real, nominal = 1.25e+005, range = [100000  150000], 1
occurrence
  va: real, nominal = 80, range = [70  90], 1 occurrence
cw = m/va;
fac2 = .16726*xcg*cw*cw*zcg - .17230*xcg*xcg*cw ...
      -3.9*xcg*cw*zcg - .28*xcg*xcg*cw*cw*zcg ...
      -.07*xcg*xcg*zcg + .29*xcg*xcg*cw*zcg ...
      + 4.9*xcg*cw - 2.7*xcg*cw*cw ...
      +.58*cw*cw - 0.25*xcg*xcg - 1.34*cw ...
      +100.1*xcg -14.1*zcg - 1.91*cw*cw*zcg ...
      +1.12*xcg*zcg + 24.6*cw*zcg ...
      +.45*xcg*xcg*cw*cw - 46.85
UMAT: 1 Rows, 1 Columns
    m: real, nominal = 1.25e+005, range = [100000  150000], 18
occurrences
  va: real, nominal = 80, range = [70  90], 8 occurrences
 xcg: real, nominal = 0.23, range = [0.15  0.31], 18 occurrences
  zcg: real, nominal = 0.105, range = [0  0.21], 1 occurrence
```

The result of the high-order polynomial is an inefficient representation involving 18 copies of `m`, 8 copies of `va`, 18 copies of `xcg` and 1 copy of `zcg`. Simplify the expression, using the `'full'` simplification algorithm

```
fac2s = simplify(fac2,'full')
UMAT: 1 Rows, 1 Columns
    m: real, nominal = 1.25e+005, range = [100000  150000], 4
occurrences
  va: real, nominal = 80, range = [70  90], 4 occurrences
 xcg: real, nominal = 0.23, range = [0.15  0.31], 2 occurrences
  zcg: real, nominal = 0.105, range = [0  0.21], 1 occurrence
```

which results in a much more economical representation.

Alternatively, change the `AutoSimplify` property of each parameter to `'full'` before forming the polynomial.

```
m.AutoSimplify = 'full';
xcg.AutoSimplify = 'full';
zcg.AutoSimplify = 'full';
va.AutoSimplify = 'full';
```

You can form the polynomial, which immediately gives a low order representation.

```
cw = m/va;
fac2f = .16726*xcg*cw*cw*zcg - .17230*xcg*xcg*cw ...
      -3.9*xcg*cw*zcg - .28*xcg*xcg*cw*cw*zcg ...
      -.07*xcg*xcg*zcg + .29*xcg*xcg*cw*zcg ...
      + 4.9*xcg*cw - 2.7*xcg*cw*cw ...
      +.58*cw*cw - 0.25*xcg*xcg - 1.34*cw ...
      +100.1*xcg -14.1*zcg - 1.91*cw*cw*zcg ...
      +1.12*xcg*zcg + 24.6*cw*zcg ...
      +.45*xcg*xcg*cw*cw - 46.85
UMAT: 1 Rows, 1 Columns
    m: real, nominal = 1.25e+005, range = [100000  150000], 4
occurrences
  va: real, nominal = 80, range = [70  90], 4 occurrences
 xcg: real, nominal = 0.23, range = [0.15  0.31], 2 occurrences
  zcg: real, nominal = 0.105, range = [0  0.21], 1 occurrence
```

Create two real parameters, `da` and `dx`, and a 2-by-3 matrix, `ABmat`, involving polynomial expressions in the two real parameters [2].

```
da = ureal('da',0,'Range',[-1 1]);
dx = ureal('dx',0,'Range',[-1 1]);
a11 = -.32 + da*(.8089 + da*(-.987 + 3.39*da)) + .15*dx;
a12 = .934 + da*(.0474 - .302*da);
a21 = -1.15 + da*(4.39 + da*(21.97 - 561*da*da)) ...
     + dx*(9.65 - da*(55.7 + da*177));
a22 = -.66 + da*(1.2 - da*2.27) + dx*(2.66 - 5.1*da);
b1 = -0.00071 + da*(0.00175 - da*.00308) + .0011*dx;
b2 = -0.031 + da*(.078 + da*(-.464 + 1.37*da)) + .0072*dx;
ABmat = [a11 a12 b1;a21 a22 b2]
```

```
UMAT: 2 Rows, 3 Columns
  da: real, nominal = 0, range = [-1  1], 19 occurrences
  dx: real, nominal = 0, range = [-1  1], 2 occurrences
```

Use `'full'` simplification to reduce the complexity of the description.

```
ABmatsimp = simplify(ABmat,'full')
UMAT: 2 Rows, 3 Columns
  da: real, nominal = 0, range = [-1  1], 7 occurrences
  dx: real, nominal = 0, range = [-1  1], 2 occurrences
```

Alternatively, you can set the parameter's `AutoSimplify` property to `'full'`.

```
da.AutoSimplify = 'full';
dx.AutoSimplify = 'full';
```

Now you can rebuild the matrix

```
a11 = -.32 + da*(.8089 + da*(-.987 + 3.39*da)) + .15*dx;
a12 = .934 + da*(.0474 - .302*da);
a21 = -1.15 + da*(4.39 + da*(21.97 - 561*da*da)) ...
     + dx*(9.65 - da*(55.7 + da*177));
a22 = -.66 + da*(1.2 - da*2.27) + dx*(2.66 - 5.1*da);
b1 = -0.00071 + da*(0.00175 - da*.00308) + .0011*dx;
b2 = -0.031 + da*(.078 + da*(-.464 + 1.37*da)) + .0072*dx;
ABmatFull = [a11 a12 b1;a21 a22 b2]
UMAT: 2 Rows, 3 Columns
  da: real, nominal = 0, range = [-1  1], 7 occurrences
  dx: real, nominal = 0, range = [-1  1], 2 occurrences
```

Algorithm

`simplify` uses heuristics along with one-dimensional model reduction algorithms to partially reduce the dimensionality of the representation of an uncertain matrix or system.

Limitations

Multidimensional model reduction and realization theory are only partially complete theories. The heuristics used by `simplify` are that - heuristics. The order in which expressions involving uncertain elements are built up, eg., distributing across addition and multiplication, can affect the details of the representation (i.e., the number of occurences of a `ureal` in an uncertain matrix). It is possible that `simplify`'s naive methods cannot completely resolve

these differences, so one may be forced to work with “nonminimal” representations of uncertain systems.

References

[1] Varga, A. and G. Looye, “Symbolic and numerical software tools for LFT-based low order uncertainty modeling,” *IEEE International Symposium on Computer Aided Control System Design,* 1999, pp. 5-11.

[2] Belcastro, C.M., K.B. Lim and E.A. Morelli, “Computer aided uncertainty modeling for nonlinear parameter-dependent systems Part II: F-16 example,” *IEEE International Symposium on Computer Aided Control System Design,* 1999, pp. 17-23.

See Also

`umat`	Creates an uncertain matrix object
`uss`	Creates an uncertain system object
`ucomplex`	Creates an uncertain complex parameter
`ureal`	Creates an uncertain real parameter
`uss`	Creates an uncertain system

skewdec

Purpose

Form skew-symmetric matrix

Syntax

```
x = skewdec(m,n)
```

Description

`skewdec(m,n)` forms the m-by-m skew-symmetric matrix

$$\begin{bmatrix} 0 & -(n-1) & -(n-2) & \dots \\ (n+1) & 0 & -(n-3) & \dots \\ (n+2) & (n+3) & 0 & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

This function is useful to define skew-symmetric matrix variables. In this case, set `n` to the number of decision variables already used.

See Also

`decinfo`	Describe how the entries of a matrix variable X relate to the decision variables
`lmivar`	Specify the matrix variables in an LMI problem

Purpose

Slow and fast modes decomposition

Syntax

```
[G1,G2] = slowfast(G,cut)
```

Description

`slowfast` computes the slow and fast modes decompositions of a system $G(s)$ such that

$$G(s) = [G(s)]_s + [G(s)]_f$$

where $[G(s)]_s := (\hat{A}_{11}, \hat{B}_1, \hat{C}_1, \hat{D}_1)$ denotes the slow part of $G(s)$, and $[G(s)]_f := (\hat{A}_{22}, \hat{B}_2, \hat{C}_2, \hat{D}_2)$ denotes the fast part. The variable cut denotes the index where the modes will be split.

`stabproj` employs the algorithm in [1] as follows:

Find a unitary matrix V via the ordered Schur decomposition routines `blksch` or `rschur` such that

$$A = V^T A V = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ 0 & \hat{A}_{22} \end{bmatrix}$$

Based on the style of ordered Schur form, you get $\left|\lambda_i(\hat{A}_{11})\right| < \left|\lambda_i(\hat{A}_{22})\right|$.

Finally solving the matrix equation for X

$$\hat{A}_{11}X - X\hat{A}_{22} + \hat{A}_{12} = 0$$

you get the state-space projections

$$[G(s)]_s := (\hat{A}_{11}, \hat{B}_1, \hat{C}_1, \hat{D}_1)$$

and

$$[G(s)]_f := (\hat{A}_{22}, \hat{B}_2, \hat{C}_2, \hat{D}_2)$$

where

$$\begin{bmatrix}\hat{B}_1\\ \hat{B}_2\end{bmatrix} := \begin{bmatrix}I & -X\\ 0 & I\end{bmatrix} VB$$

and

$$\left[\hat{C}_1 \middle| \hat{C}_2\right] := CV^T\begin{bmatrix}I & X\\ 0 & I\end{bmatrix}$$

References

[1] M.G. Safonov, E.A. Jonckheere, M. Verma and D.J.N. Limebeer, “Synthesis of Positive Real Multivariable Feedback Systems”, *Int. J. Control*, vol. 45, no. 3, pp. 817-842, 1987.

See Also

`schur`	Schur decomposition
`modreal`	Modal form realization

Purpose

Remove singleton dimensions for `umat` objects

Syntax

```
B = squeeze(A)
```

Description

`B = squeeze(A)` returns an array `B` with the same elements as `A` but with all the singleton dimensions removed. A singleton is a dimension such that `size(A,dim)==1`. 2-D arrays are unaffected by squeeze so that row vectors remain rows.

See Also

`permute`	Permutes array dimensions.
`reshape`	Changes size of matrix

uss/ssbal

Purpose

Scale state/uncertainty while preserving uncertain input/output map of uncertain system

Syntax

```
usysout = ssbal(usys)
usysout = ssbal(usys,Wc)
usysout = ssbal(usys,Wc,FSflag)
usysout = ssbal(usys,Wc,FSflag,BLTflag)
```

Description

`usysout = ssbal(usys)` yields a system whose input/output and uncertain properties are the same as `usys`, a `uss` object. The numerical conditioning of `usysout` is usually better than that of `usys`, improving the accuracy of additional computations performed with `usysout`. `usysout` is a `uss` object. The balancing algorithm uses `mussv` to balance the constant uncertain state-space matrices in discrete time. If `usys` is a continuous-time uncertain system, the uncertain state-space is mapped by using a bilinear transformation into discrete time for balancing.

`usysout = ssbal(usys,wc)` defines the critical frequency `wc` for the bilinear prewarp transformation from continuous time to discrete time. The default value of `wc` is 1 when the nominal uncertain system is stable and `1.25*mxeig` when it is unstable. `mxeig` corresponds to the value of the real, most positive pole of `usys`.

`usysout = ssbal(usys,wc,FSflag)` sets the scaling flag `FSflag` to handle repeated uncertain parameters. Setting `FSflag=1` uses full matrix scalings to balance the repeated uncertain parameter blocks. `FSflag=0`, the default, uses a single, positive scalar to balance the repeated uncertain parameter blocks.

`usysout = ssbal(usys,wc,FSflag,BLTflag)` sets the bilinear transformation flag, `BLTflag`. By default, `BLTflag=1` and transforms the continuous-time system `usys` to a discrete-time system for balancing. `BLTflag=0` results in balancing the continuous-time state-space data from `usys`. Note that if `usys` is a discrete-time system, no bilinear transformation is performed.

`ssbal` does not work on an array of uncertain systems. An error message is generated to alert you to this.

Example

Consider a two-input, two-output, two-state uncertain system with two real parameter uncertainties, `p1` and `p2`.

```
p2=ureal('p2',-17,'Range',[-19 -11]);
p1=ureal('p1',3.2,'Percentage',0.43);
A = [-12 p1;.001 p2];
B = [120 -809;503 24];
C = [.034 .0076; .00019 2];
usys = ss(A,B,C,zeros(2,2))
USS: 2 States, 2 Outputs, 2 Inputs, Continuous System
  p1: real, nominal = 3.2, variability = [-0.43  0.43]%, 1
occurrence
  p2: real, nominal = -17, range = [-19  -11], 1 occurrence
usys.NominalValue
a =
          x1     x2
   x1    -12    3.2
   x2  0.001    -17

b =
         u1    u2
   x1   120  -809
   x2   503    24

c =
            x1       x2
   y1    0.034   0.0076
   y2  0.00019        2

d =
       u1  u2
   y1   0   0
   y2   0   0

Continuous-time model.
ssbal is used to balance the uncertain system usys.

usysout = ssbal(usys)
USS: 2 States, 2 Outputs, 2 Inputs, Continuous System
  p1: real, nominal = 3.2, variability = [-0.43  0.43]%, 1
occurrence
  p2: real, nominal = -17, range = [-19  -11], 1 occurrence
```

```
usysout.NominalValue
a =
             x1         x2
   x1       -12     0.3302
   x2  0.009692        -17

b =
           u1      u2
   x1  0.7802   -5.26
   x2    31.7   1.512

c =
            x1       x2
   y1    5.229   0.1206
   y2  0.02922    31.74

d =
       u1  u2
   y1   0   0
   y2   0   0

Continuous-time model.
```

See Also

`canon`	Forms canonical state-space realizations
`c2d`	Converts continuous-time models to discrete-time
`d2c`	Converts discrete-time models to continuous-time
`mussv`	Sets bounds on the Structure Singular Value (μ)
`mussvextract`	Extracts compressed data returned from `mussv`
`ss2ss`	Changes state coordinates for state-space models

Purpose

Stable and antistable projection

Syntax

```
[G1,G2,m] = stabproj(G)
```

Description

`stabproj` computes the stable and antistable projections of a minimal realization $G(s)$ such that $G(s) = [G(s)]_- + [G(s)]_+$ where $[G(s)]_- := (\hat{A}_{11}, \hat{B}_1, \hat{C}_1, \hat{D}_1)$ denotes the stable part of $G(s)$, and $[G(s)]_+ := (\hat{A}_{22}, \hat{B}_2, \hat{C}_2, \hat{D}_2)$ denotes the antistable part. The variable `m` returns the number of stable eigenvalues of A.

Algorithm

`stabproj` employs the algorithm in [1] as follows:

Find a unitary matrix V via the ordered Schur decomposition routines `blksch` or `rschur` such that

$$A = V^T A V = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ 0 & \hat{A}_{22} \end{bmatrix}$$

Based on the style of ordered Schur form, you can get a stable $\hat{A}_{11}$ and an antistable $\hat{A}_{22}$.; $|\lambda_i(\hat{A}_{11})| < |\lambda_i(\hat{A}_{22})|$ for the case of `slowfast`.

Finally solving the matrix equation for X

$$\hat{A}_{11}X - X\hat{A}_{22} + \hat{A}_{12} = 0$$

you get the state-space projections $[G(s)]_-$ or $[G(s)]_s := (\hat{A}_{11}, \hat{B}_1, \hat{C}_1, \hat{D}_1)$

and $[G(s)]_+$ or $G(s)]_{us} := (\hat{A}_{22}, \hat{B}_2, \hat{C}_2, \hat{D}_2)$

where

$$\begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \end{bmatrix} := \begin{bmatrix} I & -X \\ 0 & I \end{bmatrix} VB$$

and

$$\left[\hat{C}_1 | \hat{C}_2\right] := CV^T \begin{bmatrix} I & X \\ 0 & I \end{bmatrix}$$

References

[1] M.G. Safonov, E.A. Jonckheere, M. Verma and D.J.N. Limebeer, “Synthesis of Positive Real Multivariable Feedback Systems”, *Int. J. Control*, vol. 45, no. 3, pp. 817-842, 1987.

See Also

`schur`	Schur decomposition
`modreal`	Modal form realization

Purpose

Construct array by stacking uncertain matrices, models, or arrays

Syntax

```
umatout = stack(arraydim,umat1,umat2,...)
usysout = stack(arraydim,usys1,usys2,...)
```

Description

`stack` constructs an uncertain array by stacking uncertain matrices, models, or arrays along array dimensions of an uncertain array.

`umatout = stack(arraydim,umat1,umat2,...)` produces an array of uncertain matrices, `umatout`, by stacking (concatenating) the `umat` matrices (or `umat` arrays) `umat1`, `umat2`,... along the array dimension `arraydim`. All models must have the same number of columns and rows. The column/row dimensions are not counted in the array dimensions.

`umatout = stack(arraydim,usys1,usys2,...)` produces an array of uncertain models, `ufrd` or `uss`, or `usysout`, by stacking (concatenating) the `ufrd` or `uss` matrices (or `ufrd` or `uss` arrays) `usys1`, `usys2`,... along the array dimension `arraydim`. All models must have the same number of columns and rows (the same input/output dimensions). Note that the input/output dimensions are not considered for arrays.

Example

Consider `usys1` and `usys2`, two single-input/single-output `uss` models:

```
zeta = ureal('zeta',1,'Range',[0.4 4]);
wn = ureal('wn',0.5,'Range',[0.3 0.7]);
P1 = tf(1,[1 2*zeta*wn wn^2]);
P2 = tf(zeta,[1 10]);
```

You can stack along the first dimension to produce a 2-by-1 `uss` array.

```
stack(1,P1,P1)
USS: 2 States, 1 Output, 1 Input, Continuous System [array, 2 x 1]
    wn: real, nominal = 0.5, range = [0.3  0.7], 3 occurrences
  zeta: real, nominal = 1, range = [0.4  4], 1 occurrence
```

You can stack along the second dimension to produce a 1-by-2 `uss` array.

```
stack(2,P1,P2)   % produces a 1-by-2 USS array.
USS: 2 States, 1 Output, 1 Input, Continuous System [array, 1 x 2]
    wn: real, nominal = 0.5, range = [0.3  0.7], 3 occurrences
  zeta: real, nominal = 1, range = [0.4  4], 1 occurrence
```

You can stack along the third dimension to produce a 1-by-1-by-2 uss array.

```
stack(3,P1,P2)   % produces a 1-by-1-by-2 USS array.
USS: 2 States, 1 Output, 1 Input, Continuous System [array, 1 x 1
x 2]
    wn: real, nominal = 0.5, range = [0.3  0.7], 3 occurrences
  zeta: real, nominal = 1, range = [0.4  4], 1 occurrence
```

See Also

`append`	Groups models by appending their inputs and outputs
`blkdiag`	Groups models by appending their inputs and outputs
`horzcat`	Performs horizontal concatenation
`vertcat`	Performs vertical concatenation

Purpose

Singular value decomposition of `frd` object

Syntax

```
S = svd(X)
[U,S,V] = svd(X)
```

Description

`S = svd(X)` operates on `X.ReponseData` at each frequency to construct `S`.

`[U,S,V] = svd(X)` produces a diagonal `frd S` that has the same dimensions as `X` and includes positive diagonal elements in decreasing order. `U` and `V` are unitary matrices and `frd` objects, such that `X = U*S*V'`. For more information, see the built-in `svd` command.

See Also

`schur`	Constructs a Schur decomposition
`svd`	Constructs a singular value decomposition

symdec

Purpose

Form symmetric matrix

Syntax

```
x = symdec(m,n)
```

Description

`symdec(m,n)` forms an m-by-m symmetric matrix of the form

$$\begin{bmatrix} (n+1) & (n+2) & (n+4) & \dots \\ (n+2) & (n+3) & (n+5) & \dots \\ (n+4) & (n+5) & (n+6) & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

This function is useful to define symmetric matrix variables. `n` is the number of decision variables.

See Also

`decinfo`	Show how matrix variables depend on decision variables

Purpose

Build interconnections of certain and uncertain matrices and systems

Syntax

```
sysout = sysic
```

Description

`sysic` requires that 3 variables with fixed names be present in the calling workspace: `systemnames`, `inputvar` and `outputvar`.

`systemnames` is a `char` containing the names of the subsystems (`double, tf, zpk, ss, uss, frd, ufrd`, etc) that make up the interconnection. The names must be separated by spaces with no additional punctuation. Each named variable must exist in the calling workspace.

`inputvar` is a `char`, defining the names of the external inputs to the interconnection. The names are separated by semicolons, and the entire list is enclosed in square brackets `[ ]`. Inputs can be scalar or multivariate. For instance, a 3-component (`x,y,z`) force input can be specified with 3 separate names, `Fx, Fy, Fz`. Alternatively, a single name with a defined integer dimension can be specified, as in `F{3}`. The order of names in `inputvar` determines the order of inputs in the interconnection.

`outputvar` is a `char`, describing the outputs of the interconnection. Outputs do not have names-they are simply linear combinations of individual subsystem's outputs and external inputs. Semicolons delineate separate components of the interconnections outputs. Between semicolons, signals can be added and subtracted, and multiplied by scalars. For multivariable subsystems, arguments within parentheses specify which subsystem outputs are to be used and in what order. For instance, `plant(2:4,1,9:11)` specifies outputs `2,3,4,1,9,10,11` from the subsystem `plant`. If a subsystem is listed in `outputvar` without arguments, then all outputs from that subsystem are used.

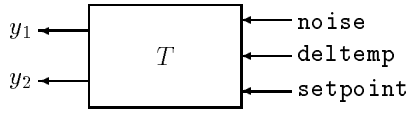
`sysic` also requires that for every subsystem name listed in `systemnames`, a corresponding variable, `input_to_ListedSubSystemName` must exist in the calling workspace. This variable is similar to `outputvar` – it defines the input signals to this particular subsystem as linear combinations of individual subsystem's outputs and external inputs.

`sysout = sysic` will perform the interconnection described by the variables above, using the subsystem data in the names found in `systemnames`. The resulting interconnection is returned in the output argument, listed above as `sysout`.

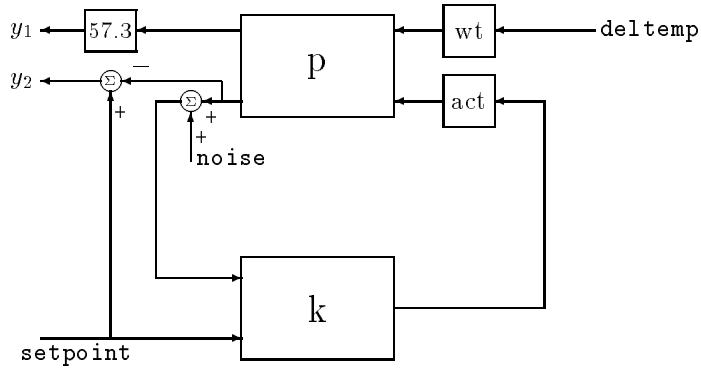
After running `sysic` the variables `systemnames, inputvar, outputvar` and all of the `input_to_ListedSubSystemName` will exist in the workspace. Setting the optional variable `cleanupsysic` to `` `yes' `` will cause these variable to be removed from the workspace after `sysic` has formed the interconnection.

Example

A simple system interconnection, identical to the system illustrated in the `iconnect` description. Consider a three-input, two-output LTI matrix T,



which has internal structure




```
P = rss(3,2,2);
K = rss(1,1,2);
A = rss(1,1,1);
W = rss(1,1,1);
systemnames = 'W A K P';
inputvar = '[noise;deltemp;setpoint]';
outputvar = '[57.3*P(1);setpoint-P(2)]';
input_to_W = '[deltemp]';
input_to_A = '[K]';
input_to_K = '[P(2)+noise;setpoint]';
input_to_P = '[W;A]';
```

```
cleanupsysic = `yes';
T = sysic;
exist(`inputvar')
```

Limitations

The syntax of `sysic` is limited, and for the most part is restricted to what is shown here. The `iconnect` interconnection object can also be used to define complex interconnections, and has a more flexible syntax.

Within `sysic`, error-checking routines monitor the consistency and availability of the subsystems and their inputs. These routines provide a basic level of error detection to aid the user in debugging.

See Also

`iconnect`	Equates expressions for `icsignal` objects

ucomplex

Purpose

Create uncertain complex parameter

Syntax

```
A = ucomplex('NAME',nominalvalue)
A = ucomplex('NAME',nominalvalue,'Property1',Value1,...
         'Property2',Value2,...)
```

Description

An uncertain complex parameter is used to represent a complex number whose value is uncertain. Uncertain complex parameters have a name (the `Name` property), and a nominal value (the `NominalValue` property).

The uncertainty (potential deviation from the nominal value) is described in two different manners:

- `Radius` (radius of disc centered at `NominalValue`)
- `Percentage` (disc size is percentage of magnitude of `NominalValue`)

The `Mode` property determines which description remains invariant if the `NominalValue` is changed (the other is derived). The default `Mode` is `'Radius'` and the default radius is 1.

Property/Value pairs can also be specified at creation. For instance,

```
B = ucomplex('B',6-j,'Percentage',25)
```

sets the nominal value to `6-j`, the percentage uncertainty to `25` and, implicitly, the `Mode` to `'Percentage'`.

Example

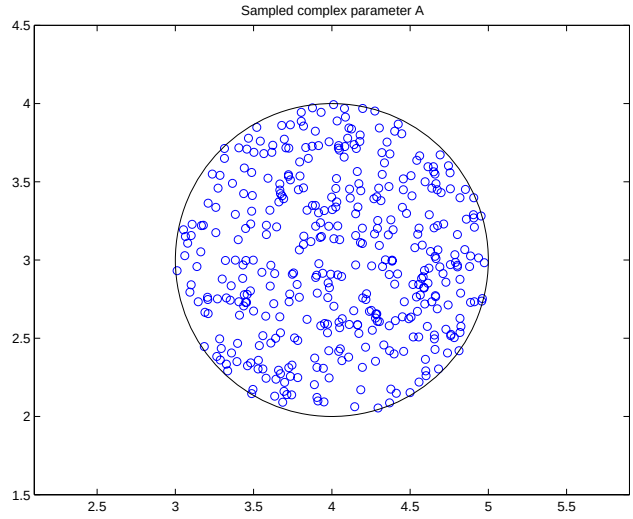
Create an uncertain complex parameter with internal name `A`. The uncertain parameter's possible values are a complex disc of radius `1`, centered at `4+3j`. The value of `A.percentage` is `20` (radius is 1/5 of the magnitude of the nominal value).

```
A = ucomplex('A',4+3*j)
Uncertain Complex Parameter: Name A, NominalValue 4+3i, Radius 1
```

You can visualize the uncertain complex parameter by sampling and plotting the data.

```
sa = usample(A,400);
w = linspace(0,2*pi,200);
circ = sin(w) + j*cos(w);
rc = real(A.NominalValue+circ);
```

```
ic = imag(A.NominalValue+circ);
plot(real(sa(:)),imag(sa(:)),'o',rc,ic,'k-')
xlim([2.5 5.5])
ylim([1.5 4.5])
axis equal
```



See Also

`get`	Gets object properties
`umat`	Creates an uncertain matrix object
`ucomplexm`	Creates an uncertain complex matrix
`ultidyn`	Creates an uncertain LTI dynamic object
`ureal`	Creates an uncertain real parameter

ucomplexm

Purpose

Create uncertain complex matrix

Syntax

```
M = ucomplexm('Name',NominalValue)
M = ucomplexm('Name',NominalValue,'WL',WLvalue,'WR',WRvalue)
M = ucomplexm('Name',NominalValue,'Property',Value)
```

Description

`M = ucomplexm('Name',NominalValue)` creates an uncertain complex matrix representing a ball of complex-valued matrices, centered at a `NominalValue` and named `Name`.

`M = ucomplexm('Name',NominalValue,'WL',WLvalue,'WR',WRvalue)` creates an uncertain complex matrix with weights `WL` and `WR`. Specifically, the values represented by `M` are all matrices `H` that satisfy `norm(inv(M.WL)*(H - M.NominalValue)*inv(M.WR)) <= 1.` `WL` and `WR` are square, invertible, and weighting matrices that quantify the size and shape of the ball of matrices represented by this object. The default values for `WL` and `WR` are identity matrices of appropriate dimensions.

Trailing Property/Value pairs are allowed, as in

```
M = ucomplexm('NAME',nominalvalue,'P1',V1,'P2',V2,...)
```

The property `AutoSimplify` controls how expressions involving the uncertain matrix are simplified. Its default value is `'basic'`, which means elementary methods of simplification are applied as operations are completed. Other values for `AutoSimplify` are `'off''`, no simplification performed, and `'full'` which applies model-reduction-like techniques to the uncertain object.

Example

Create a `ucomplexm` with the name `'F'`, nominal value `[1 2 3; 4 5 6]`, and weighting matrices `WL = diag([.1.3])`, `WR = diag([.4 .8 1.2])`.

```
F = ucomplexm('F',[1 2 3;4 5 6],'WL',diag([.1 .3]),...
   'WR',diag([.4 .8 1.2]));
```

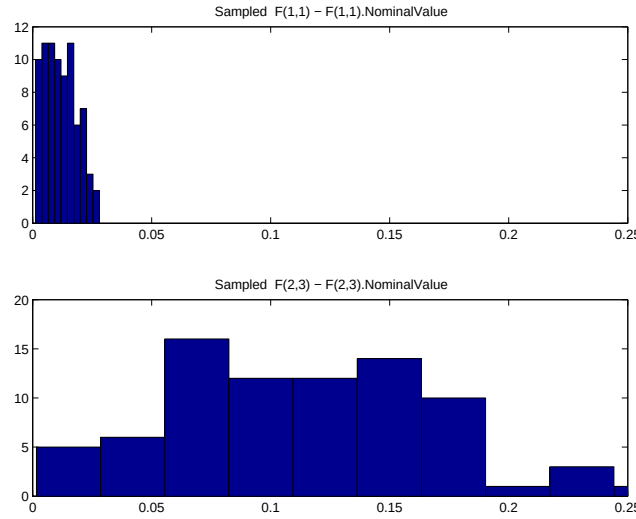
Sample the difference between the uncertain matrix and its nominal value at 80 points, yielding a 2-by-3-by-80 matrix `typicaldev`.

```
typicaldev = usample(F-F.NominalValue,40);
```

Plot histograms of the deviations in the (1,1) entry as well as the deviations in the (2,3) entry.

The absolute values of the (1,1) entry and the (2,3) entry are shown by histogram plots. Typical deviations in the (1,1) entry should be about 10 times smaller than the typical deviations in the (2,3) entry.

```
subplot(2,1,1);
hist(abs(typicaldev(1,1,:)));xlim([0 .25])
title('Sampled  F(1,1) - F(1,1).NominalValue')
subplot(2,1,2);
hist(abs(typicaldev(2,3,:)));xlim([0 .25])
title('Sampled  F(2,3) - F(2,3).NominalValue')
```



See Also

`get`	Gets object properties
`umat`	Creates an uncertain matrix object
`ucomplex`	Creates an uncertain complex parameter
`ultidyn`	Creates an uncertain LTI dynamic object
`ureal`	Creates an uncertain real parameter

udyn

Purpose

Create unstructured uncertain dynamic system object

Syntax

```
n = udyn('name',iosize);
```

Description

`n = udyn('name',iosize)` creates an unstructured uncertain dynamic system class, with input/output dimension specified by `iosize`. This object represents the class of completely unknown multivariable, time-varying nonlinear systems.

For practical purposes, these uncertain elements represent noncommuting symbolic variables (placeholders). All algebraic operations, such as addition, subtraction, and multiplication (i.e., cascade) operate properly, and substitution (with `usubs`) is allowed.

The analysis tools (e.g., `robuststab`) do not currently handle these types of uncertain elements. Therefore, these elements do not provide a significant amount of usability, and their role in the toolbox is small.

Example

You can create a `2-by-3 udyn` element and check its size and properties.

```
N = udyn('N',[2 3])
Uncertain Dynamic System: Name N, size 2x3
size(N)
ans =
     2     3
get(N)
             Name: 'N'
    NominalValue: [2x3 double]
    AutoSimplify: 'basic'
```

See Also

`ureal`	Creates an uncertain real parameter
`ultidyn`	Creates an uncertain linear time-invariant object
`ucomplex`	Creates an uncertain complex parameter
`ucomplexm`	Creates an uncertain complex matrix

Purpose

Create uncertain frequency response data (ufrd) object, or convert another model type to ufrd model

Syntax

```
usysfrd = ufrd(usys,frequency)
usysfrd = ufrd(usys,frequency,'Units',units)
usysfrd = ufrd(sysfrd)
usys = ufrd(response,frequency)
usys = ufrd(response,frequency,Ts)
usys = ufrd(response,frequency,RefSys)
usys = ufrd(response,frequency,'Units',units,Ts)
usys = ufrd(response,frequency,'Units',units,Ts,RefSys)
```

Description

Uncertain frequency response data (ufrd) models result from the conversion of an uncertain state-space (uss) system to its uncertain frequency response. ufrd models also result when frequency response data models (frd) are combined with uncertain matrices (umat).

`usysfrd = ufrd(usys,frequency,'Units',units)` converts a uss model usys to a ufrd model usysfrd by frequency response. `'Units'` specifies the units of the frequencies in frequency, which can be `'rad/s'` or `'Hz'`. If the last two arguments are omitted, the default for frequency units is `'rad/s'`.

Any of the previous syntaxes can be followed by property name/property value pairs.

`usysfrd = ufrd(usys,frequency,'Units',units,'P1',V1,'P2',V2,...)` sets the properties `P1, P2, ...` to the values `V1, V2, ...`

`usys = ufrd(response,frequency)` creates a ufrd from the response and frequency arguments. response should be a umat array, whose first array dimension (i.e., `size(response,3)`) aligns with the frequency. Note that you are unlikely to use this option.

`usysfrd = ufrd(sysfrd)` converts an frd model sysfrd to a ufrd model usysfrd with no uncertain elements.

Example

In the first example, you create a continuous-time uncertain system with both parametric uncertainty and unmodeled dynamics uncertainty. Compute the

uncertain frequency response and plot the Bode plot, using 20 random samples, with a color choice of red for random samples, and blue for nominal.

```
p1 = ureal('p1',5,'Range',[2 6]);
p2 = ureal('p2',3,'Plusminus',0.4);
p3 = ultidyn('p3',[1 1]);
Wt = makeweight(.15,30,10);
A = [-p1 0;p2 -p1];
B = [0;p2];
C = [1 1];
usys = uss(A,B,C,0)*(1+Wt*p3);
usysfrd = ufrd(usys,logspace(-2,2,60));
bode(usysfrd,'r',usysfrd.NominalValue,'b+')
```

Example 2

In this example, you convert a not-uncertain `frd` model to `ufrd` without uncertainties. You can verify the equality of the nominal value of the `ufrd` and simplified representation to the original system.

```
G = frd(tf([1 2 3],[1 2 3 4]),logspace(-2,2,40));
usys = ufrd(G)
UFRD: 1 Output, 1 Input, Continuous System, 40 Frequency points
isequal(usys.NominalValue,G)
ans =
     1
isequal(simplify(usys,'class'),G)
ans =
     1
```

See Also

`frd`	Creates or converts to frequency response data model
`ss`	Creates or converts to state-space model

Purpose

Create uncertain linear time-invariant object

Syntax

```
H = ultidyn('Name',iosize)
H =
   ultidyn('Name',iosize,'Property1',Value1,'Property2',Value2,...)
```

Description

`H = ultidyn('Name',iosize)` creates an uncertain linear, time-invariant objects are used to represent unknown dynamic objects whose only known attributes are bounds on their frequency response. Uncertain linear, time-invariant objects have a name (the `Name` property), and an input/output size (`ioSize` property).

The property `Type` is `'GainBounded'` (default) or `'PositiveReal'`, and describes in what form the knowledge about the object's frequency response is specified.

- If `Type` is `'GainBounded'`, then the knowledge is an upper bound on the magnitude (i.e., absolute value), namely `abs(H)<= Bound` at all frequencies. The matrix generalization of this is `||H||<= Bound`.
- If `Type` is `'PositiveReal'` then the knowledge is a lower bound on the real part, namely `Real(H) >= Bound` at all frequencies. The matrix generalization of this is `H+H' >= 2*Bound`

The property `Bound` is a real, scalar that quantifies the bound on the frequency response of the uncertain object as described above.

Trailing Property/Value pairs are allowed in the construction.

```
H=ultidyn('name',iosize,'Property1',Value1,'Property2',Value2,...)
```

The property `SampleStateDim` is a positive integer, defining the state dimension of random samples of the uncertain object when sampled with `usample`. The default value is 1.

The property `AutoSimplify` controls how expressions involving the uncertain matrix are simplified. Its default value is `'basic'`, which means elementary methods of simplification are applied as operations are completed. Other values for `AutoSimplify` are `'off''`, no simplification performed, and `'full'` which applies model-reduction-like techniques to the uncertain object.

Example

Example 1

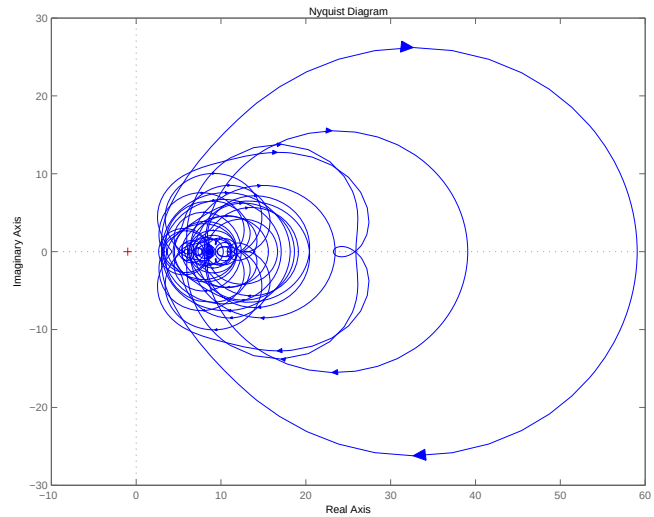
Create an `ultidyn` object with internal name `'H'`, dimensions `2-by-3`, norm bounded by `7`.

```
H = ultidyn('H',[2 3],'Bound',7)
Uncertain GainBounded LTI Dynamics: Name H, 2x3, Gain Bound = 7
```

Example 2

Create a scalar `ultidyn` object with an internal name `'B'`, whose frequency response has a real part greater than `2.5`. Change the `SampleStateDim` to `5`, and plot the Nyquist plot of `30` random samples.

```
B = ultidyn('B',[1 1],'Type','PositiveReal','Bound',2.5)
Uncertain PositiveReal LTI Dynamics: Name B, 1x1, M+M' >= 2*(2.5)
B.SampleStateDim = 5;
nyquist(usample(B,30))
```



See Also

`get`	Gets object properties
`ureal`	Creates an uncertain real parameter
`uss`	Creates an uncertain LTI system object

Purpose

Uncertain matrices

Syntax

```
h = umat(m)
```

Description

Uncertain matrices are usually created by manipulation of uncertain atoms (`ureal, ucomplex, ultidyn`, etc.), `double` matrices, and other uncertain matrices. Most standard matrix manipulations are valid, including addition, multiplication, inverse, horizontal and vertical concatenation. Specific rows/columns of an uncertain matrix can be referenced and assigned also.

The command `umat` is rarely used. There are two situations where it may be useful. If `M` is a `double`, then `H = umat(M)` recasts `M` as an uncertain matrix (`umat` object) without any uncertainties. Similarly, if `M` is an uncertain atom, then `H = umat(M)` recasts `M` as an uncertain matrix (`umat` object) whose value is merely the uncertain atom. In both cases, `simplify(H,'class')` is the same as `M`.

If `M` is a `umat`, then `M.NominalValue` is the result obtained by replacing each uncertain atom in `M` with its own nominal value.

If `M` is a `umat`, then `M.Uncertainty` is an object describing all the uncertain atoms in `M`. All atoms can be referenced and their properties modified with this `Uncertainty` gateway. For instance, if `B` is an uncertain real parameter in `M`, then `M.Uncertainty.B` accesses the uncertain atom `B` in `M`.

Example

Create 3 uncertain atoms and then a `3-by-2 umat`.

```
a = ureal('a',5,'Range',[2 6]);
b = ucomplex('b',1+j,'Radius',0.5);
c = ureal('c',3,'Plusminus',0.4);
M = [a b;b*a 7;c-a b^2]
UMAT: 3 Rows, 2 Columns
  a: real, nominal = 5, range = [2  6], 1 occurrence
  b: complex, nominal = 1+1i, radius = 0.5, 4 occurrences
  c: real, nominal = 3, variability = [-0.4  0.4], 1 occurrence
```

View the properties of `M` with `get`

```
get(M)
    NominalValue: [3x2 double]
     Uncertainty: [1x1 atomlist]
```

The nominal value of `M` is the result when all atoms are replaced by their nominal values. View the properties of `M` with `get`

```
M.NominalValue
ans =
   5.0000             1.0000 + 1.0000i
   5.0000 + 5.0000i   7.0000
  -2.0000                  0 + 2.0000i
```

Change the nominal value of `a` within `M` to 4. The nominal value of `M` reflects this change.

```
M.Uncertainty.a.NominalValue = 4;
M.NominalValue
ans =
   4.0000             1.0000 + 1.0000i
   4.0000 + 4.0000i   7.0000
  -1.0000                  0 + 2.0000i
```

Get a random sample of `M`, obtained by taking random samples of the uncertain atoms within `M`.

```
usample(M)
ans =
   2.0072             0.8647 + 1.3854i
   1.7358 + 2.7808i   7.0000
   1.3829            -1.1715 + 2.3960i
```

Select the 1st and 3rd rows, and the 2nd column of `M`. The result is a 2-by-1 `umat`, whose dependence is only on `b`.

```
M([1 3],2)
UMAT: 2 Rows, 1 Columns
  b: complex, nominal = 1+1i, radius = 0.5, 3 occurrences
```

See Also

`ureal`	Creates an uncertain real parameter
`ultidyn`	Creates an uncertain linear time-invariant object
`ucomplex`	Creates an uncertain complex parameter
`ucomplexm`	Creates an uncertain complex matrix
`usample`	Generates random samples of an uncertain object

Purpose

Plot multiple frequency response objects and doubles on same graph

Syntax

```
uplot(G1)
uplot(G1,G2)
uplot(G1,Xdata,Ydata)
uplot(G1,Xdata,Ydata,...)
uplot(G1,linetype)
uplot(G1,linetype,G2,...)
uplot(G1,linetype,Xdata,Ydata,linetype)
uplot(type,G1,linetype,Xdata,Ydata,linetype)
H = uplot(G1)
H = uplot(G1,G2)
H = uplot(G1,Xdata,Ydata)
H = uplot(G1,Xdata,Ydata,...)
H = uplot(G1,linetype)
H = uplot(G1,linetype,G2,...)
H = uplot(G1,linetype,Xdata,Ydata,linetype)
```

Description

uplot plots double and frd objects. The syntax is the same as the MATLAB® plot command except that all data is contained in frd objects, and the axes are specified by type.

The (optional) type argument must be one of

Type	Description
'iv,d'	Data versus independent variable (default)
'iv,m'	Magnitude versus independent variable
'iv,lm'	log(magnitude) versus independent variable
'iv,p'	Phase versus independent variable
'liv,m'	Magnitude versus log(independent variable)
'liv,d'	Data versus log(independent variable)
'liv,m'	Magnitude versus log(independent variable)

Type	Description
'liv,lm'	log(magnitude) versus log(independent variable)
'liv,p'	Phase versus log(independent variable)
'r,i'	Real versus imaginary (parametrize by independent variable)
'nyq'	Real versus imaginary (parametrize by independent variable)
'nic'	Nicholas plot
'bode'	Bode magnitude and phase plot

The remaining arguments of `uplot` take the same form as the MATLAB® `plot` command. Line types (for example,`'+', 'g-.', or '*r'`) can be optionally specified after any frequency response argument.

There is a subtle distinction between constants and `frd` objects with only one independent variable. A constant is treated as such across all frequencies, and consequently shows up as a line on any graph with the independent variable as an axis. A `frd` object with only one frequency point always shows up as a point. You might need to specify one of the more obvious point types in order to see it (e.g., `'+'`, `'x'`, etc.).

Example

Two SISO second-order systems are created, and their frequency responses are calculated over different frequency ranges.

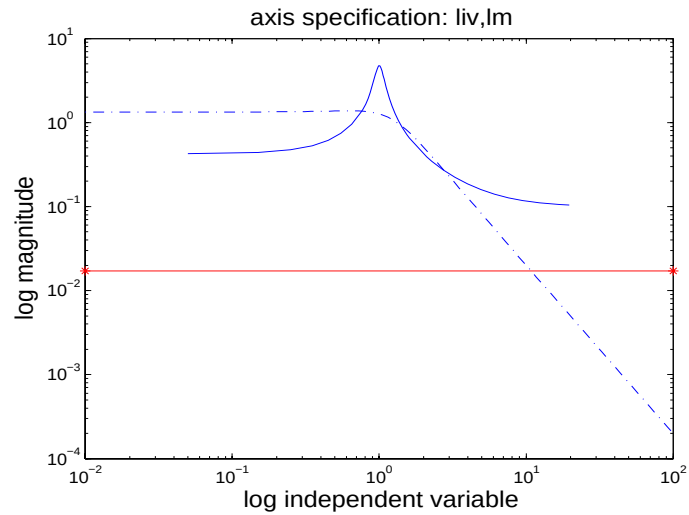
```
a1 = [-1,1;-1,-0.5];
b1 = [0;2]; c1 = [1,0]; d1 = 0;
sys1 = ss(a1,b1,c1,d1);
a2 = [-.1,1;-1,-0.05];
b2 = [1;1]; c2 = [-0.5,0]; d2 = 0.1;
sys2 = ss(a2,b2,c2,d2);
omega = logspace(-2,2,100);
sys1g = frd(sys1,omega);
omega2 = [ [0.05:0.1:1.5] [1.6:.5:20] [0.9:0.01:1.1] ];
omega2 = sort(omega2);
sys2g = frd(sys2,omega2);
```

An `frd` object with a single frequency is also created. Note the distinction between the `frd` object and the constant matrix in the subsequent plots.

```
sys3 = rss(1,1,1);
rspot = frd(sys3,2);
```

The following plot uses the `'liv,lm' plot_type` specification.

```
uplot('liv,lm',sys1g,'b-.',rspot,'r*',sys2g);
xlabel('log independent variable')
ylabel('log magnitude')
title('axis specification: liv,lm')
```



See Also

`bode`	Plots Bode frequency response
`plot`	Plots on linear axis
`nichols`	Plots Nichols frequency response
`nyquist`	Plots Nyquist frequency response
`semilogx`	Plots semi-log scale plot
`semilogy`	Plots semi-log scale plot
`sigma`	Plots singular values of a LTI system

ureal

Purpose

Create uncertain real parameter

Syntax

```
p = ureal('name',nominalvalue)
p = ureal('name',nominalvalue,'Property1',Value1,...
'Property2',Value2,...)
```

Description

An uncertain real parameter is used to represent a real number whose value is uncertain. Uncertain real parameters have a name (the `Name` property), and a nominal value (`NominalValue` property).

The uncertainty (potential deviation from `NominalValue`) is described (equivalently) in 3 different properties:

- `PlusMinus`**:** the additive deviation from `NominalValue`
- `Range`**:** the interval containing `NominalValue`
- `Percentage`**:** the percentage deviation from `NominalValue`

The `Mode` property specifies which one of these three descriptions remains unchanged if the `NominalValue` is changed (the other two descriptions are derived). The possible values for the `Mode` property are `'Range'`, `'Percentage'` and `'PlusMinus'`.

The default `Mode` is `'PlusMinus'`, and `[-1 1]` is the default value for the `'PlusMinus'` property. The range of uncertainty need not be symmetric about `NominalValue`.

The property `AutoSimplify` controls how expressions involving the uncertain matrix are simplified. Its default value is `'basic'`, which means elementary methods of simplification are applied as operations are completed. Other values for `AutoSimplify` are `'off'`, no simplification performed, and `'full'`, which applies model-reduction-like techniques to the uncertain object.

Example

Example 1

Create an uncertain real parameter and use `get` to display the properties and their values. Create uncertain real parameter object `a` with the internal name `'a'` and nominal value `5`.

```
a = ureal('a',5)
Uncertain Real Parameter: Name a, NominalValue 5, variability =
[-1  1]
```

```
get(a)
            Name: 'a'
    NominalValue: 5
            Mode: 'PlusMinus'
           Range: [4 6]
       PlusMinus: [-1 1]
      Percentage: [-20 20]
    AutoSimplify: 'basic'
```

Note that the `Mode` is `'PlusMinus'`, and that the value of `PlusMinus` is indeed `[-1 1]`. As expected, the range description of uncertainty is `[4 6]`, while the percentage description of uncertainty is `[-20 20]`.

Set the range to `[3 9]`. This leaves `Mode` and `NominalValue` unchanged, but all three descriptions of uncertainty have been modified.

```
a.Range = [3 9];
get(a)
            Name: 'a'
    NominalValue: 5
            Mode: 'PlusMinus'
           Range: [3 9]
       PlusMinus: [-2 4]
      Percentage: [-40 80]
    AutoSimplify: 'basic'
```

Example 2

Property/Value pairs can also be specified at creation.

```
b = ureal('b',6,'Percentage',[-30 40],'AutoSimplify','full');
get(b)
            Name: 'b'
    NominalValue: 6
            Mode: 'Percentage'
           Range: [4.2000 8.4000]
       PlusMinus: [-1.8000 2.4000]
      Percentage: [-30.0000 40.0000]
    AutoSimplify: 'full'
```

Note that `Mode` is automatically set to `'Percentage'`.

Example 3

Specify the uncertainty in terms of percentage, but force `Mode` to `'Range'`.

```
c = ureal('c',4,'Mode','Range','Percentage',25);
get(c)
            Name: 'c'
    NominalValue: 4
            Mode: 'Range'
           Range: [3 5]
       PlusMinus: [-1 1]
      Percentage: [-25 25]
    AutoSimplify: 'basic'
```

See Also

`ucomplex`	Creates an uncertain complex parameter
`umat`	Creates an uncertain matrix
`uss`	Creates an uncertain, linear dynamic object

Purpose

Generate random samples of uncertain object

Syntax

```
B = usample(A);
B = usample(A,N)
[B,SampleValues] = usample(A,N)
[B,SampleValues] = usample(A,Names,N)
[B,SampleValues] = usample(A,Names1,N1,Names2,N2,...)
[B,SampleValues] = usample(A,N,BW)
[B,SampleValues] = usample(A,Names,N,BW)
```

Description

`B = usample(A)` substitutes a random sample of the uncertain objects in `A`, returning a certain (i.e., not uncertain) array of size `[size(A)]`.

`B = usample(A,N)` substitutes `N` random samples of the uncertain objects in `A`, returning a certain (i.e., not uncertain) array of size `[size(A) N]`.

`[B,SampleValues] = usample(A,N)` additionally returns the specific sampled values (as a `Struct` whose field names are the names of `A`'s uncertain elements) of the uncertain elements. Hence, `B` is the same as `usubs(A,SampleValues)`.

`[B,SampleValues] = usample(A,Names,N)` samples only the uncertain elements listed in the `Names` variable (cell, or char array). If `Names` does not include all the uncertain objects in `A`, then `B` will be an uncertain object. Any entries of `Names` that are not elements of `A` are simply ignored. Note that `usample(A,fieldnames(A.Uncertainty),N)` is the same as `usample(A,N)`.

`[B,SampleValues] = usample(A,Names1,N1,Names2,N2,...)` takes `N1` samples of the uncertain elements listed in `Names1`, and `N2` samples of the uncertain elements listed in `Names2`, and so on. `size(B)` will equal `[size(A) N1 N2 ...]`.

The scalar parameter `BW` in

```
[B,SampleValues] = usample(A,N,BW)
[B,SampleValues] = usample(A,Names,N,BW)
[B,SampleValues] = usample(A,Names,N,BW)
```

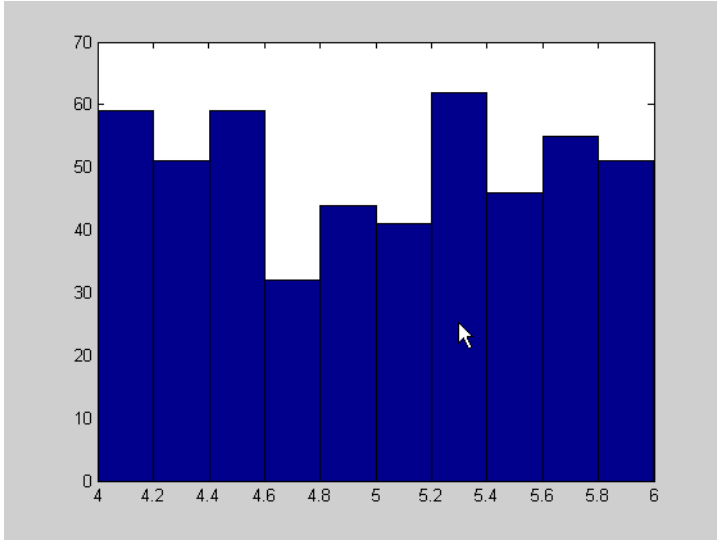
affects how `ultidyn` elements within `A` are sampled, restricting the poles of the samples. If `A` is a continuous-time `uss` or `ufrd`, then the poles of sampled

`GainBounded ultidyn` elements in `SampleValues` will each have magnitude <= `BW`. If `A` is a discrete-time, then sampled `GainBounded ultidyn` elements are obtained by Tustin transformation, using `BW/(2*TS)` as the (continuous) pole magnitude bound. In this case, `BW` should be `< 1`. If the `ultidyn` type is `PositiveReal`, then the samples are obtained by bilinearly transforming (see "Normalizing Functions for Uncertain Atoms" on page 1-60) the `GainBounded` elements described above.

Example

Sample a real parameter and plot a histogram.

```
A = ureal('A',5);
Asample = usample(A,500);
size(A)
ans =
     1     1
size(Asample)
ans =
     1     1   500
class(Asample)
ans =
double
hist(Asample(:))
```



The second example illustrates the open and closed-loop response of an uncertain plant model. You can create two uncertain real parameters and an uncertain plant.

```
gamma = ureal('gamma',4);
tau = ureal('tau',.5,'Percentage',30);
P = tf(gamma,[tau 1]);
```

Create an integral controller based on nominal plant parameter.

```
KI = 1/(2*tau.Nominal*gamma.Nominal);
C = tf(KI,[1 0]);
```

Now create an uncertain closed-loop system.

```
CLP = feedback(P*C,1); You can sample the plant at 20 values
(distributed uniformly about the tau and gamma parameter cube).

[Psample1D,Values1D] = usample(P,20);
size(Psample1D)
20x1 array of state-space models
Each model has 1 output, 1 input, and 1 state.
```

You can sample the plant `P` at 10 values in the `tau` parameter and 15 values in the `gamma` parameter.

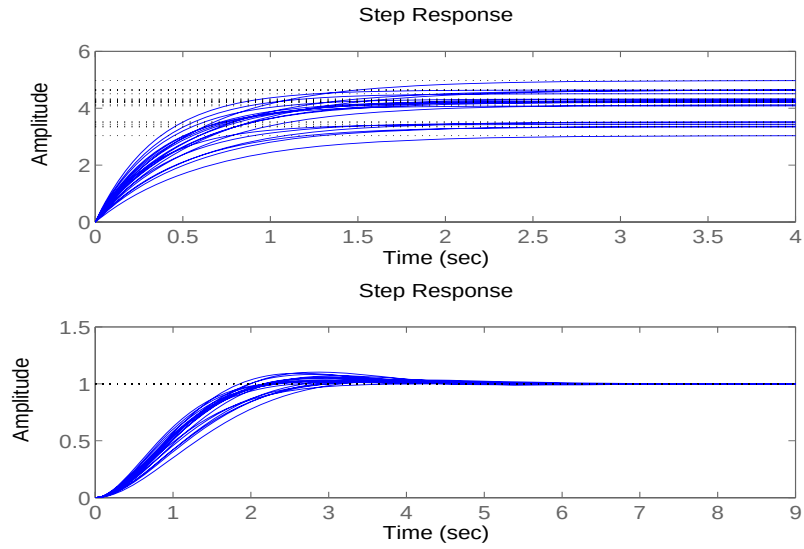
```
[Psample2D,Values2D] = usample(P,'tau',10,'gamma',15);
size(Psample2D)
10x15 array of state-space models
Each model has 1 output, 1 input, and 1 state.
```

You can plot the 1-D sampled plant step responses

```
subplot(2,1,1); step(Psample1D)
```

You can also evaluate the uncertain closed-loop at the same values, and plot the step response using `usubs`.

```
subplot(2,1,2); step(usubs(CLP,Values1D))
```



To see the effect of the bandwidth parameter, create two `ultidyn` objects

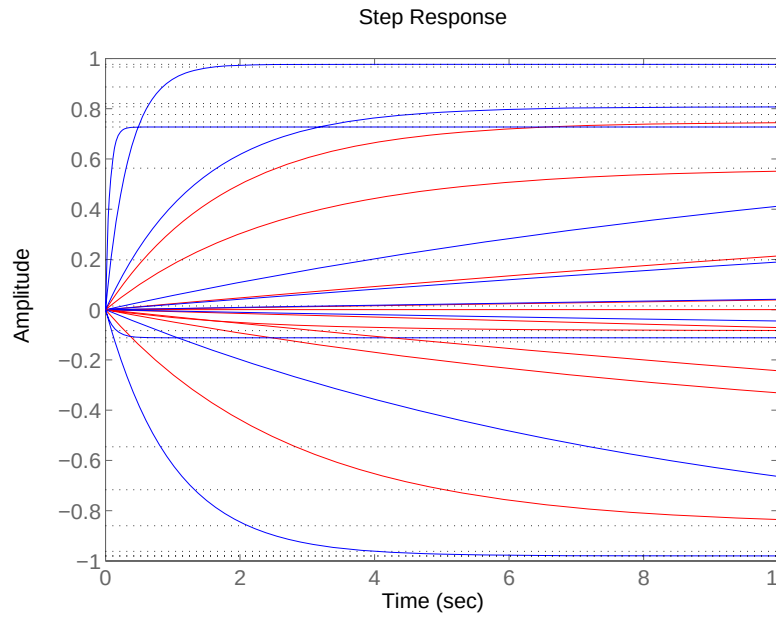
```
A = ultidyn('A',[1 1]);
B = ultidyn('B',[1 1]);
```

Sample 10 instances of each, using a bandwidth limit of 1 rad/sec on `A` and 20 rad/sec on `B`.

```
Npts = 10;
As = usample(A,Npts,1);
Bs = usample(B,Npts,20);
```

Plot 10-second step responses, for the two sample sets. Plot the slow sample (from `A`) in red, and the faster samples (from `B`.) in blue.

```
step(As,'r',Bs,'b',10)
```



See Also usubs Substitutes values for uncertain atoms

usimfill

Purpose

Helper function for USS System blocks, used to set the "User-defined Uncertainty" field or the state of the "Uncertainty value" pulldown menu.

Syntax

```
usimfill(ModelName,str)
usimfill(ModelName,'Uncertainty value','Nominal')
usimfill(ModelName,'Uncertainty value','User defined')
```

Description

The command `usimfill` allows simple control of some parameters of all `uss` System blocks in a Simulink® model.

`usimfill(ModelName,str)` pushes the string in `str` into the `Uncertainty value` name field of all `USS` System blocks in the Simulink model specified by `ModelName`.

`usimfill(ModelName,'Uncertainty value','Nominal')` sets the `Uncertainty value` pulldown menu to `Nominal` for all `USS` System blocks in the Simulink model specified by `ModelName`. Only a limited number of characters are needed to make this specification, so `usimfill(ModelName,'U','N')` accomplishes the same effect.

`usimfill(ModelName,'Uncertainty value','User defined')` sets the `Uncertainty value` pulldown menu to `User defined` for all `USS` System blocks in the Simulink model specified by `ModelName`. Only a limited number of characters are needed to make this specification, so `usimfill(ModelName,'U','U')` accomplishes the same effect.

Example

See the Robust Control Toolbox™ demo entitled “Uncertain Simulink Blocks” for a more detailed example of how to use `usimfill`.

Open the model file associated with the demo,

```
open_system('usim_model');
unc_pole = ureal('unc_pole',-5,'Range',[-10 -4]);
plant = ss(unc_pole,5,1,1);
input_unc = ultidyn('input_unc',[1 1]);
wt = makeweight(0.25,130,2.5);
sensor_gain = ureal('sensor_gain',1,'Range',[0.1 2]);
```

This has 3 `USS` System blocks. They are plant with a `ureal` atom named `unc_pole`; `input_unc` which is a `ultidyn` object, and `sensor_gain` which is a `ureal` atom.

Run `usimfill` on the model, filling in the field with the string `'newData'`.

```
usimfill('usim_model','newData');
```

View all of the dialog boxes, and see that the string `'newData'` has been entered.

Run `usimfill` on the model, changing the `Uncertainty Selection` to `Nominal`.

```
usimfill('usim_model','Uncertainty value','Nominal');
```

Similarly run `usimfill` on the model, changing the `Uncertainty Selection` to `User Specified Uncertainty`.

```
usimfill('usim_model','Uncertainty value','User defined');
```

Now generate a random sample of the uncertain atoms, and run the simulation

```
newData = usimsamp('usim_model',120);
sim('usim_model');
```

See Also

`usample`	Generate random samples of uncertain object
`usiminfo`	Find USS System blocks within Simulink model
`usimsamp`	Generate random instance of all USS System blocks in a Simulink model
`usubs`	Substitutes values for uncertain atoms

usiminfo

Purpose

Find USS System blocks within specified Simulink® model and checks for consistency

Syntax

```
[cflags,allupaths,allunames,upaths,unames,csumchar] =
  usiminfo(sname, silent)
```

Description

The command `usiminfo` returns information regarding the locations of all USS System blocks within a Simulink model and determines if these conpatiblilty conditions are satisfied. It is possible to have uncertain objects of the same name through out a Simulink model. The helper functions `usimsamp` and `usimfill` assume that these are the same uncertainty. Hence uncertain objects of the same name should have the same object properties and `Uncertainty value` in the USS System pull-down menu. `usiminfo` provides information about the uncertainty in the Simulink diagram `sname`.

The following describes the input and outputs arguments of `usiminfo`:

Inputs:

`sname`	Simulink diagram name
`silent`	Display inconsistencies between uncertain atoms if isn't empty. Default is empty.

Outputs:

`cflag`	Compatibility flag set to 1 if all uncertainties are consistent, set to 0 if an uncertainty definition(s) is consistent and set to -1 if common uncertainties in different blocks have different `Uncertainty value`.
`allupaths`	Path names of USS System blocks in the model (cell).
`allunames`	Uncertainties names in Simulink model (cell).
`upaths`	Path names associated with each `allunames` entry (cell).
`unames`	Uncertainty names associated with each `allupaths` entry (cell).
`csumchar`	Character array with description of uncertainties and their associated block path names. Empty if there is a conflict with `unames`.

See Also

`usample`	Generate random samples of uncertain object
`usimfill`	Fills in the `Uncertainty Variable Name` field or sets the `Uncertainty Selection` pull down menu in `USS` Simulink blocks
`usimsamp`	Generate random instance of all USS System blocks in a Simulink model
`usubs`	Substitutes values for uncertain atoms

usimsamp

Purpose

Generate random instance of all uncertain atoms present in all USS System blocks of Simulink® model

Syntax

```
sample = usimsamp(ModelName)
sample = usimsamp(ModelName,BW)
```

Description

The command `usimsamp` samples a Simulink model. Note that if the model contains any USS System blocks, then the model can be interpreted as an uncertain Simulink model. The sample generated by `usimsamp` is a scalar structure, with `fieldnames` corresponding to the uncertain atoms within all of the USS System blocks, and the values are specific random samples of the atoms.

For `ultidyn` atoms, the magnitude of the sampled poles can be limited using an optional second bandwidth argument, `BW`. See `usample` for more information on this parameter.

Example

See the Robust Control Toolbox™ demo called "Uncertain Simulink Blocks" for a more detailed example of how to use `usimsamp`.

Open the model file associated with the demonstration,

```
open_system('usim_model');
```

This has 3 USS System blocks. They are `plant` with a `ureal` atom named `unc_pole`; `input_unc` which is a `ultidyn` object, and `sensor_gain` which is a `ureal` atom.

Run `usimsamp` on the model, yielding a structure as described above.

```
unc_pole = ureal('unc_pole',-5,'Range',[-10 -4]);
plant = ss(unc_pole,5,1,1);
input_unc = ultidyn('input_unc',[1 1]);
wt = makeweight(0.25,130,2.5);
sensor_gain = ureal('sensor_gain',1,'Range',[0.1 2]);
data = usimsamp('usim_model')
data =
      input_unc: [1x1 ss]
    sensor_gain: 0.9935
       unc_pole: -4.1308
```

See Also

`usample`	Generate random samples of uncertain object
`usimfill`	Fills in the `Uncertainty Variable Name` field or sets the `Uncertainty Selection` pull down menu in `USS` Simulink blocks
`usiminfo`	Find USS System blocks within Simulink® model
`usubs`	Substitutes values for uncertain atoms

Purpose

Specify uncertain state space models or convert LTI model to uncertain state space model

Syntax

```
usys = uss(a,b,c,d)
usys = uss(a,b,c,d,Ts)
usys = uss(d)
usys = uss(a,b,c,d,Property,Value,...)
usys = uss(a,b,c,d,Ts,Property,Value,...)
usys = uss(sys)
```

Description

`uss` creates uncertain state-space models (`uss` objects) or to convert LTI models to the `uss` class.

`usys = uss(a,b,c,d)` creates a continuous-time uncertain state-space object. The matrices `a, b, c` and `d` can be `umat` and/or `double` and/or uncertain atoms. These are the 4 matrices associated with the linear differential equation model to describe the system.

`usys = uss(a,b,c,d,Ts)` creates a discrete-time uncertain state-space object with sampling time `Ts`.

`usys = uss(d)` specifies a static gain matrix and is equivalent to `usys = uss([],[],[],d)`.

Any of these syntaxes can be followed by property name/property value pairs.

`usys = uss(a,b,c,d,'P1',V1,'P2',V2,...)` set the properties `P1, P2, ...` to the values `V1, V2, ...`

`usys = uss(sys)` converts an arbitrary `ss`, `tf` or `zpk` model `sys` to an uncertain state space object without uncertainties. Both `usys.NominalValue` and `simplify(usys,'class')` are the same as `ss(sys)`.

Example

You can first create two uncertain atoms and use them to create two uncertain matrices. These four matrices can be packed together to form a 1-output, 1-input, 2-state continuous-time uncertain state space system.

```
p1 = ureal('p1',5,'Range',[2 6]);
p2 = ureal('p2',3,'Plusminus',0.4);
A = [-p1 0;p2 -p1];
```

```
B = [0;p2];
C = [1 1];
usys = uss(A,B,C,0);
```

In the second example, you can convert a not-uncertain `tf` model to an uncertain state-space model without uncertainties. You can verify the equality of the nominal value of the `usys` object and simplified representation to the original system.

```
G = tf([1 2 3],[1 2 3 4]);
usys = uss(G)
USS: 3 States, 1 Output, 1 Input, Continuous System
isequal(usys.NominalValue,ss(G))
ans =
     1
isequal(simplify(usys,'class'),ss(G))
ans =
     1
```

See Also

`frd`	Creates or converts to frequency response data model
`ss`	Creates or converts to state-space model

usubs

Purpose

Substitute given values for uncertain elements of uncertain objects

Syntax

```
B = usubs(M,atomname1,value1,atomname2,value2,...)
B = usubs(M,{atomname1;atomname2;...},{value1;value2;...})
B = usubs(M,StrucArray)
```

Description

`usubs` is used to substitute a specific value for an uncertain element of an uncertain object. The value can itself be uncertain. It needs to be the correct size, but otherwise can be of any class, and can be an array. Hence, the result can be of any class. In this manner, uncertain elements act as symbolic placeholders, for which specific values (which can also contain other placeholders too) can be substituted.

`B = usubs(M,atomname1,value1,atomname2,value2,...)` sets the atoms in `M`, identified by `atomname1`, `atomname2`, etc., to the values in `value1`, `value2`, etc., respectively.

`B = usubs(M,ElementName1,value1,ElementName2,value2,...)` sets the elements in `M`, identified by `ElementName1`, `ElementName2`, etc., to the values in `value1`, `value2`, etc. respectively.

The `names` and `values` can also be grouped in cell arrays, as

```
B = usubs(M,atomname1,value1,...)
```

In this case, if the `value` cell is 1-by-1, then that value is substituted for all the listed atoms. For this situation, it is not required that the value be in a cell array.

`value` can also be the string `'NominalValue'` or `'Random'` (or only partially specified) in which case the nominal value, or a random instance of the atom is used.

Combinations of the above syntaxes are also allowed, so that

```
B = usubs(M,{atomname1;atomname2},{value1;value2})
```

is allowed.

The `names` and `values` can also be grouped in a structure, with its field names constituting the `Names`, and the field values constituting the `Values`. In the following function call, `StrucArray` is a structure with field names and values.

```
B = usubs(M,StrucArray)
```

Robustness analysis commands such as `wcnorm, wcgain` and `robuststab` return the offending uncertain element values in this manner. `usample`, which randomly samples uncertain objects, also returns the sample points in this manner.

Example

Create an uncertain matrix and perform identical substitution in two different manners.

```
p = ureal('p',5);
m = [1 p;p^2 4];
size(m)
ans =
     2     2
m1 = usubs(m,'p',5)
m1 =
     1     5
    25     4
NamesValues.p = 5;
m2 = usubs(m,NamesValues)
m2 =
     1     5
    25     4
m1 - m2
ans =
     0     0
     0     0
```

You can make an array-valued substitution using the structure-based syntax,

```
NamesValues.p = rand(1,1,6);
m3 = usubs(m,NamesValues);  % 2-by-2-by-6
size(m3)
ans =
     2     2     6
```

You can use `usubs` to substitute for individual uncertainties. Create three uncertain real parameters, and form a simple `2-by-2` uncertain matrix with the parameters

```
a = ureal('a',5); b = ureal('b',3); c = ureal('c',1);
```

```
m = [a b;c a*b*c];
```

You can perform a single parameter substitution and check the results

```
m1 = usubs(m,'a',10);
simplify(m1(1,1))
ans =
    10
simplify(10*m1(1,2)*m1(2,1) - m1(2,2))
ans =
     0
```

You can replace one real parameter with a transfer function, and other parameters with `doubles`. You can do this using two different forms of the syntax and check that the results are identical.

```
m2 = usubs(m,'a',tf([5],[1 1]),'b',2.6,'c',1.3);
nv.a = tf([5],[1 1]);
nv.b = 2.6;
nv.c = 1.3;
m3 = usubs(m,nv);
norm(m2-m3,'inf')
ans =
     0
```

In `m`, replace `'a'` with `'b'`, obtaining `'b'` directly from `m` :

```
m4 = usubs(m,'a',m.Uncertainty.b);
```

See Also

`gridureal`	Grids uncertain real parameters over their range
`usample`	Generates random samples of an atom
`simplify`	Simplify representation of uncertain objects

Purpose

Calculate bounds on worst-case gain of uncertain system

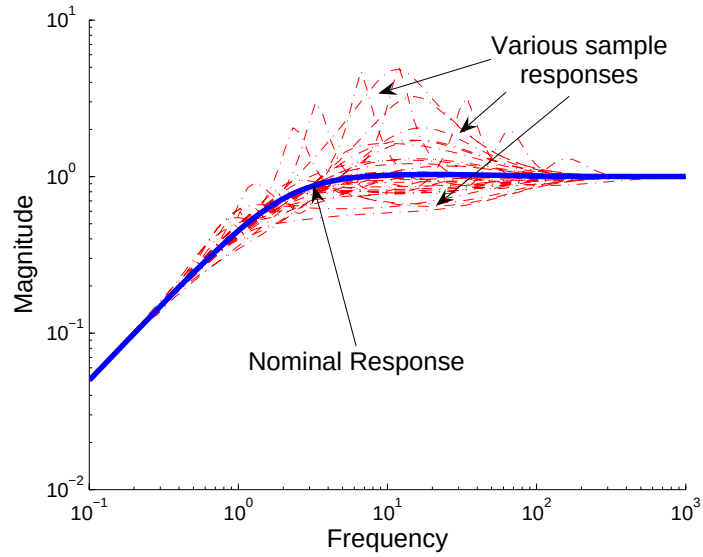
Syntax

```
[maxgain,wcu,info] = wcgain(sys)
[maxgain,wcu,info] = wcgain(sys,opts)
```

Description

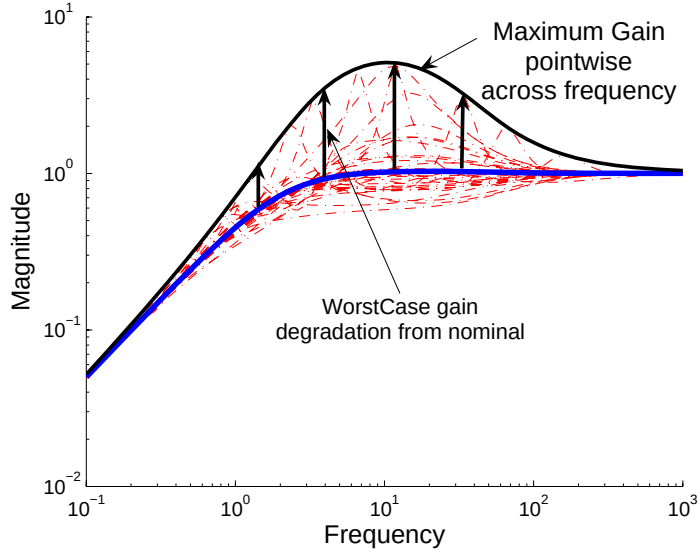
The gain of an uncertain system generally depends on the values of its uncertain elements. Here “gain” refers to the frequency response magnitude. Determining the maximum gain over all allowable values of the uncertain elements is referred to as a *worst-case gain* analysis. This maximum gain is called the *worst-case gain*.

The following figure shows the frequency response magnitude of many samples of an uncertain system model.



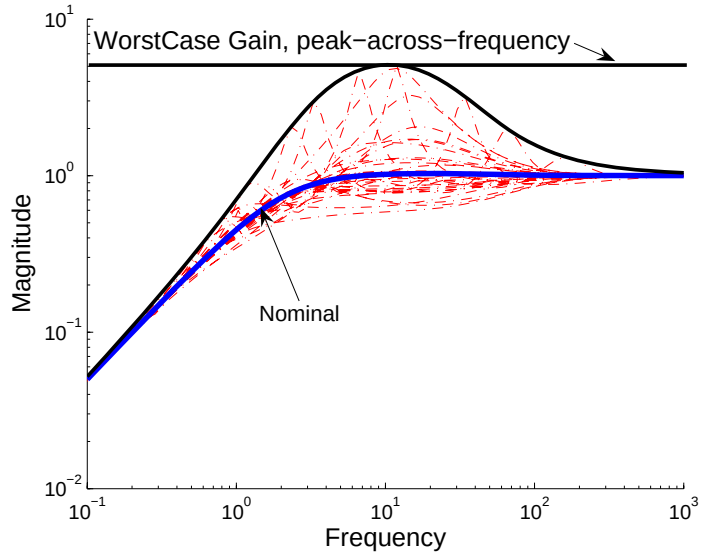
`wcgain` can perform two types of analysis on uncertain systems.

- A *pointwise-in-frequency* worst-case gain analysis yields the frequency-dependent curve of maximum gain, shown in the figure below.



This plot shows the maximum frequency-response magnitude at each frequency due to the uncertain elements within the model.

- A *peak-over-frequency* worst-case gain analysis only aims to compute the largest value of the frequency-response magnitude across all frequencies. During such an analysis, large frequency ranges can be quickly eliminated from consideration, thus reducing the computation time.



The default analysis performed by `wcgain` is *peak-over-frequency*. You can control which analysis is performed by using the `wcgopt` options object. For multi-input, multi-output systems, the gain is the maximum singular value of the frequency response matrix.

As with other *uncertain-system* analysis tools, only bounds on the worst-case gain are computed. The exact value of the worst-case gain is guaranteed to lie between these upper and lower bounds.

The computation used in `wcgain` is a frequency-domain calculation. If the input system `sys` is an uncertain frequency response object (`ufrd`), then the analysis is performed on the frequency grid within the `ufrd`. If the input system `sys` is an uncertain state-space object (`uss`), then an appropriate frequency grid is generated (automatically), and the analysis performed on that frequency grid. In all descriptions below, N denotes the number of points in the frequency grid.

Basic Syntax

Suppose `sys` is an `ufrd` or `uss` with M uncertain elements. Calculate the worst-case gain of

```
[maxgain,maxgainunc] = wcgain(sys)
```

maxgain is a structure with the following fields

Field	Description
LowerBound	Lower bound on worst-case gain, positive scalar.
UpperBound	Upper bound on worst-case gain, positive scalar. If the nominal value of the uncertain system is unstable, then maxgain.LowerBound and maxgain.UpperBound equal ∞.
CriticalFrequency	The critical value of frequency at which maximum gain occurs (this is associated with maxgain.LowerBound).

maxgainunc is a structure containing values of uncertain elements that maximize the system gain. There are *M* field names, which are the names of uncertain elements of sys. The value of each field is the corresponding value of the uncertain element, such that when combined lead to the gain value in maxgain.LowerBound. The command

```
norm(usubs(sys,maxgainunc),'inf')
```

shows the gain.

Example

Create a plant with nominal model of an integrator, and include additive unmodeled dynamics uncertainty of a level of 0.4 (this corresponds to 100% model uncertainty at 2.5 rad/s).

Design a proportional controller K_1 that puts the nominal closed-loop bandwidth at 0.8 rad/s. Roll off K_1 at a frequency 25 times the nominal closed-loop bandwidth. Repeat the design for a controller K_2 that puts the nominal closed-loop bandwidth at 2.0 rad/s. In each case, form the closed-loop sensitivity function.

```
P = tf(1,[1 0]) + ultidyn('delta',[1 1],'bound',0.4);
BW1 = 0.8;
K1 = tf(BW1,[1/(25*BW1) 1]);
S1 = feedback(1,P*K1);
BW2 = 2.0;
K2 = tf(BW2,[1/(25*BW2) 1]);
```

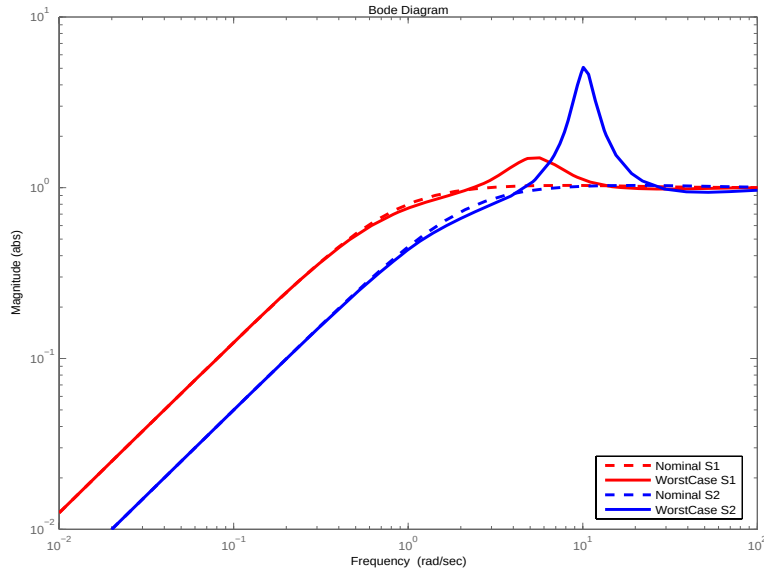
```
S2 = feedback(1,P*K2);
```

Assess the worst-case gain of the closed-loop sensitivity function.

```
[maxgain1,wcunc1] = wcgain(S1);
[maxgain2,wcunc2] = wcgain(S2);
maxgain1
maxgain1 =
           LowerBound: 1.5070e+000
           UpperBound: 1.5080e+000
    CriticalFrequency: 5.3096e+000
maxgain2
maxgain2 =
           LowerBound: 5.1024e+000
           UpperBound: 5.1034e+000
    CriticalFrequency: 1.0215e+001
```

The `maxgain` variables indicate that controller K_1 achieves better worst-case performance than K_2. Plot Bode magnitude plots of the nominal closed-loop sensitivity functions, as well as the *worst* instances, using `usubs` to replace the uncertain element with the worst value returned by `wcgain`.

```
bodemag(S1.Nom,'r--',usubs(S1,wcunc1),'r',...
S2.Nom,'b--',usubs(S2,wcunc2),'b')
```



Note that although the nominal closed-loop sensitivity resulting from K_2 is superior to that with K_1, the worst-case behavior is much worse.

Basic Syntax with Third Output Argument

A third output argument yields more specialized information, including sensitivities of the worst-case gain to the uncertain element's ranges and frequency-by-frequency information.

```
[maxgain,maxgainunc,info] = wcgain(sys)
```

The third output argument `info` is a structure with the following fields

Field	Description
`Sensitivity`	A `struct` with *M* fields. Field names are names of uncertain elements of `sys`. Values of fields are positive numbers, each entry indicating the local sensitivity of the worst-case gain in `maxgain.LowerBound` to all the individual uncertain element's uncertainty ranges. For instance, a value of 25 indicates that if the uncertainty range is enlarged by 8%, then the worst-case gain should increase by about 2%. If the `Sensitivity` property of the `wcgopt` object is `'off'`, the values are `NaN`.
`Frequency`	*N*-`by-1` frequency vector associated with analysis.
`ArrayIndex`	`1-by-1` scalar matrix whose value is 1. In more complicated situations (described later) the value of this field is dependent on the input data.

Options (e.g., turning on/off the sensitivity computation, setting the step-size in the sensitivity computation, adjusting the stopping criteria, and controlling behavior across frequency and array dimensions) can be specified using the worst-case gain analysis options `wcgopt` object. For instance, you can turn the sensitivity calculation off by executing

```
opt = wcgopt('Sensitivity','off');
[maxgain,maxgainunc,info] = wcgain(sys,opt)
```

Advanced Options: Pointwise-in-Frequency Calculations

It is also possible to perform the computation pointwise-in-frequency, determining the worst-case gain at each and every frequency point. To do this, the `wcgopt` options object must be used.

```
opt = wcgopt('FreqPtWise',1);
[maxgain,maxgainunc,info] = wcgain(sys);
```

Because the calculation is pointwise-in-frequency, many results are N-by-1 cell arrays, often containing scalar information relevant to each particular frequency. `maxgain` is a structure with the following fields:

Field	Description
`LowerBound`	Lower bound on worst-case gain, positive scalar (`frd` with N frequency points).
`UpperBound`	Upper bound on worst-case gain, positive scalar. If the nominal value of the uncertain system is unstable, then `maxgain.LowerBound` and `maxgain.UpperBound` equal ∞ (`frd` with N frequency points).
`CriticalFrequency`	Scalar. The critical value of frequency at which maximum gain occurs (this is associated with `norm(maxgain.LowerBound,inf)`).

`maxgainunc` is a N-by-1 cell array of values of uncertain elements that maximize the system gain. Each entry of the cell array is a `struct` whose M field names are the names of uncertain elements of `sys`. The maximum singular value of `usubs(sys,maxgainunc{k})` at the `k`th frequency (in `info.Frequency(k)`) is equal to `maxgain.LowerBound{k}`.

`info` is a structure with the following fields:

Field	Description
`Sensitivity`	N-by-1 cell. Each entry is a `struct` corresponding to the sensitivities in the worst-case gain at each individual frequency.
`Frequency`	N-by-1 frequency vector associated with analysis.
`ArrayIndex`	N-by-1 cell array. Each value is the 1-by-1 matrix whose numerical value is 1. In more complicated situations (described later) the value of this field is dependent on the input data.

Advanced Options: Handling Array Dimensions

If sys has array dimensions, the default behavior is to maximize over all of these dimensions as well. This can be controlled however, and it is also possible to perform the computation pointwise-in-the-array-dimensions, determining the worst-case gain at each and every grid point. Moreover, any combination of "peak-over" and "pointwise-over" is allowed. To specify the desired computation, you must use wcgopt. For concreteness, suppose that sys is a r×c×7×5×8 uncertain system (i.e., a 7-by-5-by-8 array of uncertain r output, c input systems). In order to perform the worst-case gain calculation pointwise over the 2nd and 3rd array dimensions (the slot with 5 points and the slot with 8 points), set the ArrayDimPtWise property as follows:

```
opt = wcgopt('ArrayDimPtWise',[2 3]);
```

In this case, the worst-case gain calculation is performed "pointwise" on the 5-by-8 grid, but only the "peak value" over the first array dimension (the slot with 7 points) is kept track of. For that reason, many of the results are of dimension 1-by-5-by-8.

In general, suppose that the array dimensions of sys are $d_1 \times \ldots \times d_F$ (7×5×8 in the above example). Furthermore, assume that the ArrayDimPtWise property of the wcgopt object is set to some of the integers between 1 and F. Let $e_1, e_2, \ldots, e_F$ denote the dimensions of the array on which the results are computed. By definition, if j is an integer listed in ArrayDimPtWise, then $e_j = d_j$ (all grid points in slot j are computed); otherwise, $e_j = 1$ (only the maximum in slot j is computed). In the above example, with ArrayDimPtWise set to [2 3], it follows that $e_1 = 1, e_2 = 5, e_3 = 8$.

Assume FreqPtWise is set to 'off' (you will return to that case later). In this case, the results of

```
[maxgain,maxgainunc,info] = wcgain(sys,opt)
```

are that maxgain is a structure with the following fields:

Field	Description
LowerBound	1-by-1 frd, with array dimensions $e_1 \times \ldots \times e_F$, lower bound on worst-case gain, computed pointwise over all array dimensions listed in ArrayDimPtWise property, and peaked over all others.
UpperBound	Upper bound, analogous to LowerBound
CriticalFrequency	$e_1 \times \ldots \times e_F$ array with the critical value of the frequency at which maximum gain occurs (this is associated with maxgain.LowerBound).

maxgainunc is a $e_1 \times \ldots \times e_F$ struct, containing values of uncertain elements that maximize the system gain. There are M field names, which are the names of uncertain elements of sys. The value of each field is the corresponding value of the uncertain element, such that when combined lead to the gain value in maxgain.LowerBound. The command norm(usubs(sys,maxgainunc),'inf') shows the gain, and should be identical to maxgain.LowerBound (to within the tolerance used in norm).

info is a structure with the following fields:

Field	Description
Sensitivity	$e_1 \times \ldots \times e_F$ struct array. Each entry is the local sensitivity of the worst-case gain in maxgain.LowerBound to all the individual uncertain elements uncertainty ranges.
Frequency	N–by-1 frequency vector associated with analysis.
ArrayIndex	At each value in the $e_1 \times \ldots \times e_F$ grid, there is a corresponding value in the $d_1 \times \ldots \times d_F$ grid where the maximum occurs. The variable info.ArrayIndex is an $e_1 \times \ldots \times e_F$ matrix whose value is the single-index representation of the maximizing location in the $d_1 \times \ldots \times d_F$ grid.

Advanced Options:

Array Dimension Handling with `FreqPtWise` Set to `'on'`

The final case involves array dimensions and pointwise-in-frequency calculations. Again, suppose that the array dimensions of `sys` are $d_1 \times \ldots \times d_F$. Furthermore, assume that the `ArrayDimPtWise` property of the `wcgopt` object is set to some of the integers between 1 and `F`. Let $e_1, e_2, \ldots, e_F$ denote the dimensions of the array on which the results are computed.

Because the calculation is pointwise-in-frequency, many results are `N-by-1` cell arrays, often containing $e_1 \times \ldots \times e_F$ arrays in each cell.

`maxgain` is a structure with the following fields

Field	Description
`LowerBound`	`N-by-1` cell array, `maxgain.LowerBound{k}` is a 1-by-1 `frd` with array dimensions $e_1 \times \ldots \times e_F$, and is a lower bound on worst-case gain at frequency `info.Frequency(k)`, computed pointwise over all array dimensions listed in the `ArrayDimPtWise` property, and "peaked" over all others.
`UpperBound`	Upper bound on worst-case gain, analogous to `maxgain.LowerBound`.
`CriticalFrequency`	$e_1 \times \ldots \times e_F$ array with the critical value of the frequency at which maximum gain (pointwise over all array dimensions listed in `ArrayDimPtWise` property, and "peaked" over all others) occurs.

`maxgain.CriticalFrequency` $e_1 \times \ldots \times e_F$ array with the critical value of the frequency at which maximum gain (pointwise over all array dimensions listed in the `ArrayDimPtWise` property, and "peaked" over all others) occurs.

`maxgainunc` is a `N-by-1` cell array, `k`th entry is a $e_1 \times \ldots \times e_F$ `struct`, containing values of uncertain elements that maximize the system gain at frequency `info.Frequency(k)`.

info is a structure with the following fields:

Field	Description
Sensitivity	N-by-1 cell. Each entry is the $e_1 \times \ldots \times e_F$ struct array which holds the local sensitivity of the worst-case gain at one frequency to all of the individual uncertain elements uncertainty ranges.
Frequency	N-by-1 frequency vector associated with analysis.
ArrayIndex	N-by-1 cell array, kth $e_1 \times \ldots \times e_F$, at each value in the $e_1 \times \ldots \times e_F$ grid, there is a corresponding value in the $d_1 \times \ldots \times d_F$ grid where the maximum occurs. The variable info.ArrayIndex is a $e_1 \times \ldots \times e_F$ matrix whose value is the single-index representation of the maximizing location in the $d_1 \times \ldots \times d_F$ grid.

Behavior on Not-Uncertain Systems

wcgain can also be used on not-uncertain systems (e.g., ss and frd). If sys is a single ss or frd, then the worst-case gain is simply the gain of the system (identical to norm(sys,'inf')). However, if sys has array dimensions, then the possible combinations of "peak-over" and "pointwise-over" can be used to customize the computation.

Algorithm

The worst-case gain is guaranteed to be at least as large as LowerBound (some value of allowable uncertain elements yield this gain – one instance is returned in the structure maxgainunc. The frequency at which the gain in LowerBound occurs is in CriticalFrequency. Lower bounds for wcgain are computed using a power iteration on ultidyn, ucomplex and ucomplexm uncertain atoms, (holding uncertain real parameters fixed) and a coordinate aligned search on the uncertain real parameters (while holding the complex blocks fixed).

Similarly, the worst-case gain is guaranteed to be no larger than UpperBound. In other words, for all allowable modeled uncertainty, the gain is provably less than or equal to UpperBound. These bounds are derived using the upper bound for the structured singular value, which is essentially optimally-scaled, small-gain theorem analysis. Upper bounds are obtained by solving a

semidefinite program. `wcgain` uses branch and bound on the uncertain real parameters to tighten the lower and upper bounds.

Limitations

Because the calculation is carried out with a frequency grid, it is possible (likely) that the true critical frequency is missing from the frequency vector used in the analysis. This is similar to the problem in `robuststab`. However, compared with `robuststab`, the problem in `wcgain` is less acute. Thought of as a function of problem data and frequency, the worst-case gain is a continuous function (unlike the robust stability margin, which in special cases is not; see "Regularizing Robust Stability calculations with only `ureal` uncertain elements" in the online documentation). Hence, in worst-case gain calculations, increasing the density of the frequency grid will always increase the accuracy of the answers and in the limit, answers arbitrarily close to the actual answers are obtainable with finite frequency grids.

See Also

`loopmargin`	Comprehensive analysis of feedback loops
`mussv`	Calculate bounds on the Structured Singular Value (μ)
`norm`	System norm of an LTI object
`robuststab`	Calculates stability margins of uncertain systems
`wcgopt`	Creates a `wcgain` options object
`wcsens`	Calculates worst-case sensitivities for a feedback loop
`wcmargin`	Calculates worst-case margins for a feedback loop

Purpose

Create options object for use with `wcgain`, `wcsens`, and `wcmargin`

Syntax

```
options = wcgopt
options = wcgopt('name1',value1,'name2',value2,...)
wcgopt
```

Description

`options = wcgopt` (with no input arguments) creates an options object with all the properties set to their default values.

`options = wcgopt('name1',value1,'name2',value2,...)` creates a `wcgain`, `wcsens` and `wcmargin` options object called options in which specified properties have specific values. Any unspecified property is set to its default value. It is sufficient to type only enough leading characters to define the property name uniquely. Case is ignored for property names.

`wcgopt` with no input or output arguments displays a complete list of option properties and their default values.

The following are the `wcgopt` object properties:

Object Property	Description
`Sensitivity`	Computes margin sensitivity to individual uncertainties {`off'; `on'}. Default is `'on'`
`LowerBoundOnly`	If `LowerBoundOnly` is `'on'`, then only the lower bound computation is performed. The default value is `'off'`, which implies that both upper and lower bounds for worst-case gain are computed.
`FreqPtWise`	Apply stopping criteria based on upper/lower bounds (described below) at every frequency point (as opposed to just the peak value). `FreqPtWise=1` activates the pointwise criteria. In order to only compute the peak value to within tolerance, use 0. Default = 0.

Object Property	Description
`ArrayDimPtWise`	Relevant for `uss/ufrd/ss/frd` arrays. For indices specified in `ArrayDimPtWise`, the stopping criteria based on upper/lower bounds (described below) is used at every point in array dimensions specified in `ArrayDimPtWise`, being applied to the peak value over all other array dimensions. Default = []. If `FreqPtWise==1`, the computation terminates when at least one of the following four conditions is true at *every* frequency: • `UpperBound - LowerBound <= AbsTol` • `UpperBound - LowerBound <= Reltol*UpperBound` • `UpperBound <= AGoodThreshold + MGoodThreshold*Norm(NominalValue)` • `LowerBound >= ABadThreshold + MBadThreshold*Norm(NominalValue)` If `FreqPtWise==0`, the computation terminates when any one of the following four conditions is true: • `PeakUpperBound - PeakLowerBound <= AbsTol` • `PeakUpperBound - PeakLowerBound <= Reltol*PeakUpperBound` at every frequency • `UpperBound <= AGoodThreshold + MGoodThreshold*Norm(NominalValue))` at some frequency • `LowerBound >= ABadThreshold + MBadThreshold*Norm(NominalValue)` In both situations above, the stopping condition is applied at every point in array dimensions specified in `ArrayDimPtWise`. `UpperBound` and `LowerBound` are the peak values over all other array dimensions.
`Default`	Structure with default values of all wcgopt properties.

Object Property	Description
`Meaning`	Structure. Field names are `wcgopt` properties, and values are the text description of property.
`VaryUncertainty`	Percentage variation of uncertainty used as a step size in finite-difference calculations to estimate sensitivity. Default is 25.
`AbsTol`	Upper and lower absolute stopping tolerance. Default=0.02.
`RelTol`	Upper and lower absolute stopping tolerance. Default = 0.02.
`AbsTol`	Upper and lower relative stopping tolerance. Default=0.05.
`MGoodThreshold`	Multiplicative (`UpperBound`) stopping threshold. Default = 1.04.
`AGoodThreshold`	Additive (`UpperBound`) stopping threshold. Default = 0.05.
`MBadThreshold`	Multiplicative (`LowerBound`) stopping threshold, Default = 5.
`AGoodThreshold`	Additive (`LowerBound`) stopping threshold. Default = 20.
`NTimes`	Number of restarts in lower bound search (positive integer).
`MaxCnt`	Number of cycles in lower bound search (positive integer). Default = 3.
`MaxTime`	Maximum computation time allowed (in seconds). The computation is prematurely terminated if this much real time elapses before the computation is complete. All quantities that have been computed are returned. Default = 720.

Example

You can create a `wcgopt` options object called `opt` with all default values.

```
opt = wcgopt
Property Object Values:
        Sensitivity: 'on'
     LowerBoundOnly: 'off'
         FreqPtWise: 0
     ArrayDimPtWise: []
    VaryUncertainty: 25
            Default: [1x1 struct]
            Meaning: [1x1 struct]
             AbsTol: 0.0200
             RelTol: 0.0500
     MGoodThreshold: 1.0400
     AGoodThreshold: 0.0500
      MBadThreshold: 20
      ABadThreshold: 5
             NTimes: 2
             MaxCnt: 3
            MaxTime: 720
```

The following statements change the absolute tolerance stopping criterion from `0.02` to `0.04` and the point wise over frequency test from the peak worst-case value, `opt.FreqPtWise=0`, to the worst-case value at every frequency.

```
opt = wcgopt;
opt.AbsTol = 0.04;
opt.FreqPtWise = 1;
opt
Property Object Values:
        Sensitivity: 'on'
     LowerBoundOnly: 'off'
         FreqPtWise: 1
     ArrayDimPtWise: []
            Default: [1x1 struct]
            Meaning: [1x1 struct]
    VaryUncertainty: 25
             AbsTol: 0.0400
             RelTol: 0.0500
     MGoodThreshold: 1.0400
     AGoodThreshold: 0.0500
```

```
        MBadThreshold: 20
        ABadThreshold: 5
               NTimes: 2
               MaxCnt: 3
              MaxTime: 720
```

This statement makes a single call to `wcgopt` to set the maximum computation time to 10000 seconds and disables the `Sensitivity` calculation.

```
opt = wcgopt('MaxTime',10000,'Sensitivity','off');
```

See Also

`dkitopt`	Creates a `dksyn` options object
`robopt`	Creates a `robustperf/robuststab` options object
`wcgain`	Calculates worst-case gain of a system
`wcnorm`	Calculates worst-case gain of a matrix
`wcsens`	Calculates worst-case sensitivities for a feedback loop
`wcmargin`	Calculates worst-case margins for a feedback loop

Purpose

Worst-case disk stability margins of uncertain feedback loops

Syntax

```
wcmarg = wcmargin(L)
wcmargi = wcmargin(p,c)
[wcmargi,wcmargo] = wcmargin(p,c)
wcmargi = wcmargin(p,c,opt)
[wcmargi,wcmargo] = wcmargin(p,c,opt)
```

Description

Classical gain and phase margins define the loop-at-a-time allowable, independent variations in the nominal system gain and phase for which the feedback loop retains stability. An alternative to classical gain and phase margins is the disk margin. The disk margin calculates the largest region for each channel such that for all gain and phase variations inside the region the nominal closed-loop system is stable. The guaranteed bound is calculated based on the balanced sensitivity function. See the `dmplot` and `loopmargin` Algorithm sections for more information.

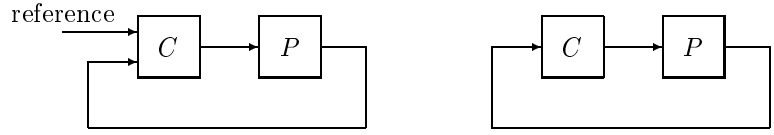
Consider a system with uncertain elements. It is of interest to determine the gain and phase margins of each individual channel in the presence of uncertainty. These margins are called worst-case margins. Worst-case margin, `wcmargin` calculates the largest disk margin such that for values of the uncertainty and all gain and phase variations inside the disk, the closed-loop system is stable. The worst-case gain and phase margin bounds are defined based on the balanced sensitivity function. Hence, results from the worst-case margin calculation imply that the closed-loop system is stable for a given uncertainty set and would remain stable in the presence of an additional gain and phase margin variation in the specified input/output channel.

`wcmargL = wcmargin(L)` calculates the combined worst-case input and output loop-at-a-time gain/phase margins of the feedback loop consisting of the loop transfer matrix `L` in negative feedback with an identity matrix. `L` must be an uncertain system, `uss` or `ufrd` object. If `L` is a `uss` object, the frequency range and number of points used to calculate `wcmargL` are chosen automatically. Note that in this case, the worst-case margins at the input and output are equal

because an identity matrix is used in feedback. `wcmarg` is a `NU`-by-`1` structure with the following fields:

Field	Description
`GainMargin`	Guaranteed bound on worst-case, single-loop gain margin at plant inputs. Loop-at-a-time analysis.
`PhaseMargin`	Loop-at-a-time worst-case phase margin at plant inputs. Units are degrees.
`Frequency`	Frequency associated with the worst-case margin (rad/s).
`Sensitivity`	`Struct` with `M` fields. Field names are names of uncertain elements of `P` and `C`. Values of fields are positive numbers, which each entry indicating the local sensitivity of the worst-case margins to all the individual uncertain element's uncertainty ranges. For instance, a value of 50 indicates that if the uncertainty range is enlarged by 8%, then the worst-case gain should increase by about 4%. If the `Sensitivity` property of the `wcgopt` object is `'off'`, the values are `NaN`.

`[wcmargi,wcmargo] = wcmargin(P,C)` calculates the combined worst-case input and output loop-at-a-time gain/phase margins of the feedback loop consisting of `C` in negative feedback with `P`. `C` should only be the compensator in the feedback path, without reference channels, if it is a *2-DOF* architecture. That is, if the closed-loop system has a *2-DOF* architecture the reference channel of the controller should be eliminated resulting in a *1-DOF* architecture as shown in the following figure. Either `P` or `C` must be an uncertain system, `uss` or `ufrd`, or an uncertain matrix, `umat`. If `P` and `C` are `ss/tf/zpk` or `uss` objects, the frequency range and number of points used to calculate `wcmargi` and `wcmargo` are chosen automatically.



2-dof architecture 1-dof architecture

Basic Syntax

```
[wcmargi,wcmargo] = wcmargin(L)
[wcmargi,wcmargo] = wcmargin(P,C)
```

`wcmargi` and `wcmargo` are structures corresponding to the loop-at-a-time worst-case, single-loop gain and phase margin of the channel. For the single-loop transfer matrix `L` of size `N`-by-`N`, `wcmargi` is a `N`-by-`1` structure. For the case with two input arguments, the plant model `P` will have `NY` outputs and `NU` inputs and hence the controller `C` must have `NU` outputs and `NY` inputs. `wcmargi` is a `NU`-by-`1` structure with the following fields:

Field	Description
`GainMargin`	Guaranteed bound on worst-case, single-loop gain margin at plant inputs. Loop-at-a-time analysis.
`PhaseMargin`	Loop-at-a-time worst-case phase margin at plant inputs. Units are degrees.
`Frequency`	Frequency associated with the worst-case margin (rad/s).
`Sensitivity`	`Struct` with `M` fields. Field names are names of uncertain elements of `P` and `C`. Values of fields are positive numbers, which each entry indicating the local sensitivity of the worst-case margins to all the individual uncertain element's uncertainty ranges. For instance, a value of 50 indicates that if the uncertainty range is enlarged by 8%, then the worst-case gain should increase by about 4%. If the `Sensitivity` property of the `wcgopt` object is `'off'`, the values are `NaN`.

`wcmargo` is an `N`-by-1 structure for the single loop transfer matrix input and `wcmargo` is an `NY`-by-1 structure when the plant and controller are input. In both these cases, `wcmargo` has the same fields as `wcmargi`. The worst-case bound on the gain and phase margins are calculated based on a balanced sensitivity function.

`[wcmargi,wcmargo] = wcmargin(L,opt)` and

`[wcmargi,wcmargo] = wcmargin(p,c,opt)` specify options described in `opt`. (See `wcgopt` for more details on the options for `wcmargin`.)

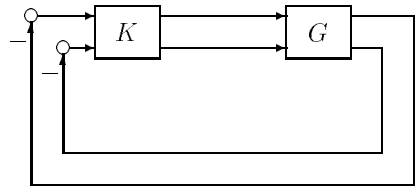
The sensitivity of the worst-case margin calculations to the individual uncertain elements is selected using the options object `opt`. To compute sensitivities, create a `wcgopt` options object, and set the `Sensitivity` property to `'on'`.

```
opt = wcgopt('Sensitivity','on');
[maxgain,maxgainunc,info] = wcgain(sys,opt)
```

Example — MIMO Loop-at-a-Time Margins

This example is designed to illustrate that loop-at-a-time margins (gain, phase, and/or distance to –1) can be inaccurate measures of multivariable robustness margins. Margins of the individual loops can be very sensitive to small perturbations within other loops.

The nominal closed-loop system considered here is shown as follows



G and K are 2-by-2 multi-input/multi-output (MIMO) systems, defined as

$$G := \frac{1}{s^2 + \alpha^2}\begin{bmatrix} s - \alpha^2 & \alpha(s+1) \\ -\alpha(s+1) & s - \alpha^2 \end{bmatrix}, \quad K = I_2$$

Set $\alpha := 10$, construct the nominal model G in state-space form, and compute its frequency response.

```
a = [0 10;-10 0];
b = eye(2);
c = [1 8;-10 1];
d = zeros(2,2);
G = ss(a,b,c,d);
K = [1 -2;0 1];
```

The nominal plant was analyzed previously using the `loopmargin` command. Based on experimental data, the gain of the first input channel, `b(1,1)`, is found to vary between 0.97 and 1.06. The following statement generates the updated uncertain model.

```
ingain1 = ureal('ingain1',1,'Range',[0.97 1.06]);
b = [ingain1 0;0 1];
Gunc = ss(a,b,c,d);
```

Because of differences between measured data and the plant model an 8% unmodeled dynamic uncertainty is added to the plant outputs.

```
unmod = ultidyn('unmod',[2 2],'Bound',0.08);
Gmod = (eye(2)+unmod)*Gunc;
Gmodg = ufrd(Gmod,logspace(-1,3,60));
```

You can use the command `wcmargin` to determine the worst-case gain and phase margins in the presences of the uncertainty.

```
[wcmi,wcmo] = wcmargin(Gmodg,K);
```

The worst-case analysis corresponds to maximum allowable disk margin for all possible defined uncertainty ranges. The worst-case single-loop margin analysis performed using `wcmargin` results in a maximum allowable gain margin variation of 1.31 and phase margin variations of ± 15.6 degs in the second input channel in the presence of the uncertainties `'unmod'` and `'ingain1'`. `wcmi(1)`

```
ans =
     GainMargin: [0.3613 2.7681]
    PhaseMargin: [-50.2745 50.2745]
      Frequency: 0.1000
    Sensitivity: [1x1 struct]
```

```
wcmi(2)
ans =
     GainMargin: [0.7585 1.3185]
    PhaseMargin: [-15.6426 15.6426]
      Frequency: 0.1000
    Sensitivity: [1x1 struct]
```

Hence even though the second channel had infinite gain margin and 90 degrees of phase margin, allowing variation in both uncertainties, `'unmod'` and `'ingain1'` leads to a dramatic reduction in the gain and phase margin. See the `loopmargin` command page example for more details

You can display the sensitivity of the worst-case margin in the second input channel to `'unmod'` and `'ingain1'` as follows:

```
wcmi(2).Sensitivity
ans =
    ingain1: 12.1865
      unmod: 290.4557
```

The results indicate that the worst-case margins are not very sensitive to the gain variation in the first input channel, `'ingain1'`, but very sensitive to the LTI dynamic uncertainty at the output of the plant.

The worst-case single-loop margin at the output results in a maximum allowable gain margin variation of 1.46 and phase margin variation of ± 21.3 degs in the second output channel in the presence of the uncertainties `'unmod'` and `'ingain1'`.

```
wcmo(1)
ans =
     GainMargin: [0.2521 3.9664]
    PhaseMargin: [-61.6995 61.6995]
      Frequency: 0.1000
    Sensitivity: [1x1 struct]
wcmo(2)
ans =
     GainMargin: [0.6835 1.4632]
    PhaseMargin: [-21.2984 21.2984]
      Frequency: 0.1000
    Sensitivity: [1x1 struct]
```

You can display the sensitivity of the worst-case margin in the second output channel to `'unmod'` and `'ingain1'` as follows:

```
wcmo(2).Sensitivity
ans =
    ingain1: 16.3435
      unmod: 392.1320
```

The results are similar to the worst-case margins at the input. However, the worst-case margins at the second output channel are even more sensitive to the LTI dynamic uncertainty than the input channel margins.

See Also

`dmplot`	Interprets disk gain and phase margins
`loopsens`	Calculates sensitivity functions of feedback loops
`loopmargin`	Performs comprehensive analysis of feedback loops
`robuststab`	Calculates stability margins of uncertain systems
`usubs`	Substitutes values for uncertain atoms
`wcgain`	Calculates worst-case gain of a system
`wcgopt`	Creates a worst-case options object
`wcsens`	Calculates worst-case sensitivity functions

Purpose

Worst-case norm of uncertain matrix

Syntax

```
maxnorm = wcnorm(m)
[maxnorm,wcu] = wcnorm(m)
[maxnorm,wcu] = wcnorm(m,opts)
[maxnorm,wcu,info] = wcnorm(m)
[maxnorm,wcu,info] = wcnorm(m,opts)
```

Description

The norm of an uncertain matrix generally depends on the values of its uncertain elements. Determining the maximum norm over all allowable values of the uncertain elements is referred to as a *worst-case norm* analysis. The maximum norm is called the *worst-case norm*.

As with other *uncertain-system* analysis tools, only bounds on the worst-case norm are computed. The exact value of the worst-case norm is guaranteed to lie between these upper and lower bounds.

Basic syntax

Suppose `mat` is a `umat` or a `uss` with M uncertain elements. The results of

```
[maxnorm,maxnormunc] = wcnorm(mat)
```

`maxnorm` is a structure with the following fields

Field	Description
`LowerBound`	Lower bound on worst-case norm, positive scalar.
`UpperBound`	Upper bound on worst-case norm, positive scalar.

`maxnormunc` is a structure that includes values of uncertain elements and maximizes the matrix norm. There are M fieldnames, which are the names of uncertain elements of `mat`. The value of each field is the corresponding value of the uncertain element, such that when jointly combined, lead to the norm value in `maxnorm.LowerBound`. The following command shows the norm:

```
norm(usubs(mat,maxnormunc))
```

Basic syntax with third output argument

A third output argument provides information about sensitivities of the worst-case norm to the uncertain elements ranges.

```
[maxnorm,maxnormunc,info] = wcgain(mat)
```

The third output argument `info` is a structure with the following fields:

Field	Description
`Sensitivity`	A `struct` with *M* fields. Fieldnames are names of uncertain elements of `sys`. Field values are positive numbers, each entry indicating the local sensitivity of the worst-case norm in `maxnorm.LowerBound` to all of the individual uncertain elements uncertainty ranges. For instance, a value of 25 indicates that if the uncertainty range is increased by 8%, then the worst-case norm should increase by about 2%. If the `Sensitivity` property of the `wcgopt` object is `'off'`, the values are `NaN`.
`ArrayIndex`	1-by-1 scalar matrix with the value of 1. In more complicated situations (described later) the value of this field depends on the input data.

Advanced options: Handling array dimensions

If `mat` has array dimensions, the default behavior is to maximize over all dimensions. It is also possible to perform the computation pointwise-in-the-array-dimensions to determine the worst-case norm at each grid point. Any combination of "peak-over" and "pointwise-over" is allowed.

To specify the desired computation, the `wcgopt` must be used. For concreteness, suppose that `mat` is an r×c×7×5×8 uncertain system (i.e., a 7-by-5-by-8 array of uncertain *r* output, *c* input systems). To perform the worst-case gain calculation pointwise over the second and third array dimensions (the slots with 5 points and 8 points, respectively), set the `ArrayDimPtWise` property:

```
opt = wcgopt('ArrayDimPtWise',[2 3]);
```

In this case, the worst-case norm calculation is performed "pointwise" on the 5-by-8 grid. Only the "peak value" in the first array dimension (the slot with 7

points) is tracked. For that reason, many of the results will be of dimension 1-by-5-by-8.

In general, suppose that the array dimensions of `sys` are $d_1 \times \ldots \times d_F$ (7×5×8 in the above example). Furthermore, assume that the `ArrayDimPtWise` property of the `wcgopt` object has been set to some of the integers between 1 and `F`. Let $e_1, e_2, \ldots, e_F$ denote the dimensions of the array on which the results are computed. By definition, if `j` is an integer listed in `ArrayDimPtWise`, then $e_j = d_j$ (all grid points in slot `j` are computed), otherwise $e_j = 1$ (only the maximum in slot `j` is computed). In the above example, with `ArrayDimPtWise` set to `[2 3]`, it follows that $e_1 = 1$, $e_2 = 5$, $e_3 = 8$.

In this case, the following command

```
[maxgain,maxgainunc,info] = wcgain(sys,opt)
```

produces the `maxgain` a structure with the following fields

Field	Description
`LowerBound`	$e_1 \times \ldots \times e_F$ matrix of lower bounds on worst-case norm, computed pointwise over all array dimensions listed in `ArrayDimPtWise` property and "peaked" over all others.
`UpperBound`	Upper bound, analogous to `LowerBound`

`maxgainunc` is a $e_1 \times \ldots \times e_F$ `struct`, containing values of uncertain elements which maximize the system norm. There are M fieldnames, which are the names of uncertain elements of `mat`. The value of each field is the corresponding value of the uncertain element, which lead to the gain value in `maxnorm.LowerBound` when jointly combined.

`info` is a structure with the following fields

Field	Description
`Sensitivity`	$e_1 \times \ldots \times e_F$ `struct` array, where each entry is the local sensitivity of the worst-case norm in `maxnorm.LowerBound` to the uncertainty range of each uncertain element.
`ArrayIndex`	At each value in the $e_1 \times \ldots \times e_F$ grid, there is a corresponding value in the $d_1 \times \ldots \times d_F$ grid where the maximum occurs. The variable `info.ArrayIndex` is an $e_1 \times \ldots \times e_F$ matrix, where the value is the single-index representation of the maximizing location in the $d_1 \times \ldots \times d_F$ grid.

Example

You can construct an uncertain matrix and compute the worst-case norm of the matrix, as well as its inverse. Your objective is to accurately estimate the worst-case, or the largest, value of the condition number of the matrix.

```
a=ureal('a',5,'Range',[4 6]);
b=ureal('b',2,'Range',[1 3]);
b=ureal('b',3,'Range',[2 10]);
c=ureal('c',9,'Range',[8 11]);
d=ureal('d',1,'Range',[0 2]);
M = [a b;c d];
Mi = inv(M);
[maxnormM] = wcnorm(M)
maxnormM =
    LowerBound: 14.7199
    UpperBound: 14.7327
[maxnormMi] = wcnorm(Mi)
maxnormMi =
    LowerBound: 2.5963
    UpperBound: 2.5979
```

The condition number of `M` must be less than the product of the two upper bounds for all values of the uncertain elements making up `M`. Conversely, the largest value of `M` condition number must be at least equal to the condition number of the nominal value of `M`. Compute these crude bounds on the worst-case value of the condition number.

```
condUpperBound = maxnormM.UpperBound*maxnormMi.UpperBound;
condLowerBound = cond(M.NominalValue);
[condLowerBound condUpperBound]
ans =
    5.0757   38.2743
```

How can you get a more accurate estimate? Recall that the condition number of an `nxm` matrix `M` can be expressed as an optimization, where a free norm-bounded matrix Δ tries to align the gains of `M` and M^{-1}

$$\kappa(M) = \max_{\substack{\Delta \in C^{m \times m} \\ \sigma_{max}(\Delta) \le 1}} (\sigma_{max}(M\Delta M^{-1}))$$

If `M` is itself uncertain, then the worst-case condition number involves further maximization over the possible values of `M`. Therefore, you can compute the worst-case condition number of an uncertain matrix by using a `ucomplexm` uncertain element, and then by using `wcnorm` to carry out the maximization.

Create a 2-by-2 `ucomplexm` object, with nominal value equal to zero.

```
Delta = ucomplexm('Delta',zeros(2,2));
```

The range of values represented by `Delta` includes 2-by-2 matrices with the maximum singular value less than or equal to 1.

You can create the expression involving `M, Delta` and `inv(M)`.

```
H = M*Delta*Mi;
```

Finally, consider the stopping criteria and call `wcnorm`. One stopping criteria for `wcnorm(H)` is based on the norm of the nominal value of `H`. During the computation, if `wcnorm` determines that the worst-case norm is at least

```
ABadThreshold + MBadThreshold*norm(H.NominalValue)
```

then the calculation is terminated. In our case, since `H.NominalValue` equals 0, the stopping criteria is governed by `ABadThreshold`. The default value of `ABadThreshold` is 5. To keep `wcnorm` from prematurely stopping, set `ABadThreshold` to 38 (based on our crude upper bound above).

```
opt = wcgopt('ABadThreshold',38);
[maxKappa,wcu,info] = wcnorm(H,opt);
```

```
maxKappa
maxKappa =
    LowerBound: 26.9629
    UpperBound: 27.9926
```

You can verify that `wcu` makes the condition number as large as `maxKappa.LowerBound`.

```
cond(usubs(M,wcu))
ans =
   26.9629
```

Algorithm

See `wcgain`

See Also

`lti/norm`	Calculates LTI system norms
`svd`	Calculates singular value decomposition
`wcgain`	Calculates worst-case gain of a system
`wcgopt`	Creates a `wcgain` options object

Purpose

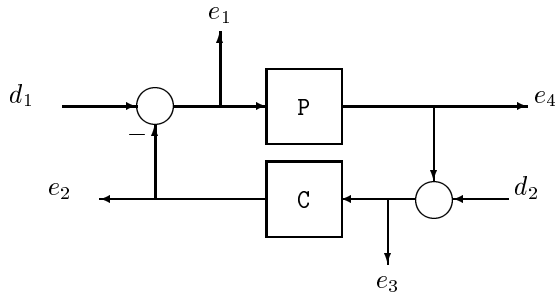
Calculate worst-case sensitivity and complementary sensitivity functions of plant-controller feedback loop

Syntax

```
wcst = wcsens(L)
wcst = wcsens(L,type)
wcst = wcsens(L,opt)
wcst = wcsens(L,type,scaling)
wcst = wcsens(L,type,scaling,opt)
wcst = wcsens(P,C)
wcst = wcsens(P,C,type)
wcst = wcsens(P,C,opt)
wcst = wcsens(P,C,type,scaling)
wcst = wcsens(P,C,type,scaling,opt)
```

Description

The sensitivity function, $S=(I+L)^{-1}$, and the complementary sensitivity function, $T=L(I+L)^{-1}$, where L is the loop gain matrix associated with the input, CP, or output, PC, are two transfer functions related to the robustness and performance of the closed-loop system. The multivariable closed-loop interconnection structure, shown below, defines the input/output sensitivity, complementary sensitivity and loop transfer functions.



Description	Equation
Input sensitivity ($TF_{e1 \leftarrow d1}$)	$(I+CP)^{-1}$
Input complementary sensitivity ($TF_{e2 \leftarrow d1}$)	$CP(I+CP)^{-1}$

Description	Equation
Output sensitivity ($TF_{e3 \leftarrow d2}$)	$(I+PC)^{-1}$
Output complementary sensitivity ($-T$)$F_{e4 \leftarrow d}$)	$PC(I+PC)^{-1}$
Input loop transfer function	CP
Output loop transfer function	PC

`wcst = wcsens(L)` calculates the worst-case sensitivity and complementary sensitivity functions for the loop transfer matrix `L` in feedback in negative feedback with `an identity matrix.` If `L` is a `uss` object, the frequency range and number of points are chosen automatically.

`wcst = wcsens(P,C)` calculates the worst-case sensitivity and complementary sensitivity functions for the feedback loop `C` in negative feedback with `P. C` should only be the compensator in the feedback path, not any reference channels, if it is a *2-dof* architecture (see `loopsens`). If `P` and `C` are `ss/tf/zpk` or `uss` objects, the frequency range and number of points are chosen automatically. `wcst` is a structure with the following substructures:

Table 5-2: Fields of `wcst`

Field	Description
`Si`	Worst-case input-to-plant sensitivity function
`Ti`	Worst-case input-to-plant complementary sensitivity function
`So`	Worst-case output-to-plant sensitivity function
`To`	Worst-case output-to-plant complementary sensitivity function
`PSi`	Worst-case plant times input-to-plant sensitivity function

Table 5-2: Fields of `wcst`

Field	Description
`CSo`	Worst-case compensator times output-to-plant sensitivity function
`Stable`	1 if nominal closed loop is stable, 0 otherwise. `NaN` for `frd/ufrd` objects.

Each sensitivity substructure is a structures with five fields `MaximumGain, BadUncertainValues, System, BadSystem, Sensitivity` derived from the outputs of `wcgain`.

Table 5-3: Fields of Si, So, Ti, To, PSi, CSo

Field	Description
`MaximumGain`	struct with fields `LowerBound`, `UpperBound` and `CriticalFrequency`. `LowerBound` and `UpperBound` are bounds on the unweighted maximum gain of the uncertain sensitivity function. `CriticalFrequency` is the frequency at which the maximum gain occurs.
`BadUncertainValues`	`Struct`, containing values of uncertain elements which maximize the sensitivity gain. There are `M` fluidness, which are the names of uncertain elements of sensitivity function. The value of each field is the corresponding value of the uncertain element, such that when jointly combined, lead to the gain value in `MaximumGain.LowerBound`.
`System`	Uncertain sensitivity function (`ufrd` or `uss`).

Table 5-3: Fields of Si, So, Ti, To, PSi, CSo

Field	Description
`BadSystem`	Worst-case system based on the uncertain object values in `BadUncertainValues`. `BadSystem` is defined as `BadSystem=usubs(System, BadUncertainValues)`.
`Sensitivity`	`Struct` with `M` fields, fieldnames are names of uncertain elements of system. Values of fields are positive numbers, each entry indicating the local sensitivity of the maximum gain to all of the individual uncertain elements uncertainty ranges. For instance, a value of 50 indicates that if the uncertainty range is enlarged by 8%, then the maximum gain should increase by about 4%. If the `'Sensitivity'` property of the `wcgopt` object is `'off'`, the values are `NaN`.

`wcst = wcsens(L,type)` and `wcst = wcsens(P,C,type)` allows selection of individual Sensitivity and Complementary Sensitivity functions, `type`, as `'Si','Ti','So','To','PSi','CSo'` corresponding to the sensitivity and complementary sensitivity functions. Setting `type` to `'S'` or `'T'` selects all sensitivity functions (`'Si','So','PSi','CSo'`) or all complementary sensitivity functions (`'Ti','To'`). Similarly, setting `type` to `'Input'` or `'Output'` selects all input Sensitivity functions (`'Si','Ti','PSi'`) or all output sensitivity functions (`'So,'To','CSo'`). `'All'` selects all six Sensitivity functions for analysis (default). `type` may also be a cell containing a collection of strings, i.e. `'Si','To'`, as well as a comma separated list.

`wcst = wcsens(L,type,scaling)` and `wcst = wcsens(P,C,type,scaling)` adds a `scaling` to the worst-case sensitivity analysis. `scaling` is either the character strings `'Absolute'` (default), `'Relative'` or a `ss/tf/zpk/frd` object. The default scaling `'Absolute'` calculates bounds on the maximum gain of the uncertain sensitivity function. The `'Relative'` scaling finds bounds on the maximum relative gain of the uncertain sensitivity function. That is, the maximum relative gain is the largest ratio of the worst-case gain and the nominal gain evaluated at each frequency point in the analysis, Similarly if `scaling` is a `ss/tf/zpk/frd` object, bounds on the maximum scaled gain of the

uncertain sensitivity function are found. If `scaling` is `'Relative'` or a `ss/tf/zpk/frd` object, the worst-case analysis peaks over frequency. If `scaling` is an object, its input/output dimensions should be 1-by-1 or dimensions compatible with `P` and `C`. `type` and `scaling` can also be combined in a cell array, e.g.

```
wcst = wcsens(P,C,{'Ti','So'},'Abs','Si','Rel','PSi',wt)
```

`wcst = wcsens(P,C,opt)` or `wcst = wcsens(P,C,type,scaling,opt)` specifies options for the worst-case gain calculation as defined by `opt`. (See `wcgopt` for more details on the options for `wcsens`.)

The sensitivity of the worst-case sensitivity calculations to the individual uncertain components can be determined using the options object `opt`. To compute the sensitivities to the individual uncertain components, create a `wcgopt` options object, and set the `Sensitivity` property to `'on'`.

```
opt = wcgopt('Sensitivity','on');
wcst = wcsens(P,C,opt)
```

Example

The following constructs a feedback loop with a first order plant and a proportional-integral controller. The time constant is uncertain and the model also includes an multiplicative uncertainty. The nominal (input) sensitivity function has a peak of 1.09 at omega = 1.55 rad/sec. Since the plant and controller are single-input / single-output, the input/output sensitivity functions are the same.

```
delta = ultidyn('delta',[1 1]);
tau = ureal('tau',5,'range',[4 6]);
P = tf(1,[tau 1])*(1+0.25*delta);
C=tf([4 4],[1 0]);
looptransfer = loopsens(P,C);
Snom = looptransfer.Si.NominalValue;
norm(Snom,inf)
ans =
  1.0864
```

`wcsens` is then used to compute the worst-case sensitivity function as the uncertainty ranges over its possible values. More information about the fields in `wcst.Si` can be found in the `wcgain` help. The `badsystem` field of `wcst.Si` contains the worst case sensitivity function. This worst case sensitivity has a

peak of 1.52 at omega = 1.02 rad/sec. The `maxgainunc` field of `wcst.Si` contains the perturbation that corresponds to this worst case sensitivity function.

```
wcst = wcsens(P,C)
wcst =
        Si: [1x1 struct]
        Ti: [1x1 struct]
        So: [1x1 struct]
        To: [1x1 struct]
       PSi: [1x1 struct]
       CSo: [1x1 struct]
    Stable: 1
Swc = wcst.Si.BadSystem;
omega = logspace(-1,1,50);
bodemag(Snom,'-',Swc,'-.',omega);
legend('Nominal Sensitivity','Worst-Case Sensitivity',...
  'Location','SouthEast')
norm(Swc,inf)
ans =
    1.5075
```

For multi-input/multi-output systems the various input/output sensitivity functions will, in general, be different.

Reference

J. Shin, G.J. Balas, and A.K. Packard, "Worst case analysis of the X-38 crew return vehicle flight control system," *AIAA Journal of Guidance, Dynamics and Control,* vol. 24, no. 2, March-April 2001, pp. 261-269.

See Also

`loopsens`	Calculate sensitivity functions of feedback loops
`loopmargin`	Comprehensive analysis of feedback loops
`robuststab`	Calculate stability margins of uncertain systems
`usubs`	Substitutes values for uncertain atoms
`wcgain`	Calculate worst-case gain of a system
`wcgopt`	Create a worst-case options object
`wcmargin`	Calculate worst-case margins for feedback loop

6

Block Reference

MultiPlot Graph

USS System

Purpose

Plot multiple signals

Description

The MultiPlot Graph block displays signals in a MATLAB® figure.

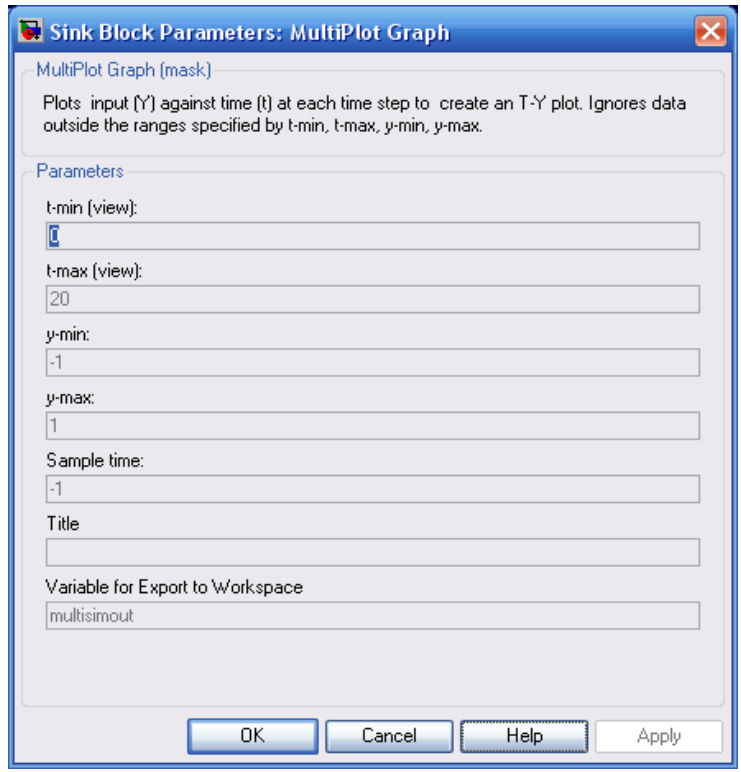
If the input signal is a vector, then each component of the vector is plotted in a separate axes. Lines are added to the axes in subsequent simulations. The most recent data is plotted in red. Older plots appear in blue. The block acts as a "hold-on, subplotter."

There are two buttons in the toolbar menu. The eraser button clears the data from all axes. The export button saves all the visible plot data to the MATLAB workspace in a variable named by the dialog box entry **Variable for Export to Workspace**. The format is a struct array, following the behavior of a `To Workspace` block, using the "Structure, With Time" save format.

The MultiPlot Graph block can be used in conjunction with the USS Simulink® block to visualize Monte Carlo and worst-case simulation time responses.

MultiPlot Graph

Dialog Box



Parameters

t-min, t-max

The parameter entries t-min and t-max are the minimum and maximum x-axis limits. t-min and t-max may be vectors corresponding to each subplot.

y-min, y-max

The parameter entries y-min and y-max are the minimum and maximum y-axis limits and similarly may be vector quantities.

Sample time

Sample time corresponds to the sample time at which to collect points.

Title

Specifies the title of the multiplot figure.

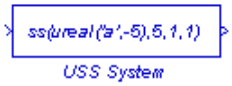
Variable for Export to Workspace

Variable name of the MATLAB® object to contain all the visible plot data exported to the MATLAB workspace. The format is a struct array, following the behavior of a To Workspace block, using the "Structure, With Time" save format.

USS System

Purpose

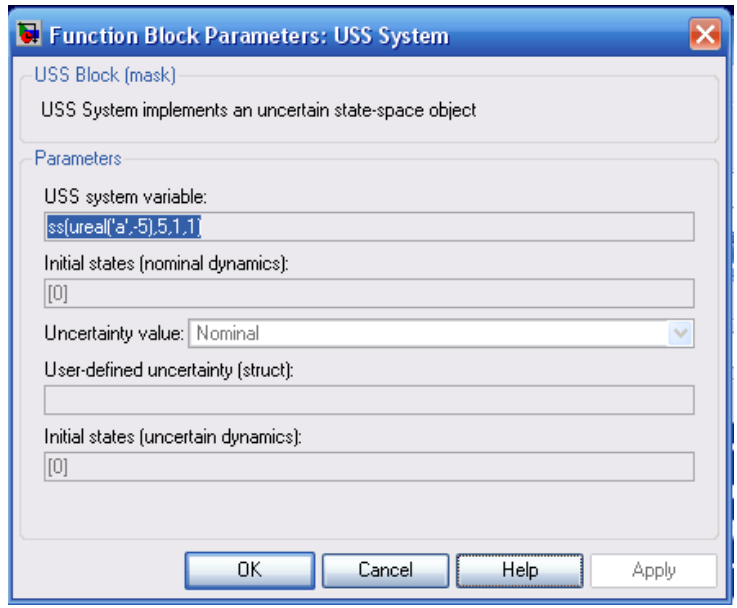
Import uncertain systems into Simulink®



Description

The USS System block accepts USS and UMAT containing `ureal` and `ultidyn` uncertain objects, as well as `ureal` and `ultidyn` objects. An instance of the uncertain system is used in the simulation or linearization. Internally, USS models are converted to their state space equivalent for evaluation.

Dialog Box



Parameters

USS system variable

The uncertain object (USS, UMAT, `ureal`, or `ultidyn`) is entered in the USS system variable.

Initial States (nominal dynamics)

If the nominal value for the USS system variable has states, then the initial condition for these states is entered in `Initial states (nominal dynamics)`.

Uncertainty value

The values for the uncertain elements are controlled by the `Uncertainty value` menu. If `Nominal` is selected, then the nominal value of the uncertain object is used. If you select `User defined` is selected, then you must enter a MATLAB® structure in the `User-defined uncertainty (struct)` dialog box. The field names of the structure should correspond to the names of the uncertain atoms within the USS system variable , while the values of the fields are the values used for the uncertain objects (using the command usubs). If `User defined` is selected from the `Uncertainty value` pop-up menu, then a MATLAB structure must be entered in the `User-defined uncertainty (struct)` dialog box. The field names of the structure should correspond to the names of the uncertain atoms within the USS system variable , while the values of the fields are the values used for the uncertain objects (using the command usubs). If some of these values are SS objects, then these states are refered to as uncertainty states.

The order of the uncertainty states is determined by the order of atoms in the Uncertainty property of the USS system variable. The state dimension is determined by the actual data in the `User-defined uncertainty` structure. Any extra fields in the `User-defined uncertainty` structure are ignored.

User-defined uncertainty (struc)

If `User defined` is selected from the `Uncertainty value` pop-up menu, then the structure data entered in `User-defined uncertainty (struct)` must contain fields corresponding to every uncertain atom of the the USS system variable. Extra fields are ignored. `usimsamp` generates a random instance of each atom in a Simulink model. It returns a structure, suitable for entry in `User-defined uncertainty (struct)`.

Initial states (uncertain dynamics)

The initial condition for the uncertainty states is entered in `Initial states (uncertain dynamics)`.

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